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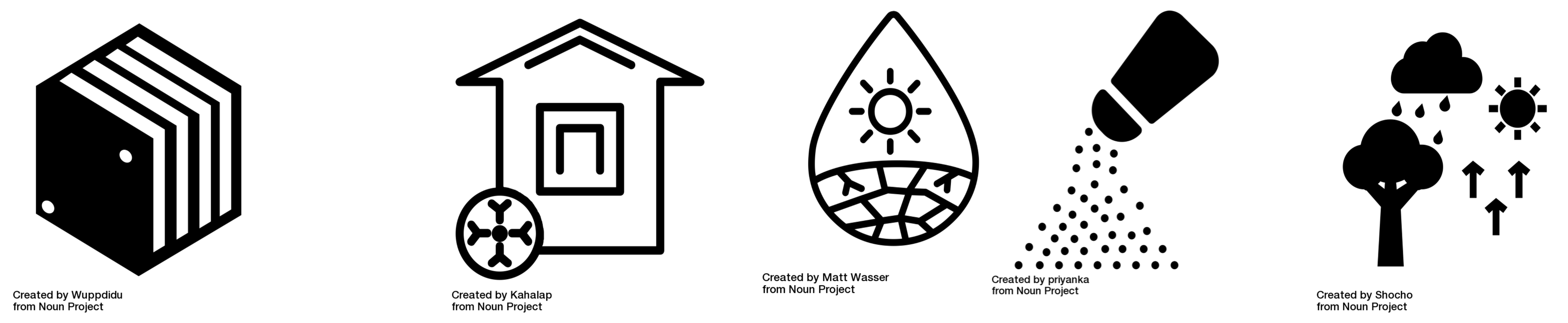
Coupling free flow and porous-medium flow: comparison of non-refined, globally-refined and locally-refined axiparallel free-flow grids

Physical Problem

Where does free flow coupled to porous medium flow occur?

- Fuel Cells
- Buildings/Urban Areas
- Salinization
- Evaporation



Finite-Volume Staggered Grid Discretization

Our Model

Navier Stokes Equations

Free Flow

Mass Balance

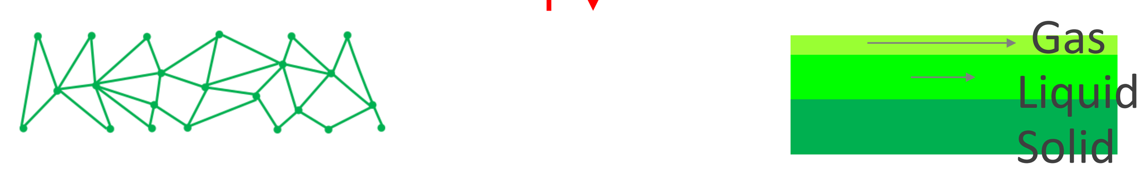
$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \boldsymbol{u}) - q_{\mathrm{p}} = 0$$

Momentum Balance

$$\frac{\partial (\varrho \boldsymbol{u})}{\partial t} + \nabla \cdot (\varrho \boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}) - \nabla \cdot (\mu (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}})) + \nabla p - \varrho \boldsymbol{g} - q_{\mathrm{u}} = 0$$

