Modeling of Hyperconcentrated Sediment-Laden Floods in Lower Yellow River

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Abstract: This paper presents a rapid forecast model for simulating hyperconcentrated sediment-laden floods in the Lower Yellow River. The model is a hybrid of a conventional one-dimensional mathematical model for unsteady sediment-laden flow and an artificial neural networks model for encapsulation of numerical results. The former provides detailed river flood routing information under typical scenarios, whereas the latter extracts modeling outputs from the former and establishes a station-specific model for efficient flood forecasting. Three typical floods that occurred in the Lower Yellow River in 1977, 1982, and 1996 are simulated. Not only the hybrid model predictions are found to be in close agreement with measured data, but also the computational speed is significantly enhanced. It is found that sediment transport is of significance with regard to the flooding behavior of hyperconcentrated flows. Therefore, the model presented herein is of particular use for rivers with high sediment concentration.

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Introduction

The Yellow River, the second largest river in China (Fig. 1), is noted for its high sediment concentration; in fact, the maximum recorded sediment concentration in the Yellow River to date has been 1,650 kg/m³ (Chien 1988). In cases where the Yellow River has a sediment concentration above 100 kg/m³, Chen et al. (2000) categorize the flood as a hyperconcentrated sediment-laden flood (HCSLF). This type of flood is quite different from a low-concentrated sediment-laden flood (LCSLF) with regard to flow characteristics, flood routing, and consequential effects on sediment transport and river bed deformation.

In the Lower Yellow River, the consequences of a HCSLF on sediment transport and river deformation, such as channel erosion and deposition, are quite different to that of LCSLF. Fig. 2 presents the spatially averaged mean deposition rate of sediment against the spatially averaged mean sediment concentration obtained by analyzing field data from stations located between Tiexie and Gaocun. A polynomial fit to the rather limited data is also given in Fig. 2 indicating that the deposition rate is fairly low at about $0.3 \times 10^{11}$ kg for sediment concentrations below about $60$ kg/m³. At higher sediment concentrations the deposition rate appears to increase nonlinearly. It is quite obvious that the amount of deposition increases more rapidly during HCSLF than LCSLF events. From Tiexie to Gaocun there are many measurement stations, and it is well established that the river cross-section changes form often during a HCSLF. This is typical behavior of a braided river dominated by flood events, and implies that there may be a potential increase of flood risk in the whole Lower Yellow River. Under these circumstances, it is necessary to be able to predict consequences very quickly at any specific station. Therefore, rapid forecasting of sediment concentration, discharge, and water level, which are closely related to the consequences of floods, is vital.

Many difficulties arise in the prediction, prevention, and control of hyperconcentrated floods due to the uncertainties encountered in modeling turbulent free surface flows with high sediment concentration. For example, there are no explicit rating curves expressing the relationship between water level and flow discharge in the Lower Yellow River due to changes in sediment concentration during floods (Fig. 3).

To date, many mathematical models (e.g., Cunge et al. 1980; Lin et al. 1983; Lin and Falconer 1996) have been developed for low-concentrated sediment-laden flows, but few are applicable to hyperconcentrated floods. To describe the complex stage discharge behavior, such as shown in Fig. 3, new approaches are needed to take into account sediment effects in hyperconcentrated floods. Wei (1990) developed a two-dimensional mathematical model for sediment-laden flow in the Lower Yellow River. Zhang et al. (2001) analyzed characteristics of hyperconcentrated floods in the Lower Yellow River and proposed a one-dimensional sediment-laden flow model.

Conventional hydrodynamic and sediment transport models are satisfactory for reproducing typical (LCSLF) scenarios, but they are not very efficient for real time prediction of HCSLF. In recent years, the artificial neural networks (ANN) model approach has found wide applications in engineering science. Minns and

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Fig. 1. Lower Yellow River downstream of Sanmenxia

Hall (1996) used an ANN model to study the dynamic variation of precipitation and runoff processes in catchments. Hsu et al. (1995) compared results obtained from ANN to conventional mathematical models, and indicated that ANN is more efficient for the prediction of runoff characteristics. Li and Li (1999) established an ANN model for sediment-laden flows in rivers. Yonas and Michael (1999) investigated several types of ANN-agent architectures and proved their ability in encapsulating site-specific knowledge and data. Ni and Xue (1999) developed an integrated ANN and hydrodynamic models associated with the geographical information system to control flood diversion in the Jingjiang diversion area. The aforementioned studies demonstrate the apparent advantages of ANN in rapid prediction, feedback, and process control through encapsulation of knowledge from numerical models in large computational domains. In the present paper, an ANN is used to extract information from conventional hydrodynamic and sediment models, and the resulting hybrid model used to enhance the simulation of hyperconcentrated floods in the Lower Yellow River. The hybrid model provides detailed hydraulic information at any section under consideration and facilitates rapid response to varying flow and sediment conditions.

