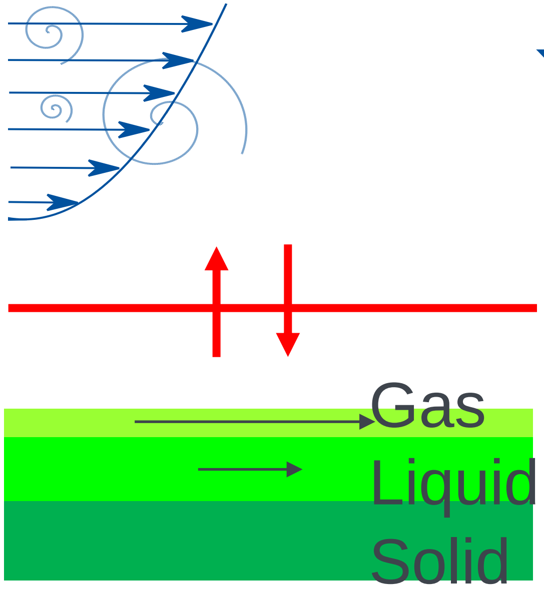
University of Stuttgart
Institute for Modelling Hydraulic and Environmental Systems
Department of Hydromechanics and Modelling of Hydrosystems

Coupling Free Flow and Porous-Medium Flow: Comparison of Non-Refined, Globally-Refined and Locally-Refined Axiparallel Free-Flow Grids

Melanie Lipp, Martin Schneider, Rainer Helmig



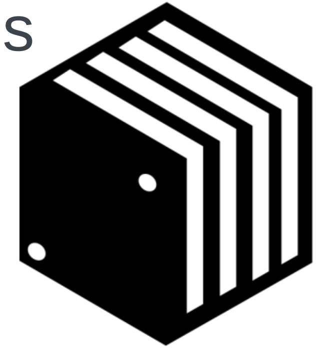
DROPIT

SFB 1313

Physical problem

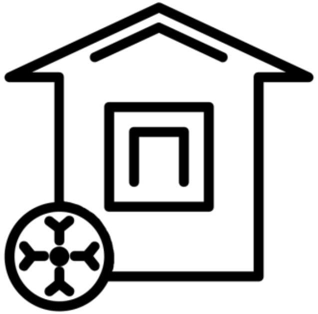
Motivation

- Fuel Cells



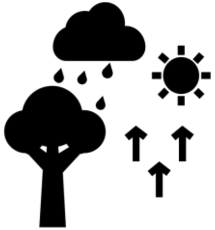
Created by Wuppdidu
from Noun Project

- Buildings/Urban Areas



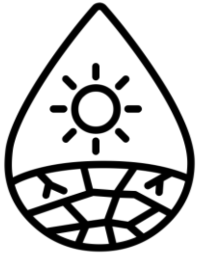
Created by Kahalap
from Noun Project

- Evaporation



Created by Shocho
from Noun Project

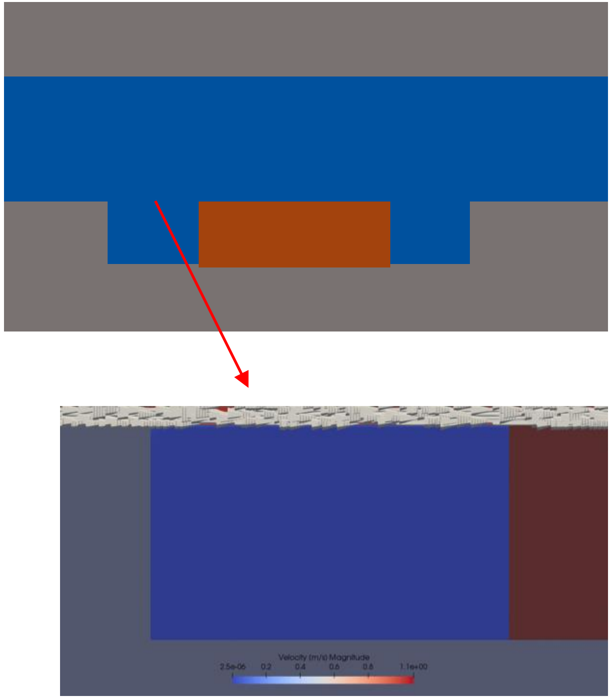
- Salinization



Created by Matt Wasser
from Noun Project

Created by priyanka
from Noun Project

Complex Flow Field



Free-Flow Regime

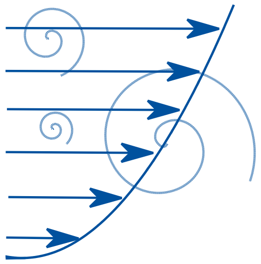
Porous-Medium-Flow Regime

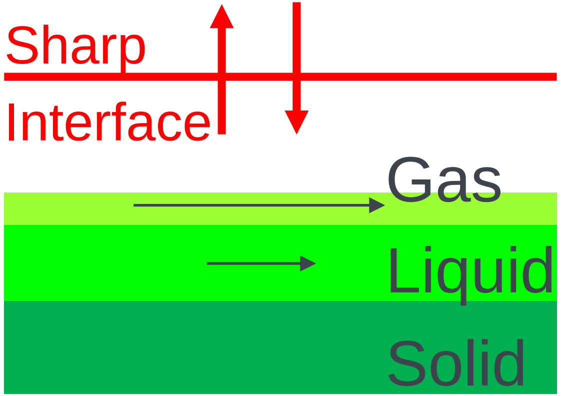
[Coltman, E., Lipp, M., Vescovini, A., & Helmig, R. (2020). *Transport in Porous Media*, *134*(2), 275--301.]

Model

Model

Free Flow





Porous Medium Flow

Pore network model

Representative Elementary Volume -scale model

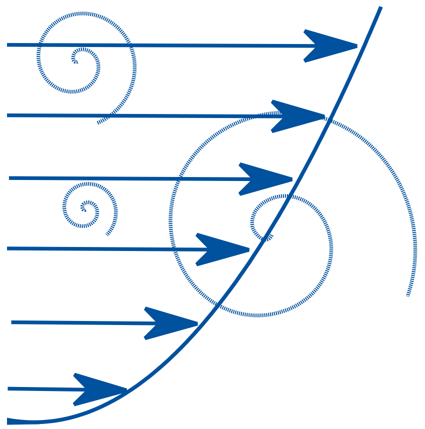
[Weishaupt et al. 2019, J Comput Phys X]

[Baber et al. 2012, IMA J Math]

Free Flow – Navier Stokes Equations

Mass Balance

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \boldsymbol{u}) - q_{\mathrm{p}} = 0$$



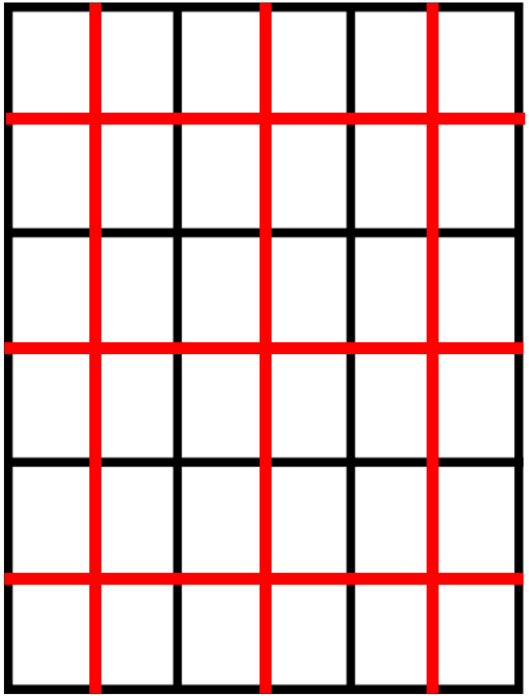
Momentum Balance

$$\frac{\partial (\varrho \boldsymbol{u})}{\partial t} + \nabla \cdot (\varrho \boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}) - \nabla \cdot (\mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}})) + \nabla p - \varrho \boldsymbol{g} - q_{\mathrm{u}} = 0$$