Sharp Interface

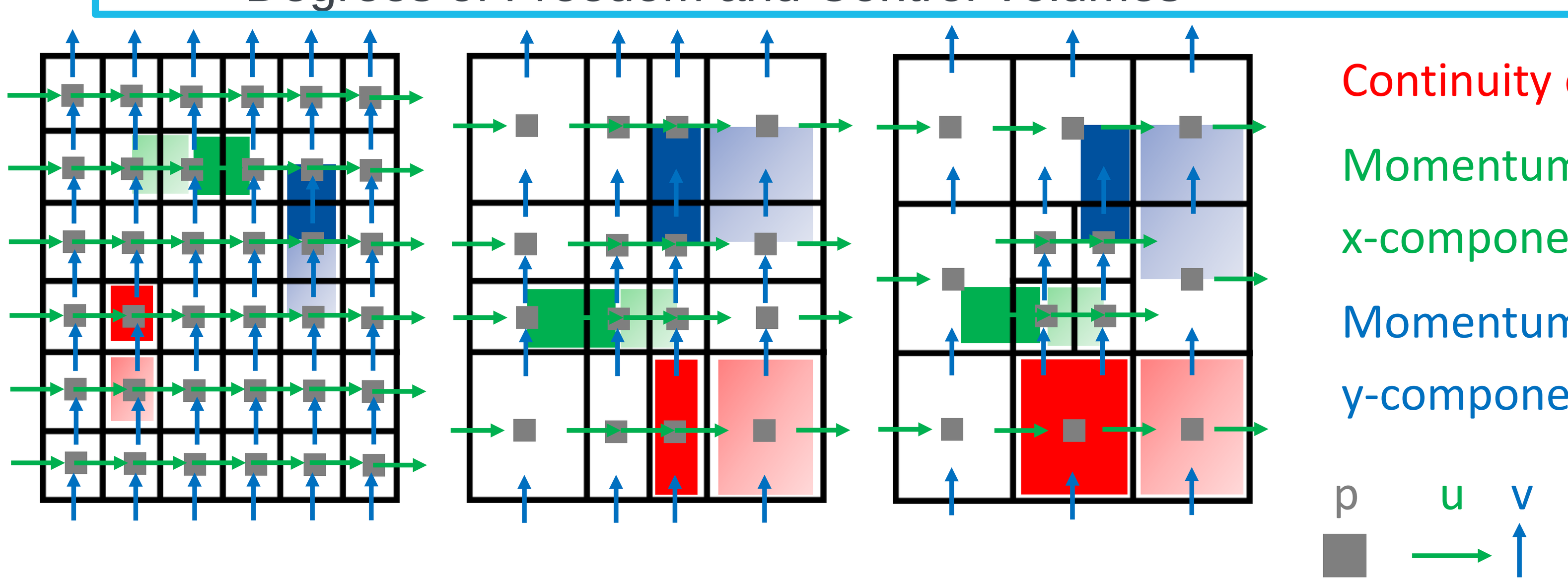
Porous Medium Flow



Pore network model [Weishaupt et al. 2019, J Comput Phys X]

Representative Elementary Volume-scale model [Baber et al. 2012, IMA J Math]

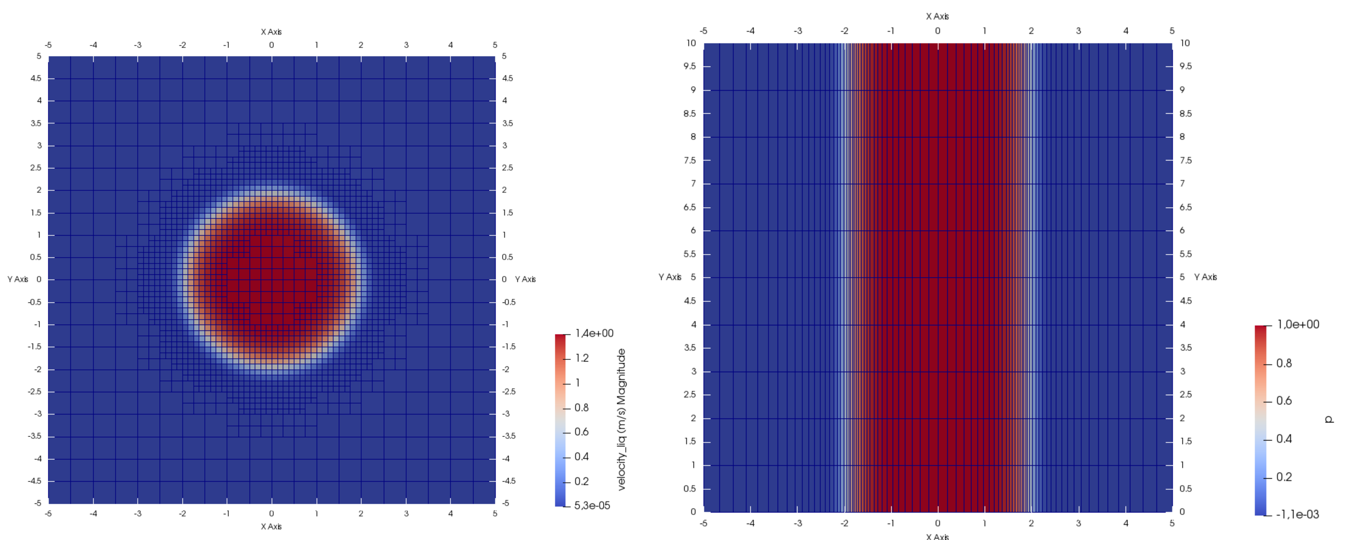
Degrees of Freedom and Control Volumes



[Lipp, M. and R. Helmig. A locally-refined locally-conservative quadtree finite-volume staggered-grid scheme. In G. Lamanna, S. Tonini, G.E. Cossali, and B. Weigand, editors, Droplet Interactions and Spray Processes, volume 121 of Fluid Mechanics and Its Applications, pages 149–159. Springer, 2020. ISBN 978-3-030-33337-9.]