Numerical Modeling

Basic Equations and Parameters

The basic equations of one-dimensional unsteady sediment-laden flow are as follows:

\[
\frac{\partial Q_i}{\partial t} + \frac{\partial (Q_i^2)}{\partial x} - q_{LI} = 0
\]  
(1)

\[
\frac{\partial Q_i}{\partial t} + \frac{\partial (Q_i^2)}{\partial x} + \alpha_1 Q_i \frac{Q_i}{A_i} + \alpha_2 Q_i q_{LI} + g A_i \left( \frac{\partial Z_i}{\partial x} + \frac{Q_i^2}{K_i} \right) = 0
\]  
(2)

\[
\frac{\partial (A_i S_i)}{\partial t} + \frac{\partial (A_i V_i S_i)}{\partial x} = \sum_{j=1}^{m} (K_{ij} \alpha_{s,i,j} f_j b_j S_{ij} \omega_{ij}) - \sum_{j=1}^{m} (K_{ij} \alpha_{s,i,j} f_j b_j S_{ij} \omega_{ij}) - S_L q_{LI} = 0
\]  
(3)

\[
\frac{\partial Z_{bij}}{\partial t} - \frac{K_{ij} \alpha_{s,j}}{\gamma_0} \omega_{ij} f_j b_j S_{ij} - S_{rij} = 0
\]  
(4)

Eqs. (1) and (2) are continuity and momentum equations for fluid flow, respectively, and Eqs. (3) and (4) are the continuity equation for sediment transport and riverbed deformation, respectively. In Eq. (3) the first two terms relate to advection of suspended sediment, the third and fourth terms relate to sediment fluxes, and the fifth (source) term corresponds to the source-tye lateral sediment inflows to the system. In these equations, \(i=1,2,\ldots,N\) and \(N\) is the total number of cross sections along the river. \(\Phi_i(\Phi = Q_i A_i q_{LI}, \ldots)\) denotes the quantity at the \(i\)th cross section; each cross section is divided into several subsections, and \(\Phi_{ij}(\Phi = K_{ij} \alpha_{s,i,j} f_j b_j S_{ij} \omega_{ij})\) stands for the quantity at the \(j\)th subsection of the \(i\)th cross section. \(Q_i=\text{water discharge}; A_i=\text{cross sectional area}; t=\text{time}; x=\text{coordinate in flow direction}; q_{LI}=\text{lateral water inflow per unit length}; Z=\text{water stage}; g=\text{gravitational acceleration}; \alpha_1 \text{ and } \alpha_2=\text{momentum correction factor}; K=\text{discharge factor related to riverbed roughness}; b=\text{width}; S=\text{suspended sediment concentration by volume}; S_{ij}=\text{lateral sediment inflow per unit length}; V=\text{velocity of water flow}; Z_b=\text{bed elevation}; \alpha_s=\text{saturated recovery coefficient}; \omega=\text{settling velocity of sediment particles}; \gamma_0 = \text{weight of sediment}; S_{rij}=\text{sediment carrying capacity}; K_{ij}=\text{comprehensive coefficient}; f_j=\text{subsaturation coefficient}. The saturated recovery coefficient \(\alpha_s\) denotes the ratio between...
the bottom and average concentrations under equilibrium conditions, which is not a constant in this model and is evaluated using the following empirical expression obtained by integrating the sediment concentration over water depth (Zhang and Zhang 1992).

