An XFEM approach for the simulation of fractured porous-media systems

International Research Training Group nupus

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Motivation

- Flow in fractured porous-media systems is often dominated by their heterogeneities and discontinuities. Such systems characterise many applications, e.g.
  - CO_2 capture and storage,
  - reservoir engineering,
  - groundwater resource management.

Figure 1: Fractured rock, Pliezhausen, GER

- At the scale of interest the material properties differ in orders of magnitude for the fracture network and the surrounding rock matrix. Furthermore, the characteristic flow behaviour of the whole system depends crucially on both the fractures and the rock matrix.
- The exact fracture structure on the field scale cannot be determined. Thus the fracture-network model has to be stochastically generated. To get meaningful results several (>100) realisations have to be simulated.

⇒ *The discrete fractured porous-medium model has to be meshed fast and produce accurate results.*

Goals

Development of robust, flexible and consistent weakly coupled schemes for:

- porous media and fracture networks of codimension one and
- overlapping non-conforming grids for the rock matrix and the fracture network.

The implementation of these schemes will be

- integrated into the porous-media simulation toolbox DuMu^x and
- based on DUNE (the Distributed and Unified Numerics Environment).

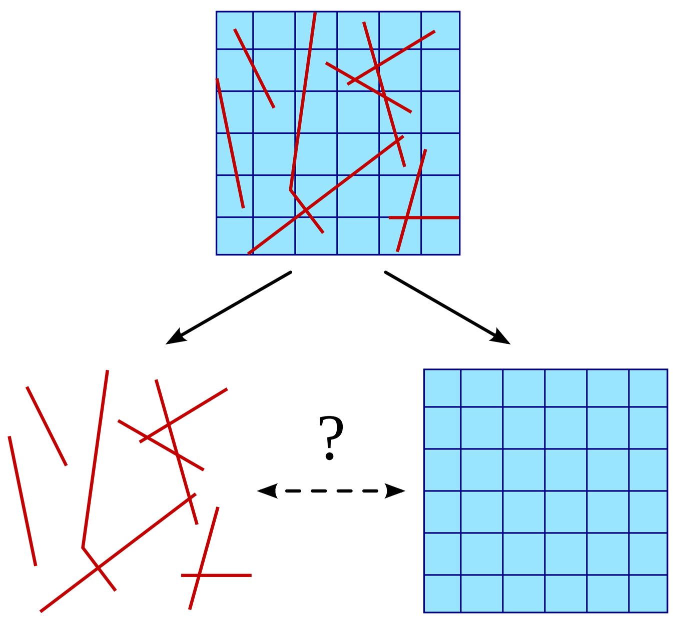

Figure 2: Coupling between the fracture network and the rock matrix

Fracture Handling

There exist different approximations for different points of interest:

- fracture-network models (e.g. parallel-plate)
- fracture-matrix models, *Sandve and Nordbotten[3]*
- continua models (e.g. multi-porosity/permeability), *Tatomir et al.[4]*

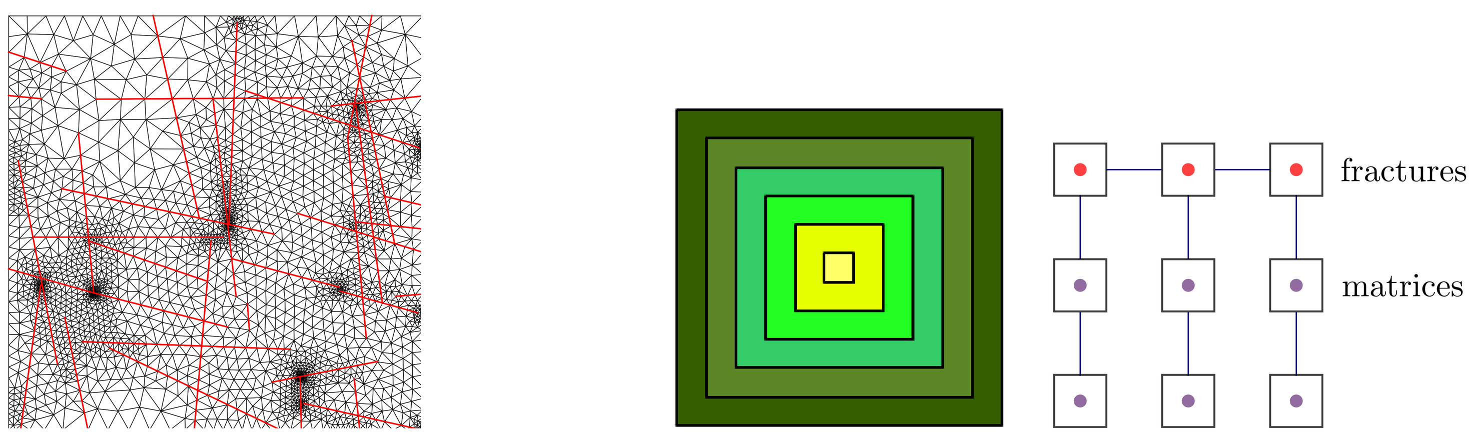

Figure 3: a) shows a lower dimensional discrete fracture matrix model and b) a schematic multi-continua model which can be derived from a)

To compare the different approaches, regarding

- accuracy (h-related, analytical solution, fine scale equidimensional solution) and
- speed (iterations, condition number, actual implementation)

three test cases are chosen: single fracture, fracture bands, Hydrocoin 1988.