Examples

Example A: Pure Free-Flow - Supergaussian Peak



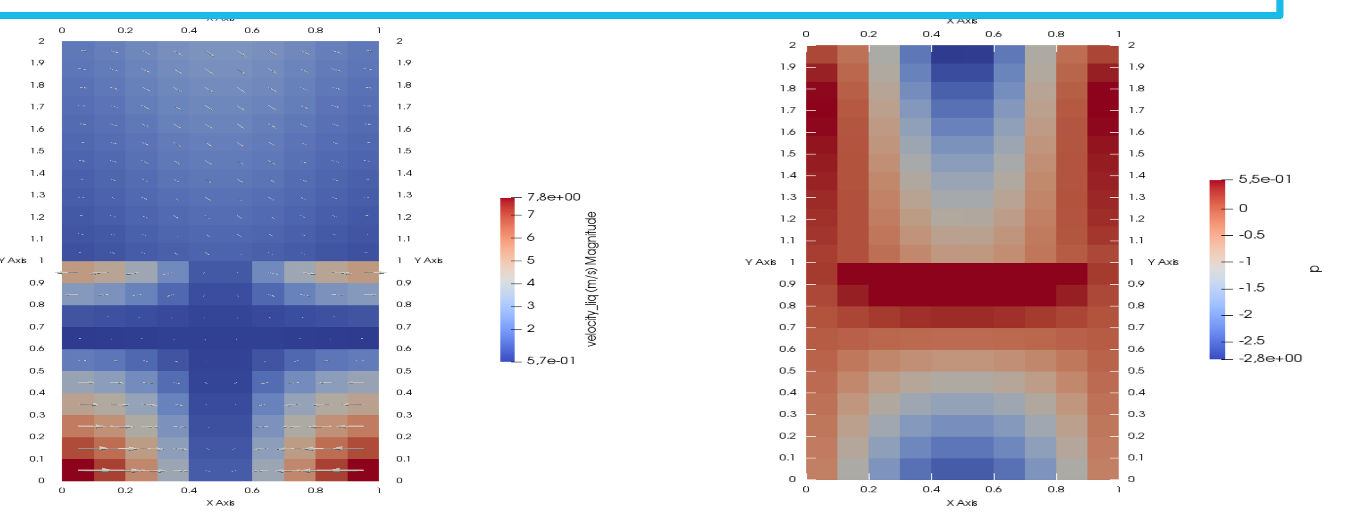
	Local				Global		
	#dofs	p	u	v	p	u	v
Without Refinement	6165	3.65e-02	9.22e-03	9.22e-03	1.02e-03	5.32e-05	2.60e-05
With Refinement	6120	1.45e-02	5.48e-03	5.48e-03	6.83e-04	3.44e-05	1.75e-05

In this example with refinement better ☺

Example B: Free-Flow Coupled to Porous Medium (Representative Elementary Volume Scale)

Manufactured solution

[Schneider, Martin, et al. "Coupling staggered-grid and MPFA finite volume methods for free flow/porous-medium flow problems." *Journal of Computational Physics* 401 (2020): 109012.]



	L2(p),abs	L2(p),rel	L2(vx),abs	L2(vx),rel	L2(vy),abs	L2(vy,rel)
uniform	5,08E-01	3,33E-01	4,72E-03	3,09E-03	9,01E-03	8,34E-03
globally	4,32E-01	2,83E-01	4,25E-03	2,78E-03	8,67E-03	8,02E-03
	Better Than Uniform					
locally	4,92E-01	3,23E-01	4,79E-03	3,14E-03	7,95E-03	7,35E-03
	Better Than Uniform / Worse Than Grading		Worse Than All		Better Than All	

Example C: Free-Flow Coupled to Porous Medium (Pore-network model)
Channel flow over 3x3 Pore-network