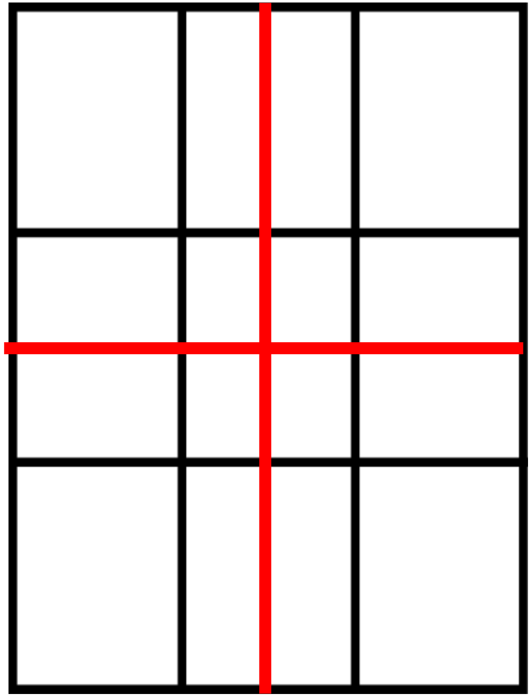
Finite-Volume Staggered Grid Discretization

Refinement Types

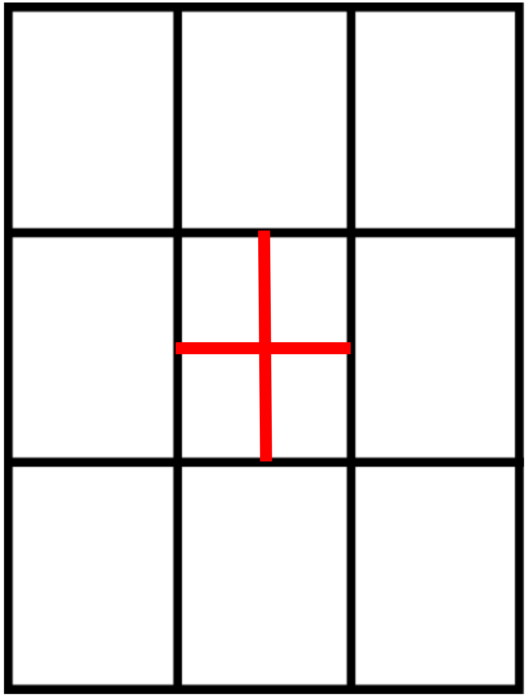
Everything Fine



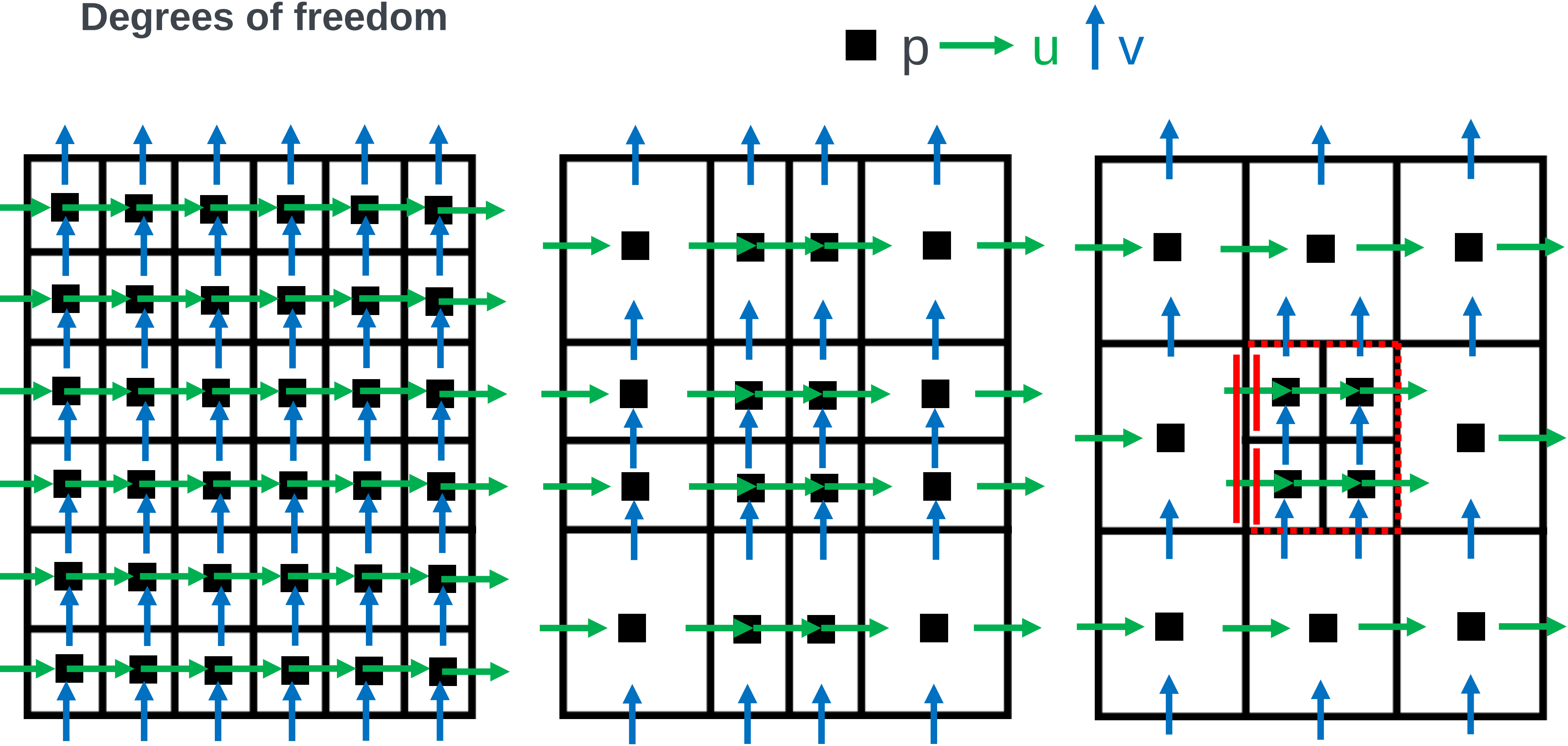
Global Refinement



Local Refinement

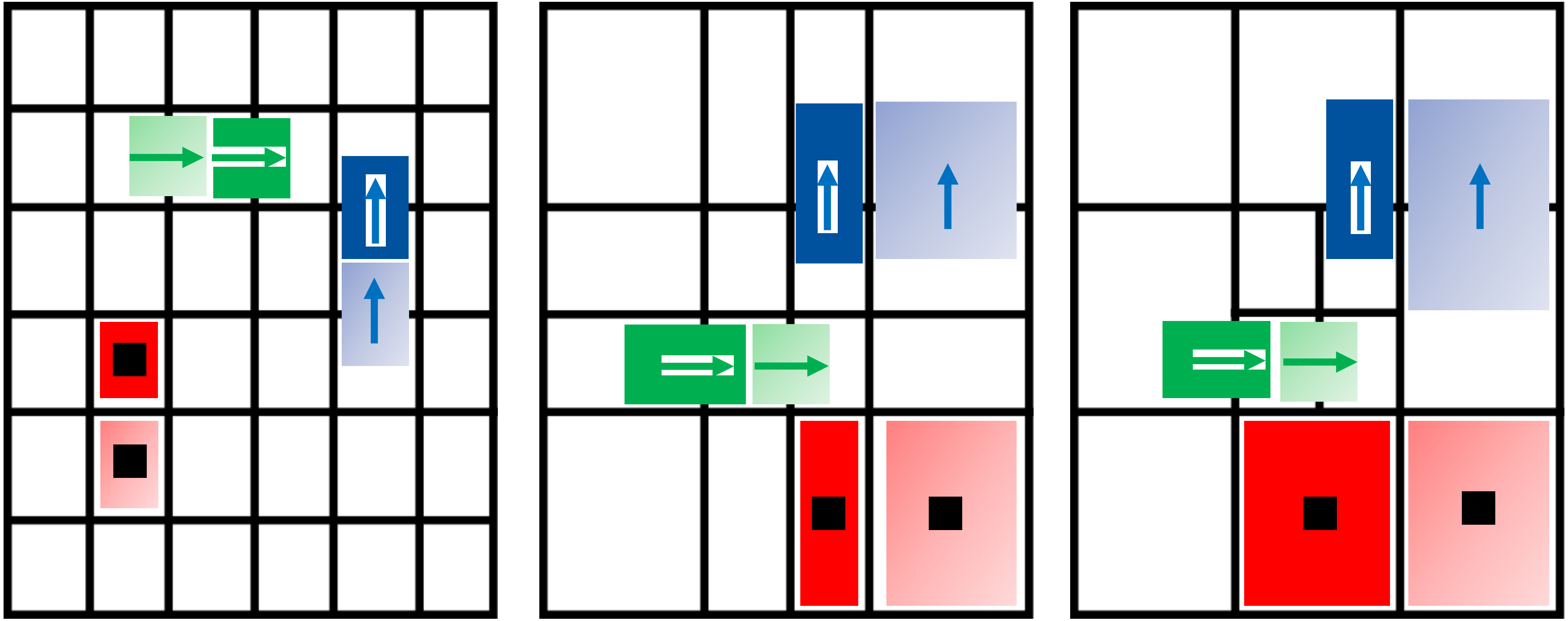


Degrees of freedom



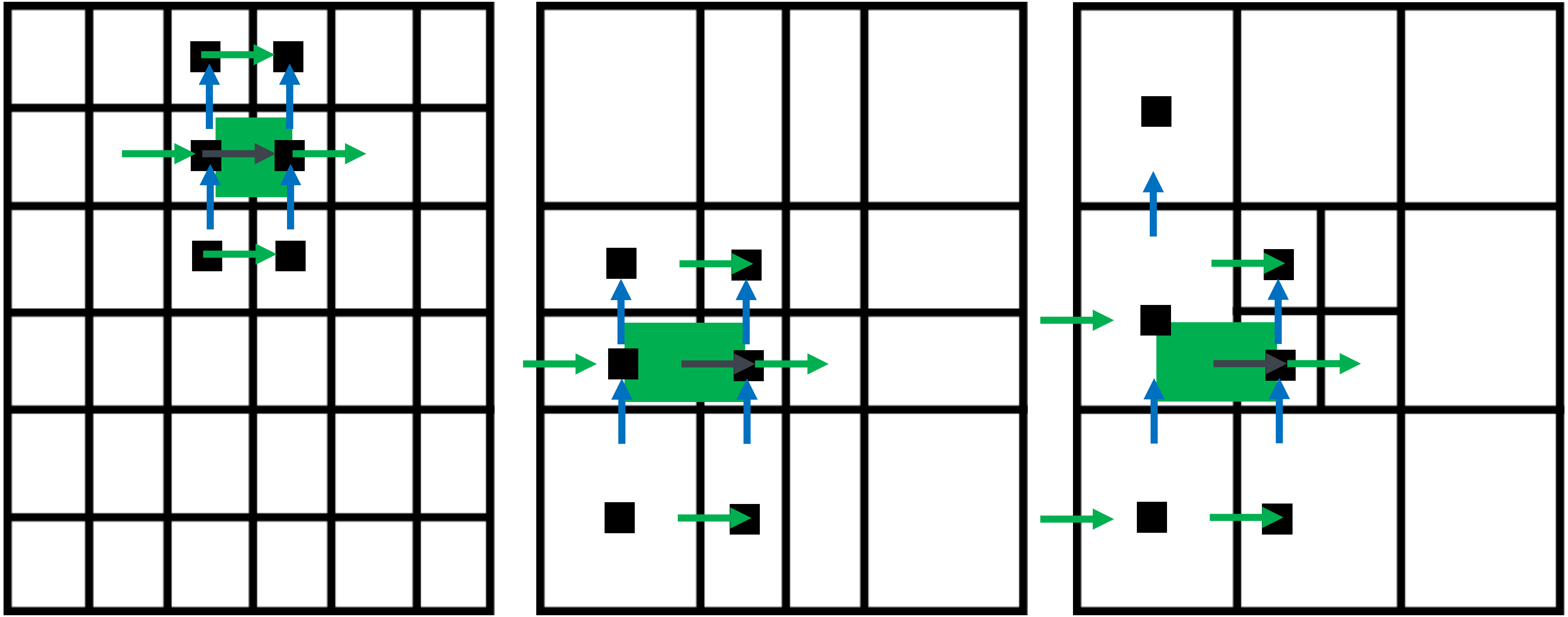
Control volumes

Continuity equation, Momentum equation x-component, Momentum equation y-component