\[ \alpha = \frac{1}{N_0} \exp \left( 8.21 \frac{\omega}{\kappa u_s} \right) \]  

in which

\[ N_0 = \int_0^1 \left( \frac{\sqrt{s}}{c_s C} \eta \right) \exp \left( \frac{5.33 \omega}{\kappa u_s} \right) \frac{1}{\eta - 1} d\eta \]  

and

\[ f \left( \frac{\sqrt{s}}{c_s C} \right) = 1 - \frac{3\pi \sqrt{s}}{8c_s C} \left( \frac{\sqrt{s}}{\eta - 1} + \arcsin \sqrt{s} \right) \]  

Here, \( \kappa, C \) and \( u_s \) = Karman constant, Chezy resistance coefficient, and friction velocity, respectively; \( \eta \) = relative depth; \( c_s \) = vortex coefficient (\( c_s = 0.375k \)); the Karman universal constant \( \kappa = 0.4 \div 1.68(0.365 - S_i) \sqrt{S_i} \), and, \( K_i \) = comprehensive coefficient. Based on similarity theory and movable bed models, \( K_1 \) is derived as follows (Zhang and Zhang 1992):

\[ K_1 = \frac{1}{2.65} K^{1.5} \left( \frac{1.5}{V^{1.5}} \right) ^{1.14} \]  

The subsaturation coefficient \( f_i \) is deduced from numerous experimental data and can be estimated from (Zhang et al. 2001)

\[ f_i = \left( \frac{S_i}{S} \right) ^{0.1/\arctan(S/S_i)} \]  

To ensure that the model is capable of simulating both normal sediment-laden flows and hyperconcentrated flows in the Lower Yellow River, the following equation for the sediment carrying capacity is introduced (Zhang et al. 2001):

\[ S_s = 2.5 \left[ \frac{(0.0022 + S_i) V^3}{\kappa (\gamma_s - \gamma_m) g h \omega} \right]^{-0.62} \]  

where \( S_i \) = particle volume fraction; \( h \) = water depth; \( D_{50} \) = median diameter of bed materials; \( \gamma_s \) = specific weight of sediment; and \( \gamma_m \) = specific weight of muddy water. The sediment settling velocity \( \omega \) and Manning’s roughness coefficient \( n \) are two important parameters in the modeling process. Allowing for the special conditions that pertain to the Lower Yellow River, the sediment settling velocity can be expressed as (Zhang et al. 2001)

\[ \omega = \omega_0 (1 - 1.25S_i) \left[ 1 + \frac{S_i}{2.25(D_{50})} \right] ^{3.5} \]  

and the Manning’s coefficient can be estimated from (Zhang et al. 2001)

\[ n = \frac{h^{1/6}}{\sqrt{r}} \left[ \frac{D_{50}^{1/3}}{c_s h} \right] \left( \frac{\delta_s}{h} \right) ^{0.77} \left[ 0.49 \left( \frac{\delta_s}{h} \right) ^{0.15} + 3 \pi \left( \frac{1 - \delta_s}{h} \right) ^{0.2} \right] ^{1/2} \]  

in which \( \delta_s \) = friction thickness, defined as

\[ \delta_s = D_{50}(1 + 10^{(8 - 13D_{50}^{1.1} - 1.5F^2)}) \]  

The suspended sediment concentration profile and the distribution of the sediment particle’s median diameter in the transverse di-

\[ \frac{S}{S_i} = C \left( \frac{h}{h_i} \right) ^{0.1} \left( \frac{V_i}{V} \right) ^{0.1} \]  

and

\[ C_1 = \frac{Q}{\int_a^b q_i (h_i/h) ^{0.1} (V_i/V) ^{0.1} d} \]  

\[ C_2 = \frac{Q S_i}{\int_a^b q_i S_i (h_i/h) ^{0.1} (V_i/V) ^{0.1} d} \]  

It should be noted that Eqs. (1)–(4) do not directly couple the liquid and solid phases, as is in fact the case between the liquid and solid phases in hyperconcentrated flows. Various researchers (Greimann et al. 1999; Ni et al. 2000) have shown that the effects of hyperconcentration should be taken into account when the depth-averaged suspended sediment concentration exceeds 100 kg/m³. Herein, the effect of hyperconcentration is incorporated by the use of empirical parameters (such as sediment carrying capacity, etc.) in the governing equations.