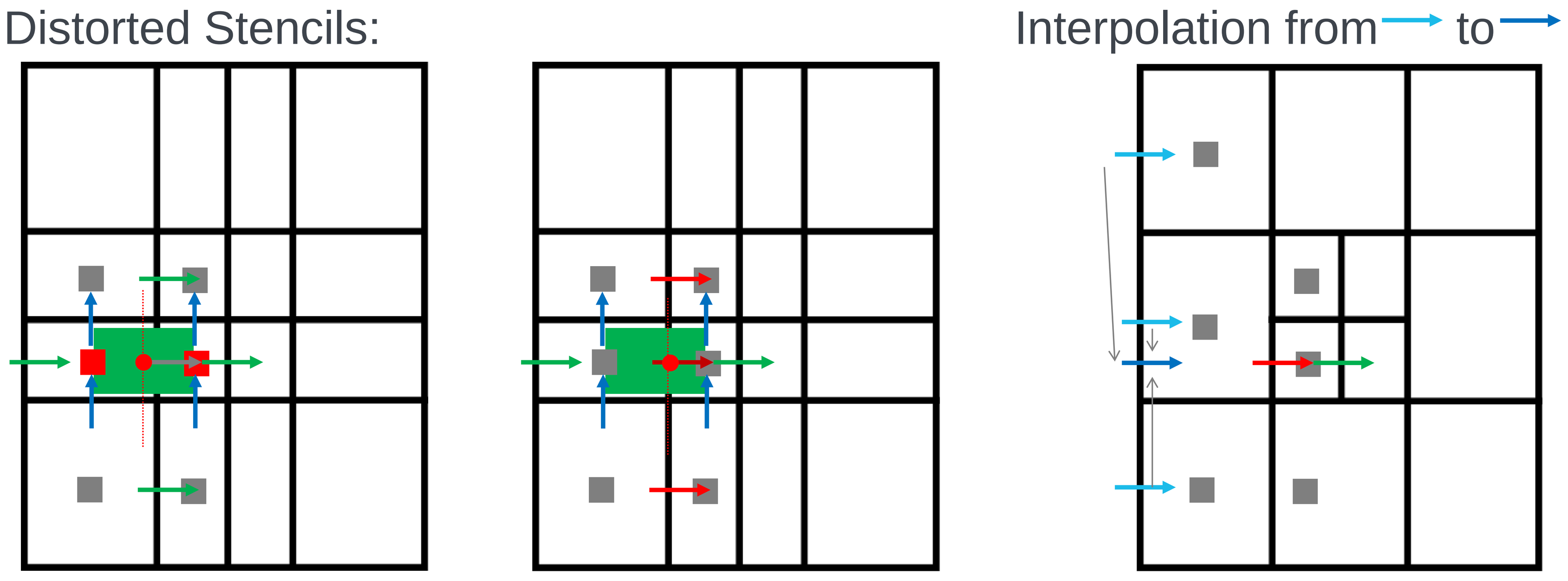
	CC Dofs	Face Dofs	Deviation from 120x40 free-flow grid	Conclusion
Uniform 30x10	300	640	2.89e-6	Best
Globally refined	300	640	3.01e-6	Middle
Locally refined	299	712	4.81e-6	Worst

Deviation = $\sqrt{\sum_{i=0}^{N} (p_{i,\text{this grid}} - p_{i,120\times 40 \text{ grid}})^2}$ Sum over Pore Bodies

Discussion

Distorted stencils, interpolations and local truncation errors contribute to the results we get.

Distorted Stencils: Interpolation from → to → :



Local truncation errors:
Grading: Superconvergence
Local Refinement:

$$\frac{\mu(x_r, y_c)\,\partial_x u(x_r, y_c) - \mu(x_l, y_c)\,\partial_x u(x_l, y_c)}{\Delta x} =$$

$$\frac{\mu_r}{\Delta x \Delta x_r} u_{rr} - \left(\frac{\mu_r}{\Delta x \Delta x_r} + \frac{\mu_l}{\Delta x \Delta x_l}\right) u_c + \frac{\mu_l}{\Delta x \Delta x_l} u_{ll} + \mathcal{O}(\Delta)$$

[See also: van der Plas, P. (2017). Local grid refinement for free-surface flow simulations. [Groningen]: Rijksuniversiteit Groningen.]

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