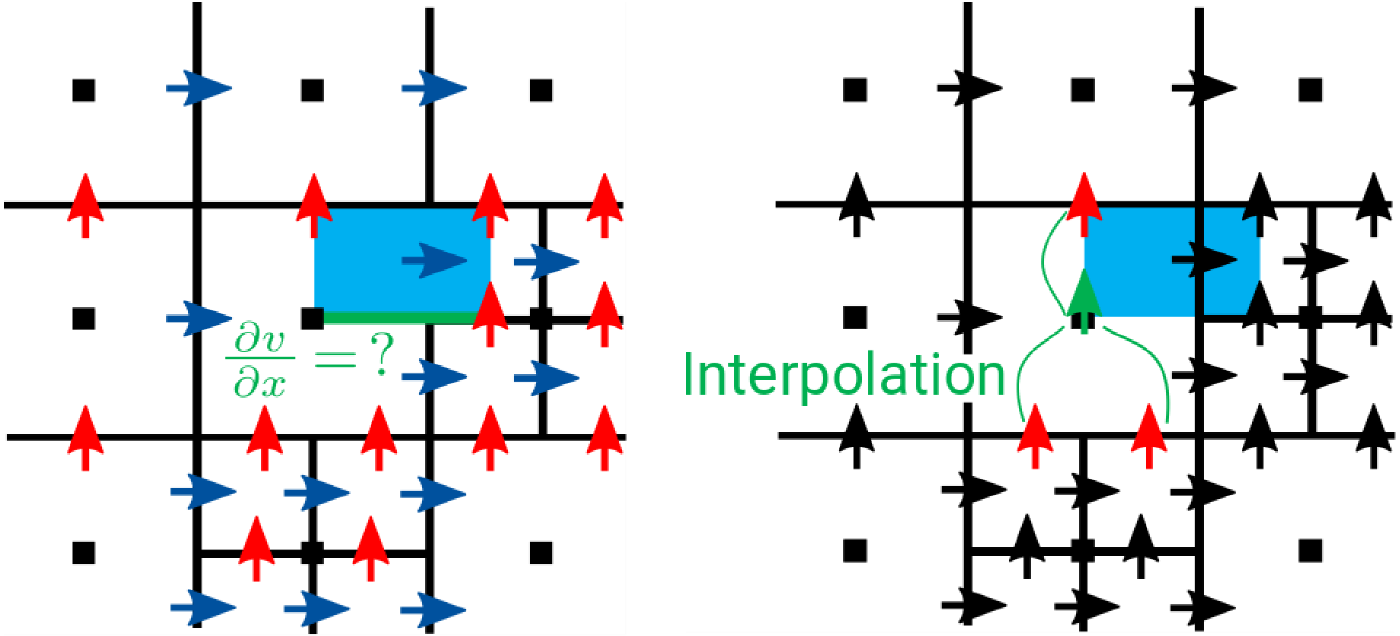


Stencils

- Momentum Equation, x-component



Local Refinement: Stencil - Interpolations



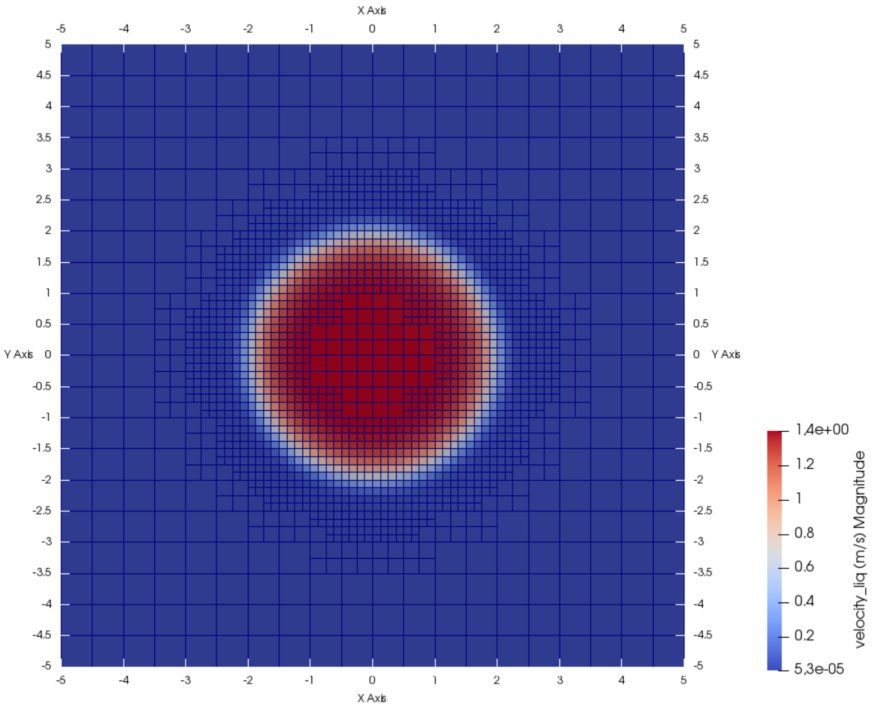
Lipp, M. and R. Helmig. A locally-refined locally-conservative quadtree finite-volume staggered-grid scheme. In G. Lamanna, S. Tonini, G.E. Cossali, and B. Weigand, editors, Droplet Interactions and Spray Processes, volume 121 of Fluid Mechanics and Its Applications, pages 149–159. Springer, 2020. ISBN 978-3-030-33337-9.

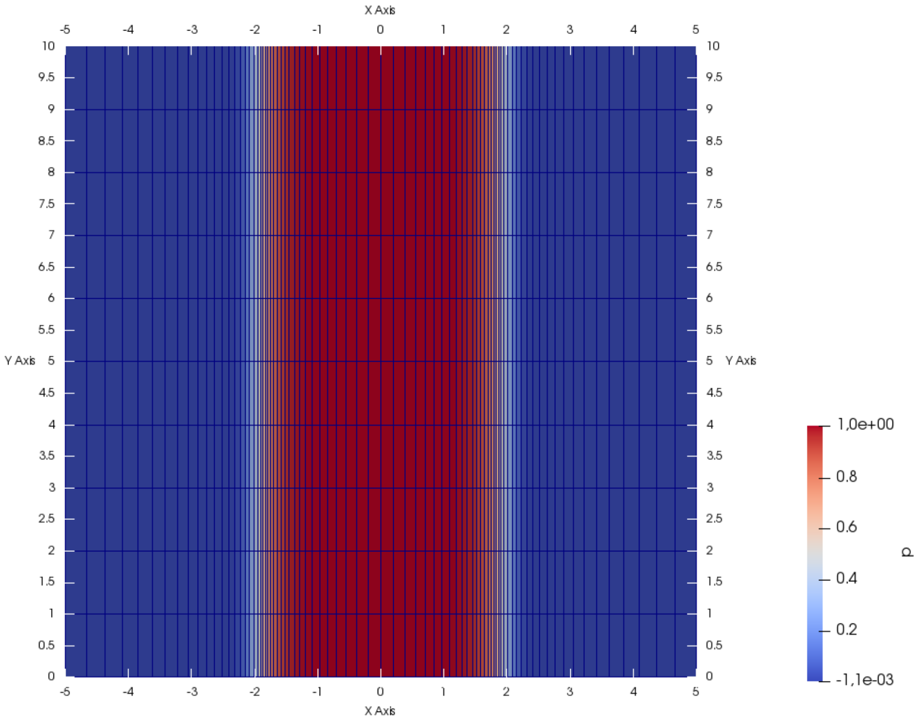
Our results

- Dune-Alugrid

Example A: Free-Flow Only: Supergaussian Peak

Numerical solution