**Numerical Method**

Preissmann four-point-difference approximations (Cunge et al. 1980) and the tridiagonal matrix algorithm are employed for solving equation systems (1) and (2). The discrete form of Eq. (3) is solved using an implicit scheme. After discretization, the riverbed deformation equation (4) becomes

\[ Z_{ij}^{n+1} = Z_{ij}^n + \Delta t \left( \frac{K_i \gamma_r}{\gamma_0} \omega_i (f_{ij} Z_{ij}^{n+1} - Z_{ij}^{n+1}) \right) \]  

in which \( \Delta t \) = time step.

**Boundary and Initial Conditions**

Boundary conditions for the equation system given by Eqs. (1) and (2) are

\[ Q_1 = Q_1(t) \text{ or } Z_1 = Z_1(t) \text{ (inlet condition)} \]  

\[ Z_2 = Z_2(t) \text{ or } Q_2 = f(Z_2) \text{ (outlet condition)} \]  

where \( Q_1 \) = inlet discharge and \( Z_1 \) = inlet water level; \( Q_2 \) and \( Z_2 \) = outlet discharge and water level, respectively.

As to the boundary conditions for the sediment transport equation, the concentration at entrance section is given. Initial conditions are determined by a series of physical parameters, such as water level, sediment concentration, and topographic features. For simplicity, these parameters are initially given estimated values. It should be noted that the approximation errors disappear quickly after the start of simulation, subject to the correct boundary conditions being implemented.
Model Validation

In 1977, 1982, and 1996, major floods occurred along the 400 km reach of the Lower Yellow River between Tiexie and Sunkou (Fig. 1). The numerical model was set up for the reach, calibrated, and validated against measured data, which is from the “Yellow River Yearbook Report” (Yellow River Conservancy Commission, 1977, 1982, and 1996). Initial and boundary conditions were determined by measured data. The topographic data are the cross sectional data along the concerned reach of the river with a distance about 10 km between two adjacent sections, obtained in the wet season before the specific flood’s coming in a year. The discharge processes at Xiaolangdi upstream Tiexie, which corresponds to the specific floods, are given as inlet condition; and the rating curves at Sunkou are given as outlet condition. For the validation tests, computational grids consisting of 41 nodes in the x direction were used with spatial increment of about $D_x = 10,000$ m. The time step was $D_t = 900$ s.

It should be addressed that the sediment makeup in the Yellow River can be well reflected by a median diameter $D_{50}$, which implies a simplification with the uniform-sediment assumption in the mobile-bed equations (Chien 1988). Moreover, previous studies have revealed that suspended load is the major part and bed-load is negligible in sediment-laden flows, particularly for the HCSLF (Chien 1988; Zhao 1996). Since the parameters were determined using numerous measured data obtained under different conditions, the modeling results could reflect the complex response of the HCSLF such as the extremely looped rating curve shown in Fig. 3 resulted from both bedform processes and hyperconcentration.

Good convergence is found with this grid system. Examples taken from the model validation tests are given in Figs. 4–6, where reasonable agreement between the calculated and measured results is evident. It is clear that the model properly describes hyperconcentrated floods even when the sediment concentration is higher than 100 kg/m$^3$ (Fig. 6). Detailed information about validation and model reliability can be seen in Zhang et al. (2001).

Radial-Basis Function–Artificial Neural Networks Model

As the rapid simulating HCSLF in the Lower Yellow River is one of the typical nonlinear processes, ANNs are introduced to speed up the simulation of the complicated HCSLF. The combination of a conventional one-dimensional mathematical model for unsteady sediment-laden flow and an ANN model for encapsulation of numerical results would be more efficient (Ni and Xue 2003). The conventional one-dimensional mathematical model provides detailed river flood routing information under typical scenarios, and the ANN model extracts numerical results from the former as training samples and validation samples to establish a station-specific model for efficient flood forecasting. The first step in the combination of the two models is to obtain detailed numerical information under typical scenarios, and the ANN model extracts numerical results from the former as training samples and validation samples from the numerical results. The third step is to train and validate the ANN model, where a neural network takes previously solved samples from the numerical model, looks for patterns in these samples, learns these patterns, and develops the ability to classify new patterns correctly. The last step is to use the ANN model to simulate if it is validated.

