An XFEM approach for the simulation of fractured porous-media systems

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Motivation

• Flow in fractured porous-media systems is often dominated by their heterogeneities and discontinuities. Such systems characterise many applications, e.g.
  • CO₂ capture and storage,
  • reservoir engineering,
  • groundwater resource management.
• At the scale of interest the material properties differ in orders of magnitude for the fracture network and the surrounding rock matrix. Furthermore, the characteristic flow behaviour of the whole system depends crucially on both the fractures and the rock matrix.
• The exact fracture structure on the field scale cannot be determined. Thus the fracture-network model has to be stochastically generated. To get meaningful results several (> 100) realisations have to be simulated.

⇒ The discrete fractured porous-medium model has to be meshed fast and produce accurate results.

Goals

Development of robust, flexible and consistent weakly coupled schemes for:
• porous media and fracture networks of codimension one and
• overlapping non-conforming grids for the rock matrix and the fracture network.

The implementation of these schemes will be
• integrated into the porous-media simulation toolbox DuMuX and
• based on DUNE (the Distributed and Unified Numerics Environment).

Fracture Handling

There exist different approximations for different points of interest:
• fracture-network models, e.g. parallel-plate
• fracture-matrix models, Sandve and Nordbotten[3]
• continua models, e.g. multi-porosity/permeability, Tatomir et al.[4]

Figure 4: Domain decomposition of the lower dimensional fracture and the rock matrix

Figure 3: 1) shows a lower dimensional discrete fracture matrix model and 2) a schematic multi-continua model which can be derived from a)

To compare the different approaches, regarding
• accuracy (h-related, analytical solution, fine scale equidimensional solution) and
• speed (iterations, condition number, actual implementation)
three test cases are chosen: single fracture, fracture bands, Hydrocoin 1988.

Literature


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Figure 8: Fracture aperture a = 10⁻² m, matrix permeability Kₘ = 1.0

Future work will consist of the implementation for additional conditions to handle
• fractures which end within the domain and
• matrix element edge-aligned fractures.

Model

Weak Formulation

The approach is based on the strong problem formulation of Martin et al.[2] for isothermal, single-phase, incompressible Darcy flow.

• A weak formulation for the porous matrix is derived, similar to D’Angelo and Scotti[1], where the pressure can be discontinuous across the fractures and pressure jumps and averages are defined: Find \( p \in \{ \phi \} \) such that for the matrix:
  \[
  (K \nabla p, \nabla \phi)_{\Omega} + \left( \frac{\alpha I}{\varepsilon^2} \right) \{ p \} \cdot \{ \phi \} = \left( \frac{\alpha I}{\varepsilon^2} \right) \{ \phi \},
  \]
  for the Darcy flow through the fracture network of codimension one:
  \[
  (K_{ij} \nabla p_i, \nabla \phi_j)_{\Omega} + \left( \frac{\alpha I}{\varepsilon^2} \right) \{ p_i \} \cdot \{ \phi_j \} = \left( \frac{\alpha I}{\varepsilon^2} \right) \{ \phi_j \},
  \]
• The coupling conditions are given by:
  \[
  \begin{align*}
  a \cdot n &= \frac{\alpha I}{\varepsilon^2} \{ p \} - p_i , \\
  p &= \frac{\alpha I}{\varepsilon^2} \{ a \cdot n \} ,
  \end{align*}
  \]

Discrete Model

• The interface problem (discontinuities) is handled by an XFEM-based (eXtended Finite Element Method) approach.
• Additional function spaces are introduced at elements which contain at least one fracture part.

Figure 5: 2D XFEM porous-matrix grid (grey) with enriched elements (blue) and lower dimensional fracture (red)

Figure 6: Additional function space with additional (enriched) degrees of freedom (yellow)

The global solution consists of the combined standard degrees of freedom and the additional (enriched) ones
• Nodes of fracture-containing elements are enriched, i.e., they are duplicated.
• The basis functions of enriched elements are multiplied by specific shape-function multipliers \( \Psi_i \), so that discontinuities within elements can be handled.

Figure 7: 2D example of modified XFEM shape functions

Results and Outlook

The model is implemented solving the problem
• monolithically or
• iteratively.

In the future parts of the iterative implementation will be used as preconditioner for the monolithic system.