L2 Error Results

	Local				Global		
	#dofs	p	u	v	p	u	v
Without Refinement	6165	3.65 e-02	9.22 e-03	9.22 e-03	1.02 e-03	5.32 e-05	2.60 e-05
With Refinement	6120	1.45 e-02	5.48 e-03	5.48 e-03	6.83 e-04	3.44 e-05	1.75 e-05

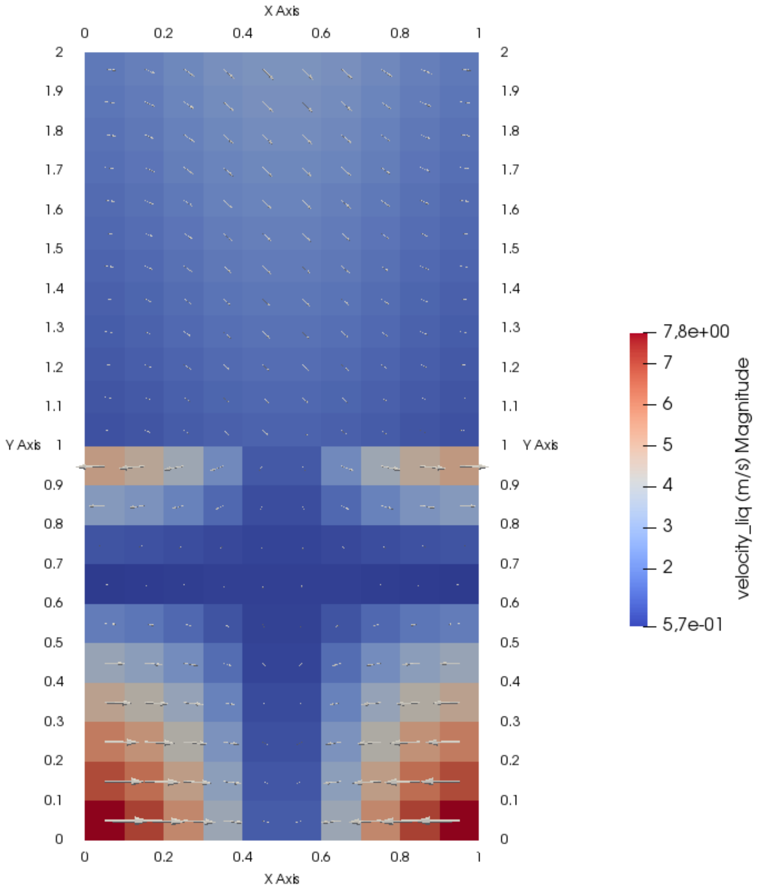
In this example better results with refinement ☺

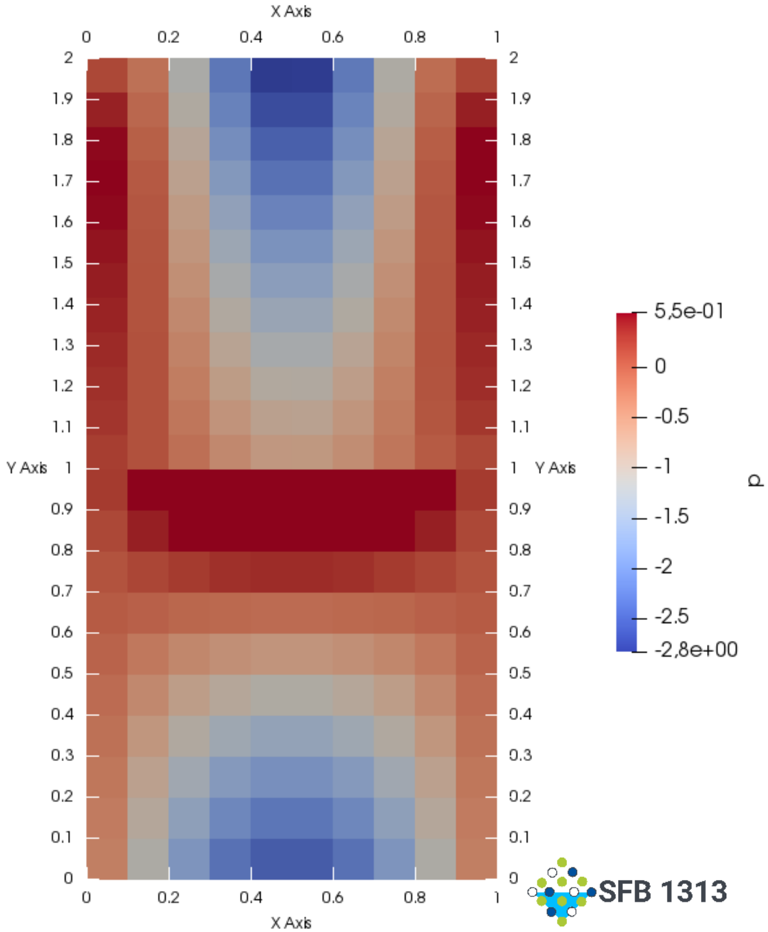
Example B: Coupling to Porous Medium (Representative Elementary Volume Scale)

Setup

Manufatured solution

[Schneider, Martin, et al. "Coupling staggered-grid and MPFA finite volume methods for free flow/porous-medium flow problems." *Journal of Computational Physics* 401 (2020): 109012.]