Among the various learning algorithms available to neural networks, the radial basis function (RBF)–ANN has the apparent advantages of learning efficiency over the more frequently used...
backpropagation ANN. The RBF–ANN is often used in fields of forecasting, regression, and function interpolation (Michael and David 1998). It is particularly appropriate to the present study, due to there being sufficient samples from hydrodynamic and sediment transport models to guarantee the accuracy of the ANN model (Lohninger 1993; Dai et al. 2000). With integration of RBF–ANN and numerical modeling based on partial-differential equations in terms of the continuum concept (Li and Li 2000), the combined model has been designed to achieve rapid simulation of hyperconcentrated floods.

The RBF–ANN for HCSLF in the Yellow River includes models for discharge, water level, and sediment concentration. It should be noted that there are two categories of the model that are considered in the present paper. The “station-specific” ANN model establishes a relationship between a single desired station and the entrance station, whereas the “one-dimensional” ANN model establishes a relationship between multiple stations and the entrance station. The system is truly time varying and has strong time-delayed phenomena. In order to simulate the time-delayed phenomena correctly, in the ANN model for discharge (e.g., at Huayuankou), the serial discharge $Q$ is sampled as a sequence of five temporal values before time $t$ as the input nodes of the ANN model. Initially, there are 12 hidden layers, which are naturally adjusted to the proper number of nodes during the ANN training process. The output layer calculates one new discharge at time $t$. So the arrangement of input, hidden, and output nodes are 5, 12, and 1, respectively. Although the station-specific model is highly efficient for discharge evaluation at a particular station under consideration, it is not suitable for simulation of the whole process at all stations, which should be accomplished by a hydrodynamic model. Nevertheless, a one-dimensional ANN model could be established through extending the number of nodes in the input layer based on a station-specific model (five discharge inputs) with an additional five inputs of distance ordinate $x$, sediment carrying capacity $S_p$, saturation recovery coefficient $a_p$, riverbed roughness $n$, and cross sectional width $b$. All the foregoing input

![Fig. 5. Comparison of computed and measured discharge](image)

![Fig. 6. Comparison of computed and measured sediment concentration at Huayuankou](image)
parameters are time dependent and are selected to denote the difference between stations. The numbers of nodes for the one-dimensional model then become 10, 12, 1, for the input layer, hidden layer, and output layer, respectively.

**Encapsulation of Station-Specific Knowledge**

For any given station, evaluated and validated values of water level and discharge at certain time steps can be extracted from the one-dimensional hydrodynamic model. To guarantee wide adaptability of the ANN model, typical Lower Yellow River boundary conditions are provided from the 1977, 1982, and 1996 flood data. The numerical model results under these representative flood scenarios provide an adequate sample of data sets for the neural networks. Thus a rapid response is established between discharge, water level and sediment concentration. For the station-specific ANN model, 250 training samples and 50 validation samples are selected from numerical results at the Huayuankou, Jiahetan, Gaocun, and Shunkou stations. For the one-dimensional ANN model, 1,000 training samples and 200 validation samples are extracted from numerical modeling results.

**Evaluation and Verification of Artificial Neural Network Model**

The system root mean square error decreases within the training process. Fig. 7 illustrates a typical error decay process (obtained using the Huayuankou station-specific ANN model). After, 5,000 iterations, the training process of the ANN models become stable, and the root mean square error only changes 0.002 from iteration 5,000 to 18,000. It is obvious that 5,000 training iterations is enough to construct a stable ANN model, but further training iterations will not lead to an overtraining state.

Fig. 8 shows the close agreement achieved between the predicted discharges obtained using the ANN model and the input (validation) measurements at Huayuankou. An indication of the performance of the training set is that the root mean square error of the ANN model is about than 7.2%. The overall precision is 92.8% (1 rms error). Figs. 9 and 10 provide validation results for water levels and sediment concentrations at Huayuankou. The validation set for the ANNs (Fig. 9) is of the same length as that for the numerical model validation (Fig. 4) because the same data set was used in both cases. It is evident that the ANN models reproduce properly the output of the numerical model, with which they were trained.

**Improvement of Computational Speed**

All computations have been performed on a PC with CPU 866 MHz and RAM of 128 M under the Windows 2000 Professional operating system. The CPU time is less than 11 s for station-specific models and less than 27 s for one-dimensional models in order to calculate discharge, water level, and sediment concentration at Huayuankou, Jiahetan, Gaocun, and Shunkou with the ANN. The CPU time for the RBF–ANN model is approxi-
mately 50 times faster than the corresponding one-dimensional hydrodynamic model. It is therefore found that the combination of the conventional hydrodynamic numerical model and ANN leads to considerable efficiency gains.