Literature

[1] *D'Angelo, C. and Scotti, A.*: A Mixed Finite Element Method for Darcy Flow in Fractured Porous Media with non-matching Grids. *ESAIM: Mathematical Modelling and Numerical Analysis*, 46 (2012), p. 465–489.

[2] *Martin, V., Jaffré, J. and Roberts, J.E.*: Modeling fractures and barriers as interfaces for flow in porousmedia. *SIAM Journal on Scientific Computing*, 26 (2005), p. 1667–1691.

[3] *Sandve, T. H. and Berre, I. and Nordbotten, J. M.*: An efficient multi-point flux approximation method for Discrete Fracture–Matrix simulations *Journal of Computational Physics, 2012.*

[4] *Tatomir, A. B. and Szymkiewicz, A. and Class, H. and Helmig, R.*: Modeling two phase flow in large scale fractured porous media with an extended multiple interacting continua method. *Computer Modeling in Engineering and Sciences, 77 (2011), p. 81–112.*

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Model

Weak Formulation

The approach is based on the strong problem formulation of *Martin et al.[2]* for isothermal, single-phase, incompressible Darcy flow.

- A weak formulation for the porous matrix is derived, similar to *D'Angelo and Scotti[1]*, where the pressure can be discontinuous across the fractures and pressure jumps and averages are defined. Find $p = (p_i, p_f)$ such that

for the matrix:

$$\sum_i (\mathbf{K}_i \nabla p_i, \nabla \phi_i)_{\Omega_i} + \left(\frac{\alpha_f}{\xi - 1/2}\{p\}, \{\phi\}\right)_\gamma + \left(\frac{\alpha_f}{2}[\![p]\!], [\![\phi]\!]\right)_\gamma = \left(\frac{\alpha_f}{\xi - 1/2} p_f, \{\phi\}\right)_\gamma$$

for the Darcy flow through the fracture network of codimension one:

$$(\mathrm{K}_{f,t}\, a \nabla_t p_f, \nabla_t \phi_f)_\gamma + \left(\frac{\alpha_f}{\xi - 1/2} p_f, \phi_f\right)_\gamma = \left(\frac{\alpha_f}{\xi - 1/2}\{p\}, \phi_f\right)_\gamma$$

- The coupling conditions are given by:

$$[\![\boldsymbol{u}\cdot n]\!] = \frac{\alpha_f}{\xi - 1/2}(\{p\} - p_f) \,,$$

$$[\![p]\!] = \frac{2}{\alpha_f}\{\boldsymbol{u}\cdot n\} \,.$$

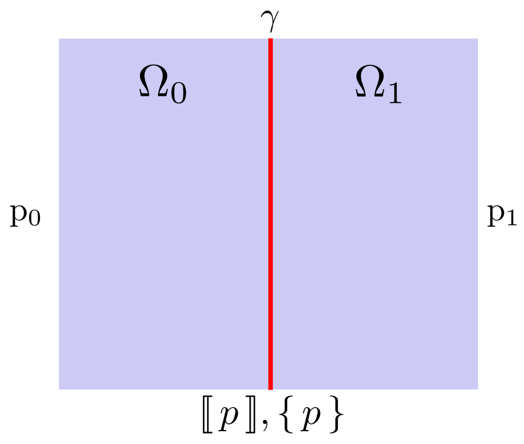

Figure 4: Domain decomposition of the lower dimensional fracture and the rock matrix

Discrete Model

- The interface problem (discontinuities) is handled by an XFEM-based (eXtended Finite Element Method) approach.
- Additional function spaces are introduced at elements which contain at least one fracture part.