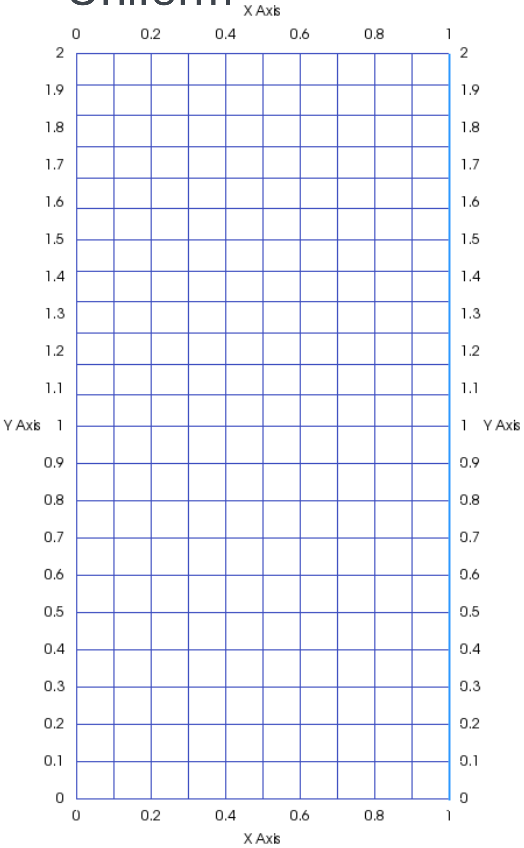




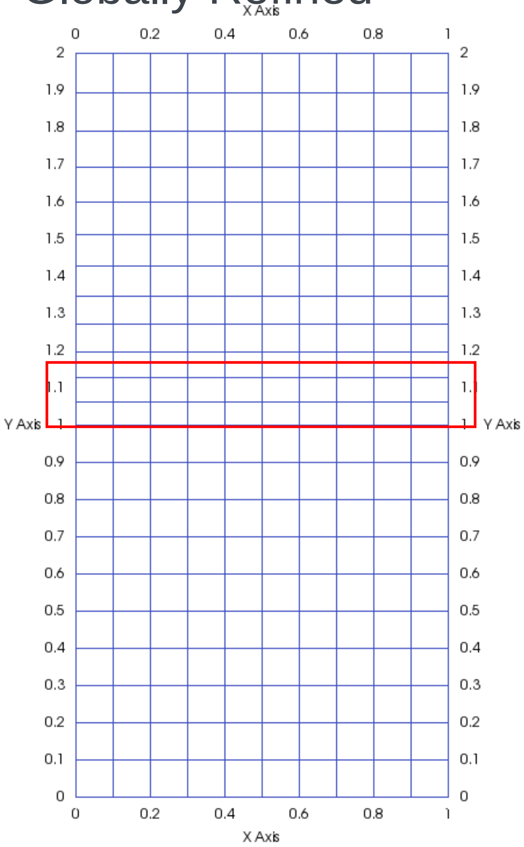
SFB 1313

Grids

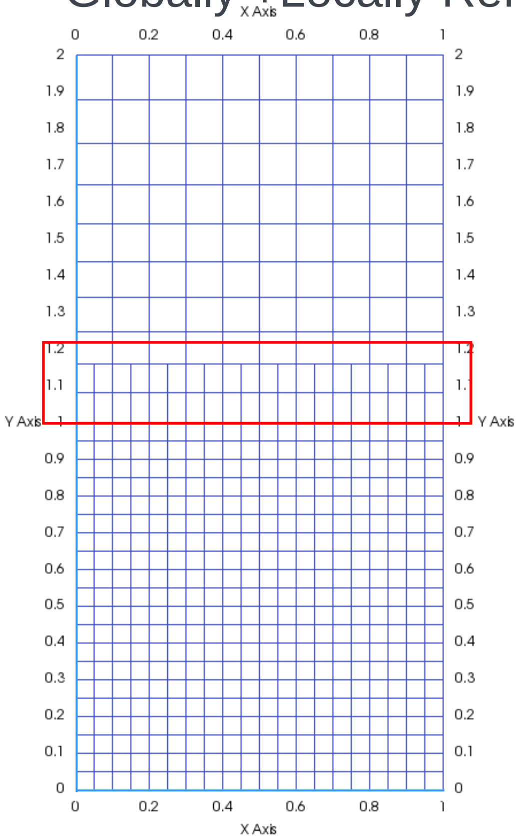
Uniform



Globally Refined



Globally +Locally Refined



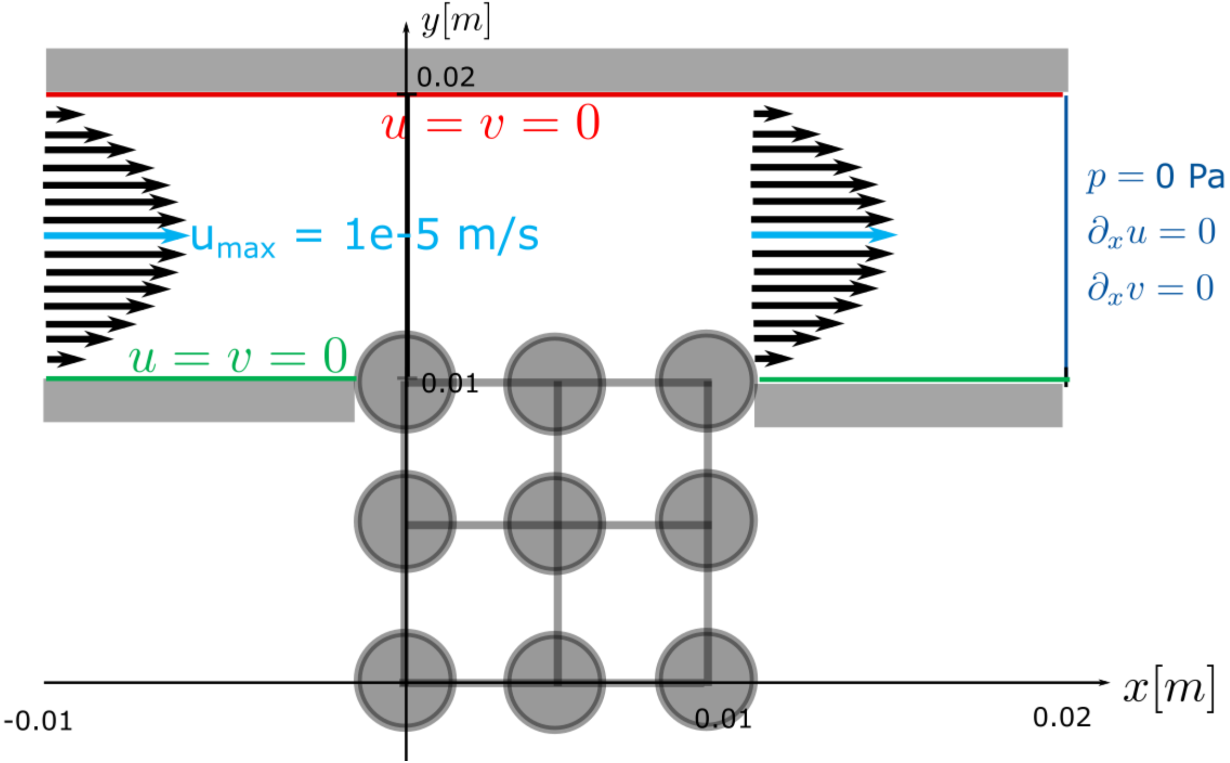
Results

For this example – errors for the freeflow solution:

	L2(p),abs	L2(p),rel	L2(vx),abs	L2(vx),rel	L2(vy),abs	L2(vy,rel)
uniform	5,08E-01	3,33E-01	4,72E-03	3,09E-03	9,01E-03	8,34E-03
globally	4,32E-01	2,83E-01	4,25E-03	2,78E-03	8,67E-03	8,02E-03
	Better Than Uniform					
locally	4,92E-01	3,23E-01	4,79E-03	3,14E-03	7,95E-03	7,35E-03
	Better Than Uniform		Worse Than All		Better Than All	
	Worse Than Grading					

Example C: Coupling to Porous Medium (Pore Network)

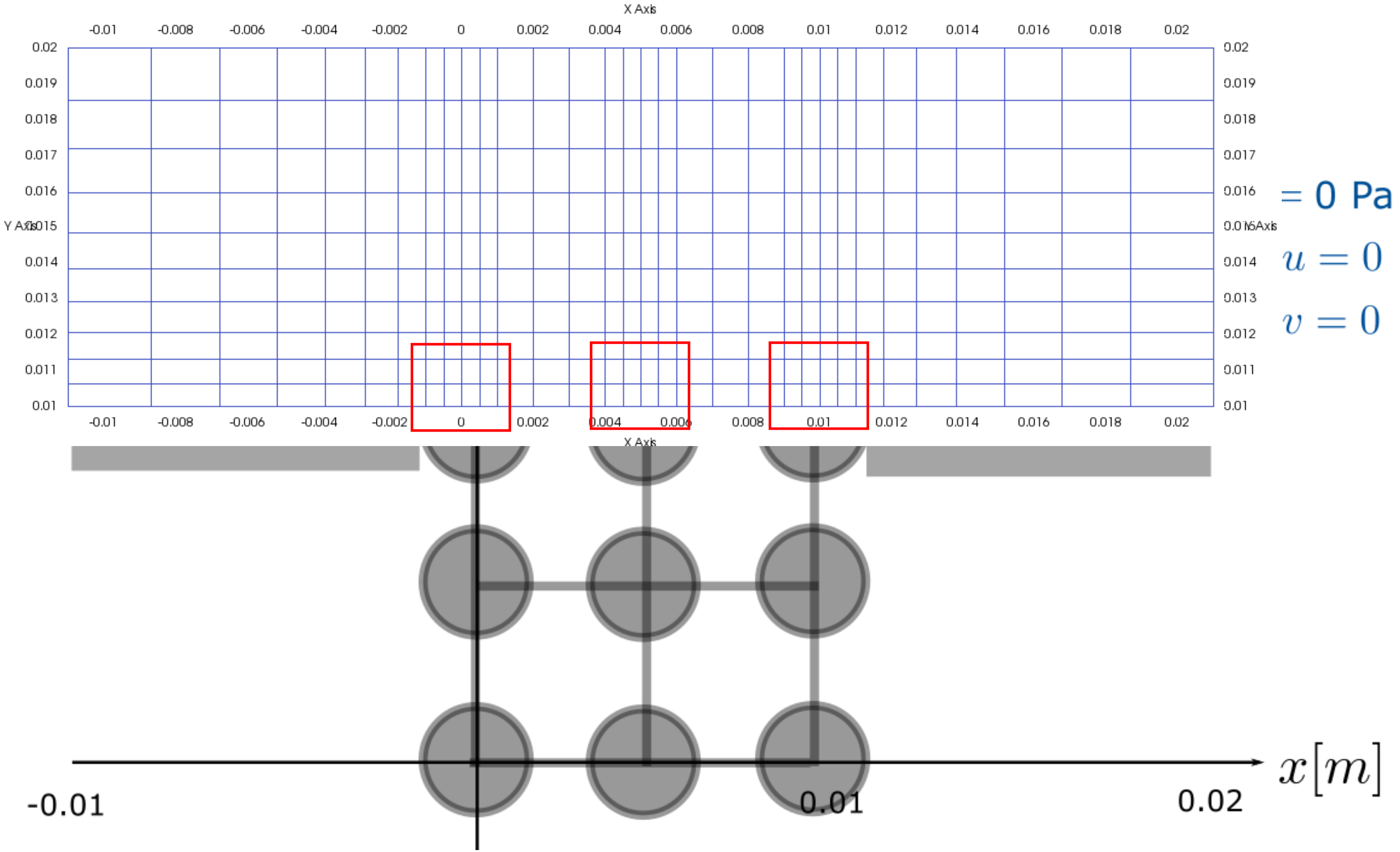
Setup



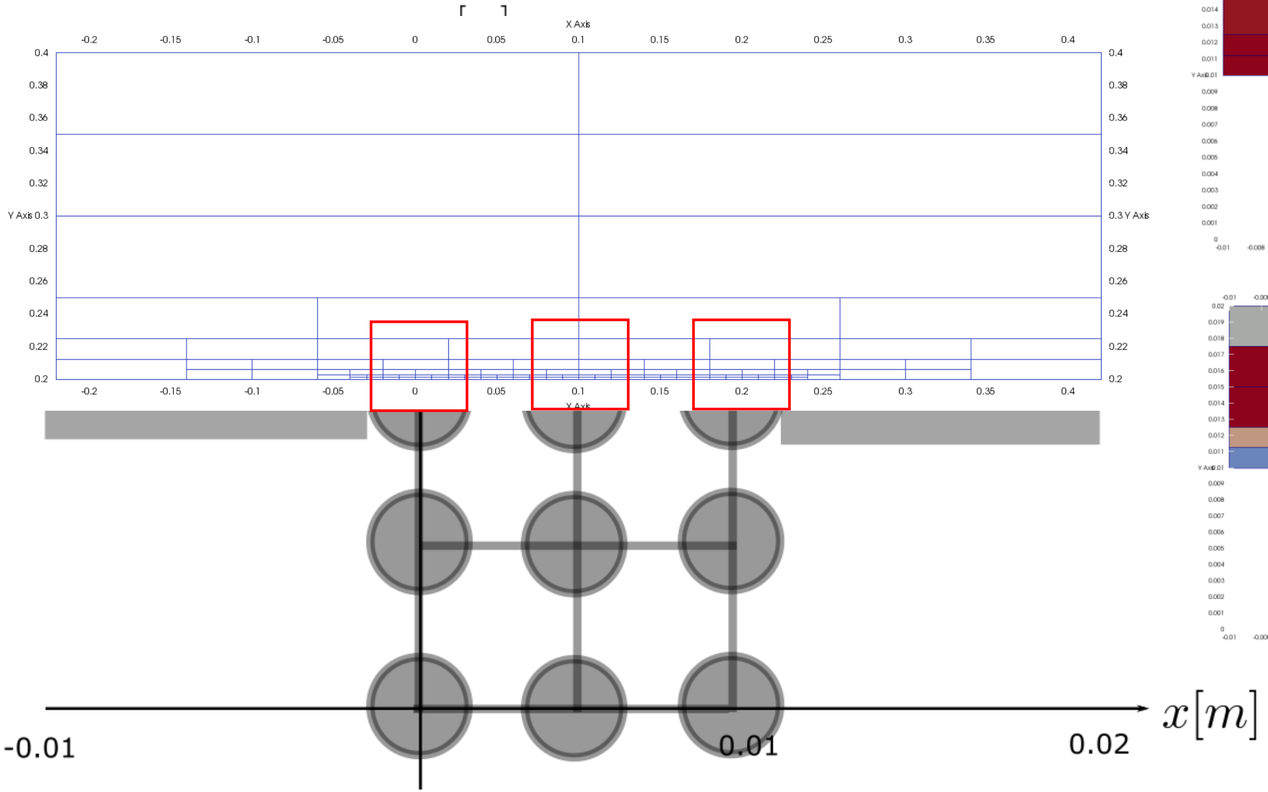
Freeflow:

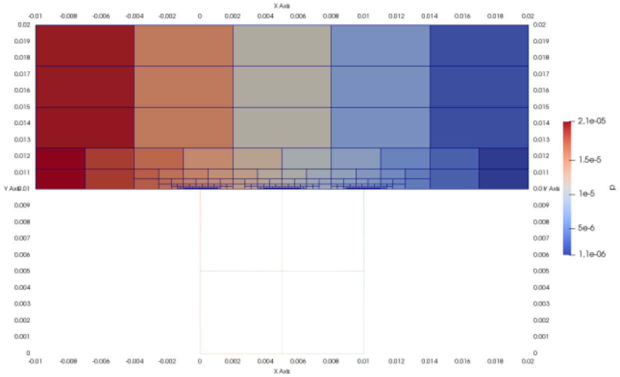
- Stationary
- Laminar Stokes flow (Re = 0.1)
- 2D
- Single Phase (Gas)
- 1 component
- Isothermal
- no gravity
- Constant density (1e3)/ Kinematic viscosity(1e-6)

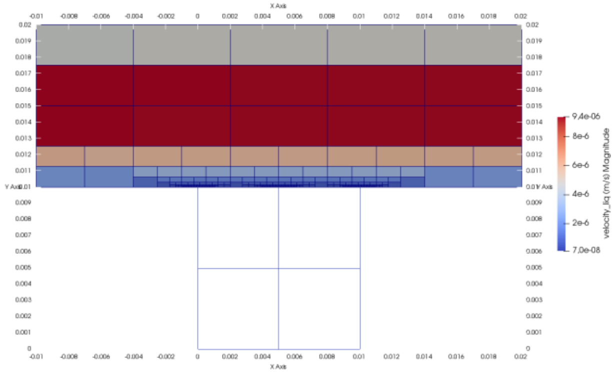
Globally Refined Grid



Locally Refined Grid







Results

Sum over Pore Bodies

$$\sqrt{\sum_{i=0}^{N} \left(p_{i,\text{this grid}} - p_{i,120\times40\text{ grid}}\right)^2}$$

	CC Dofs	Face Dofs	Deviation from 120x40 free-flow grid
Uniform 30x10	300	640	2.89e-6
Globally refined	300	640	3.01e-6
Locally refined	299	712	4.81e-6

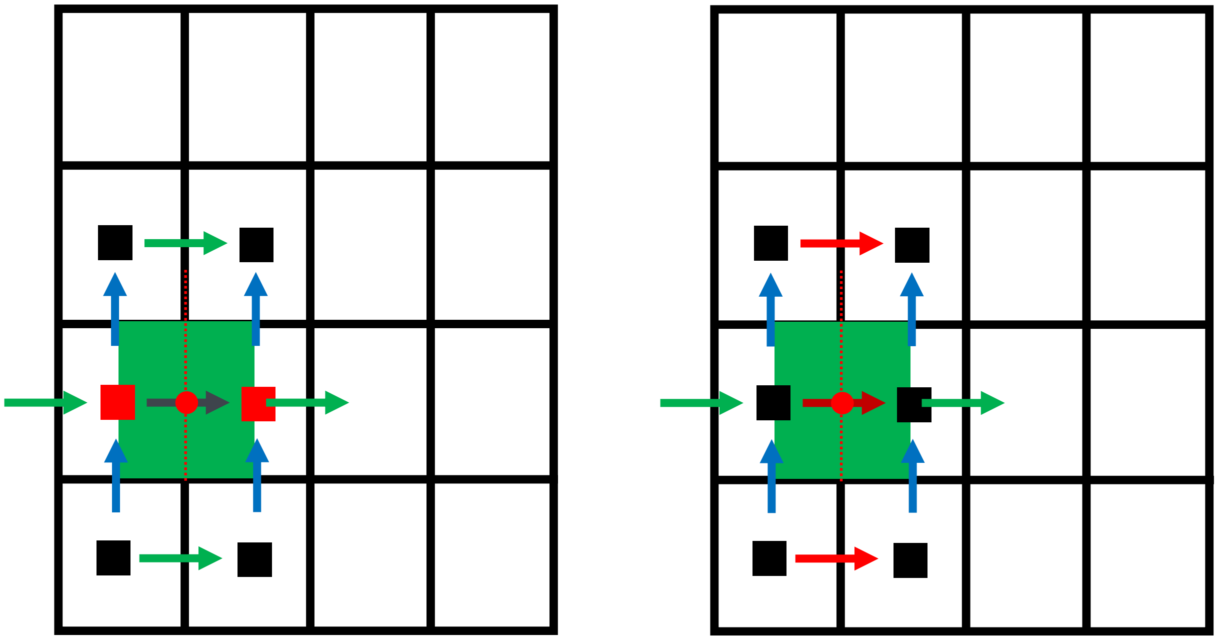
Discussion

Effects Not Covered by Local Truncation Error Analysis

- Global refinement: Superconvergence!
- Local refinement: Superconvergence disturbed?

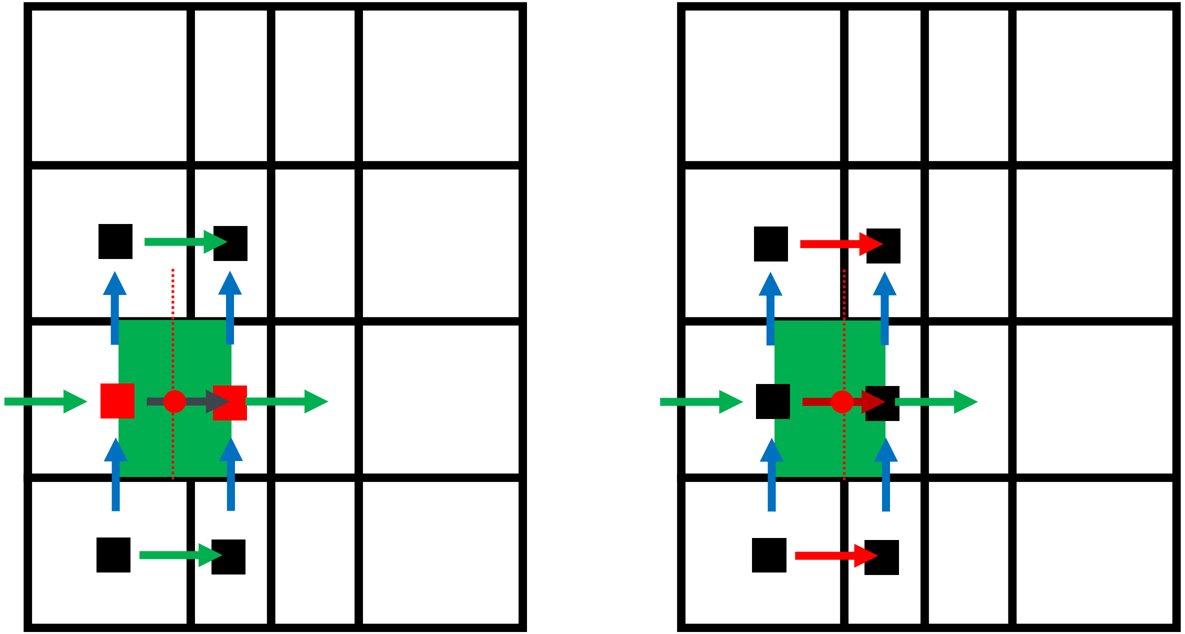
Effects Not Covered by Local Truncation Error Order

- Global+Local Refinement: Stencil Distortion Affects Local Truncation Errors



Effects Not Covered by Local Truncation Error Order

- Global+Local Refinement: Stencil Distortion Affects Local Truncation Errors



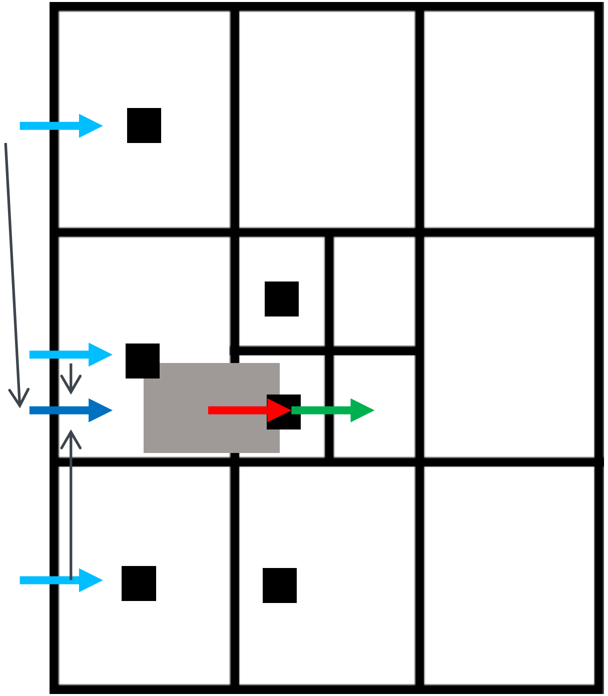
Effects within Local Truncation Error Order