Conclusions

Hyperconcentrated sediment-laden flows have very different characteristics to low-concentrated sediment laden flows, and are inherently more difficult to predict, prevent, or control. Due to differences in the solid–liquid flow physics, conventional mathematical models developed for low-concentrated sediment-laden floods are not applicable to hyperconcentrated floods. Herein, a model that integrates a conventional one-dimensional mathematical model for unsteady sediment-laden flow and an ANN has been developed. In the present uncoupled model, the effect of hyperconcentration is necessarily incorporated through the calibration of the numerical model to the field data through empirical parameters (such as sediment carrying capacity, etc.). In future work, it would be better to develop a fully coupled numerical model and use it to train the ANN model.

The model provides a rapid-response forecast tool applicable to varying flow and sediment conditions. Three typical floods occurring in 1977, 1982, and 1996 in the Lower Yellow River have been used to evaluate and validate the model. It has been demonstrated that the numerical results closely fit the measured data. The computational speed has been significantly improved, confirming the practical applicability of the hybrid model to rapid estimation of hydraulic and sediment parameters (such as discharge, water level, and sediment concentration). The present model is appropriate to both hyperconcentrated and low-concentrated sediment-laden floods. Station-specific ANN models trained with data from a specific time period may become increasingly inaccurate at predicting future scenarios if the river reach changes its bed morphology considerably due to erosion and deposition. Future work is required to develop ANN models that are retrained as and when new data are available, and are biased towards the most recent information.

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Notation

The following symbols are used in this paper:

- \( A \) = cross-sectional area (m\(^2\));
- \( B \) = flow width at free surface (m);
- \( b \) = empirical coefficient;
- \( C \) = Chézy resistance coefficient;
- \( C_1, C_2 \) = functions as defined in Eq. (15) and (17), respectively;
- \( c_a \) = \( c_a = 0.375 \kappa \), vortex coefficient;
- \( D_{50} \) = median diameter of bed material (m);
- \( d_{cp} \) = average diameter of suspended material (m);
- \( d_{50} \) = median diameter of suspended material (m);
- \( F \) = \( F = V/(gh)^{1/2} \), Froude number;
- \( f \) = function as defined in Eq. (7);
- \( f_1 \) = subsaturation coefficient as defined in Eq. (9);
- \( g \) = gravitational acceleration (m/s\(^2\));
- \( h \) = water depth (m);
- \( K \) = discharge modulus;
- \( K_i \) = comprehensive coefficient as defined in Eq. (8);
- \( N_0 \) = integral function as defined in Eq. (6);
- \( n \) = Manning’s roughness coefficient;
- \( Q \) = flow discharge (m\(^3\)/s);
- \( q \) = flow discharge per unit width (m\(^2\)/s);
- \( q_L \) = lateral water inflow per unit length (m\(^2\)/s);
- \( S \) = sediment concentration (kg/m\(^3\));
- \( S^* \) = sediment carrying capacity (kg/m\(^3\));
- \( S_L \) = lateral sediment inflow per unit length;
- \( S_v \) = particle volume fraction;
- \( t \) = time (s);
- \( u^* \) = friction velocity (m/s);
- \( V \) = flow velocity (m/s);
- \( x \) = spatial variable (m);
- \( y \) = coordinate in transversal direction (m);
- \( Z \) = water level (m);
- \( Z_b \) = bed elevation (m);
- \( \alpha^* \) = saturation recovery coefficient;
- \( \alpha_1, \alpha_2 \) = momentum correction factors;
- \( \gamma \) = specific weight of water;
- \( \gamma_m \) = specific weight of muddy water; \( \gamma_m = \gamma + S(\gamma_s - \gamma)/\gamma \);
- \( \gamma_s \) = specific weight of sediment;
- \( \gamma_0 \) = dry weight of sediment;
- \( \Delta t \) = time step (s);
- \( \delta^* \) = Kármán universal constant;
- \( \omega \) = settling velocity of sediment particles (m/s); and
- \( \omega_0 \) = settling velocity of sediment particles in clear water (m/s).

References