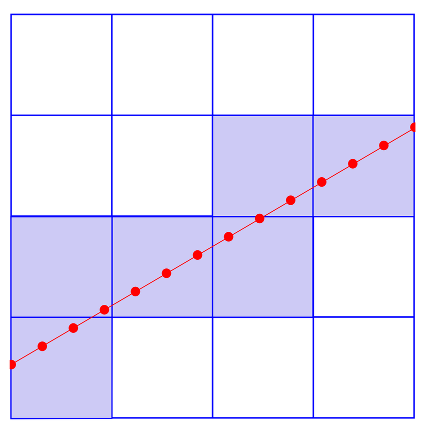

Figure 5: 2D XFEM porous-matrix grid (grey) with enriched elements (blue) and lower dimensional fracture (red)

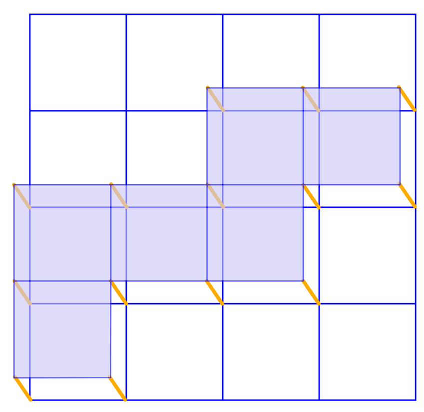

Figure 6: Additional function space with additional (enriched) degrees of freedom (yellow)

- The global solution consists of the combined standard degrees of freedom and the additional (enriched) ones
- Nodes of fracture-containing elements are enriched, i.e., they are duplicated.
- The basis functions of enriched elements are multiplied by specific shape-function mulitpliers Ψ_j, so that discontinuities within elements can be handled.

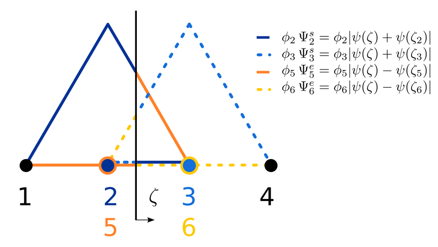

Figure 7: 1D example of modified XFEM shape functions

Results and Outlook

The model is implemented solving the problem

- monolithically or
- iteratively.

In the future parts of the iterative implementation will be used as preconditioner for the monolithic system.

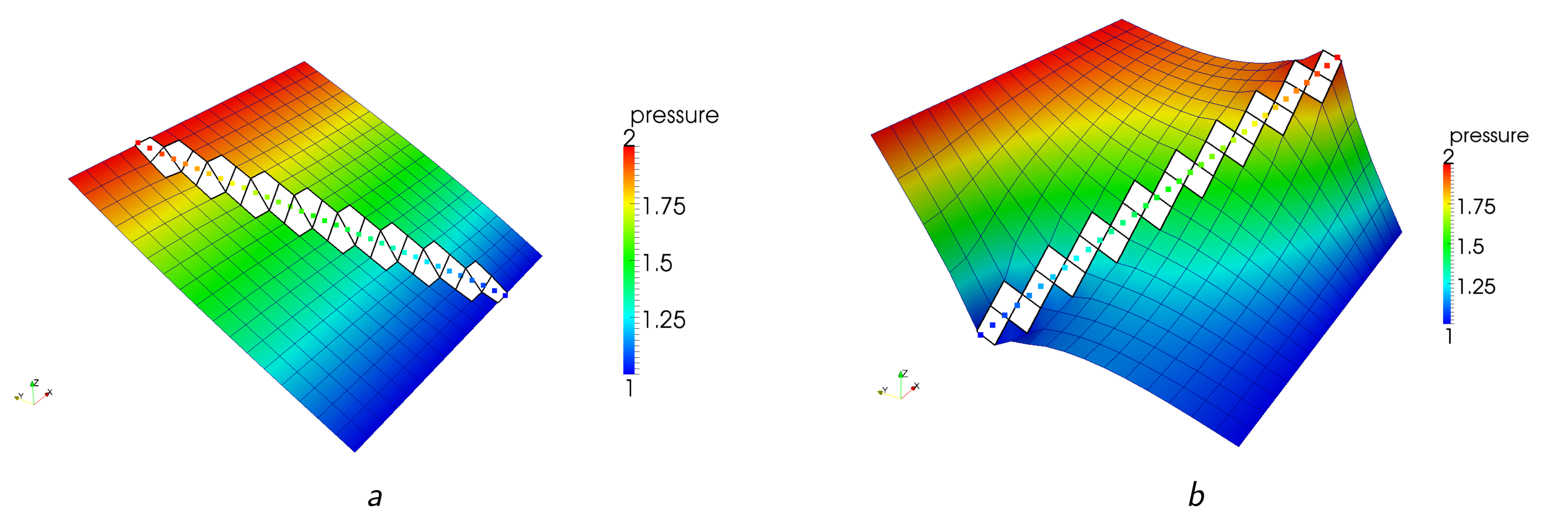

Figure 8: fracture aperture $a = 10^{-4}$, matrix permeability $\mathbf{K}_m = \mathbf{I}$,

Future work will consist of the implementation for additional conditions to handle

- fractures which end within the domain and
- matrix element edge-aligned fractures.