- Local Refinement:

  Interpolations Affect Local Truncation Errors

$$\frac{\mu(x_r, y_c)\,\partial_x u(x_r, y_c) - \mu(x_l, y_c)\,\partial_x u(x_l, y_c)}{\Delta x} =$$

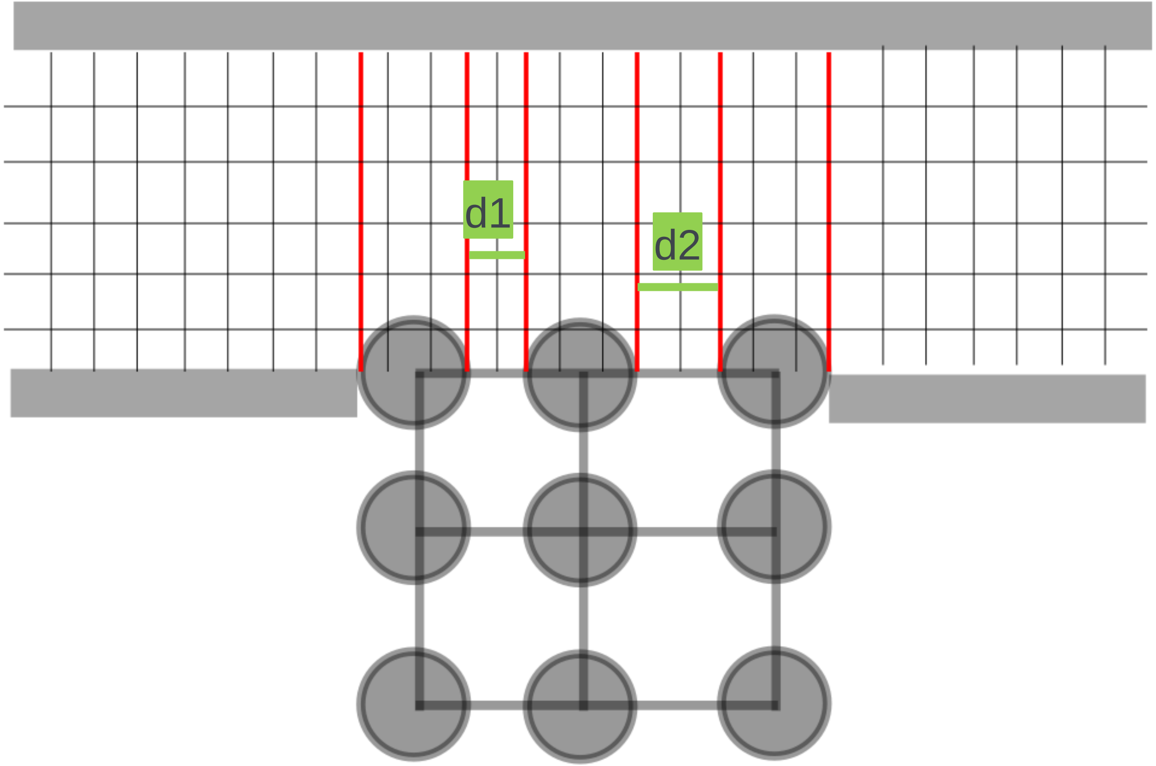
$$\frac{\mu_r}{\Delta x \Delta x_r} u_{rr} - \left(\frac{\mu_r}{\Delta x \Delta x_r} + \frac{\mu_l}{\Delta x \Delta x_l}\right) u_c + \frac{\mu_l}{\Delta x \Delta x_l} u_{ll} + \mathcal{O}(\Delta)$$



See also [van der Plas, P. (2017). Local grid refinement for free-surface flow simulations. [Groningen]: Rijksuniversiteit Groningen.]

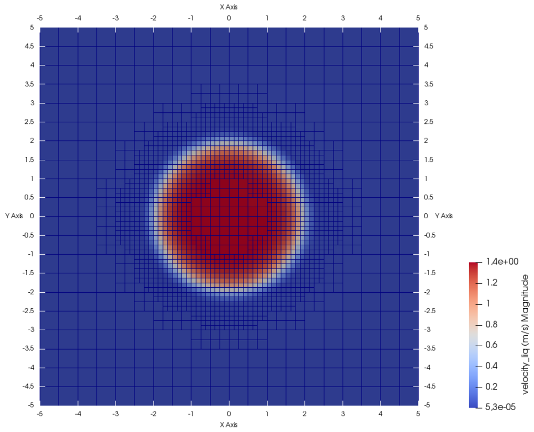
Occasionally Graded Grid Needed

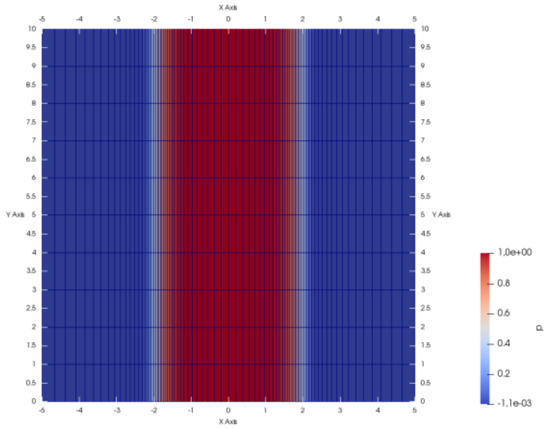
Pore-Network Model – Free-Flow



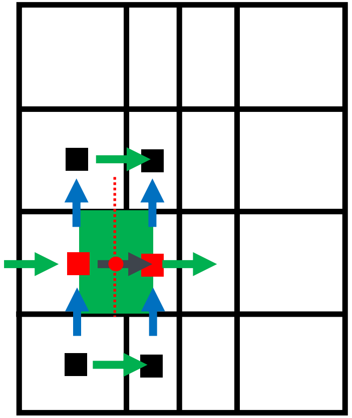
Summary

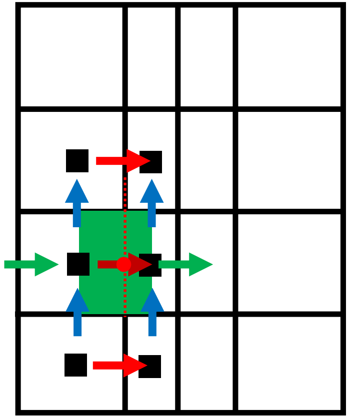
Summary



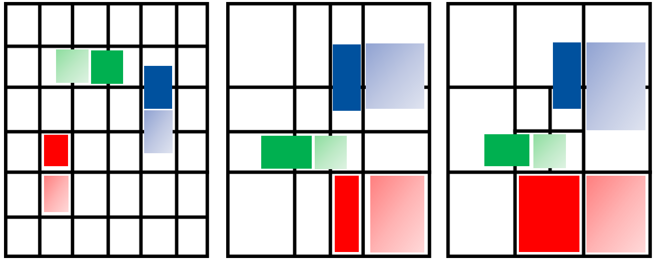


Created by Bold Yellow
from Noun Project





Thank you!



Melanie Lipp

e-mail melanie.lipp@iws.uni-stuttgart.de

phone +49 (0) 711 685-64736

www. Iws.uni-stuttgart.de/lh2/

University of Stuttgart
Institute for Modelling Hydraulic and
Environmental Systems

Pfaffenwaldring 61, 70569 Stuttgart, Germany

We thank the German Research Foundation (DFG) for supporting this work by funding SFB 1313, Project Number 327154368.

SFB 1313