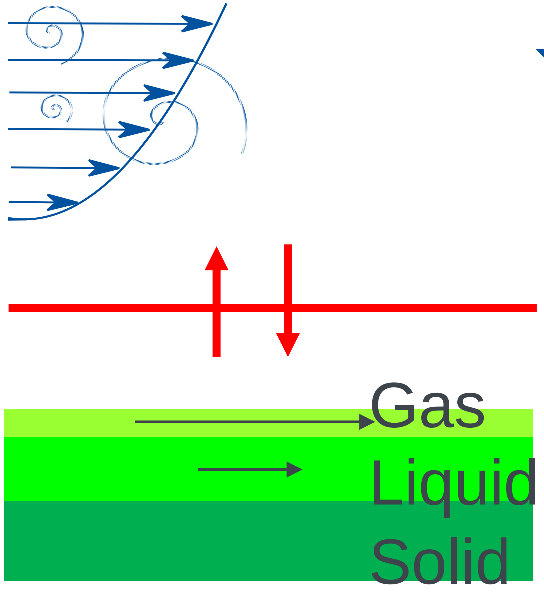
University of Stuttgart
Institute for Modelling Hydraulic and Environmental Systems

Department of Hydromechanics and Modelling of Hydrosystems

COUPLING FREE FLOW AND POROUS-MEDIUM FLOW: COMPARISON OF NON-REFINED, GLOBALLY-REFINED AND LOCALLY-REFINED AXIPARALLEL FREE-FLOW GRIDS



Melanie Lipp, Rainer Helmig, Kilian Weishaupt

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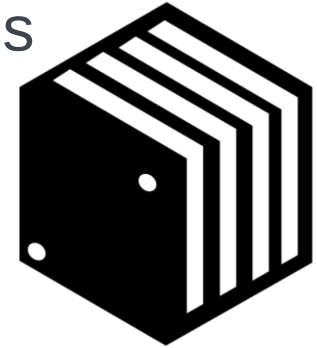
Physical problem

Motivation

- Fuel Cells



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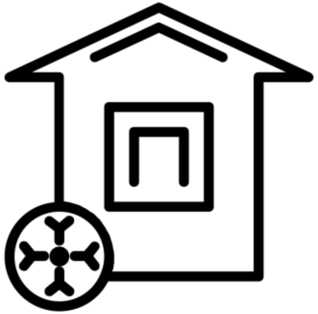
- Buildings/Urban Areas



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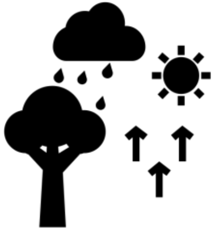
- Evaporation



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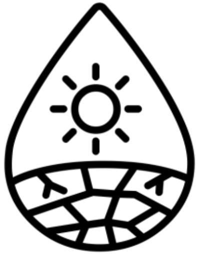
- Salinization



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Created by priyanka
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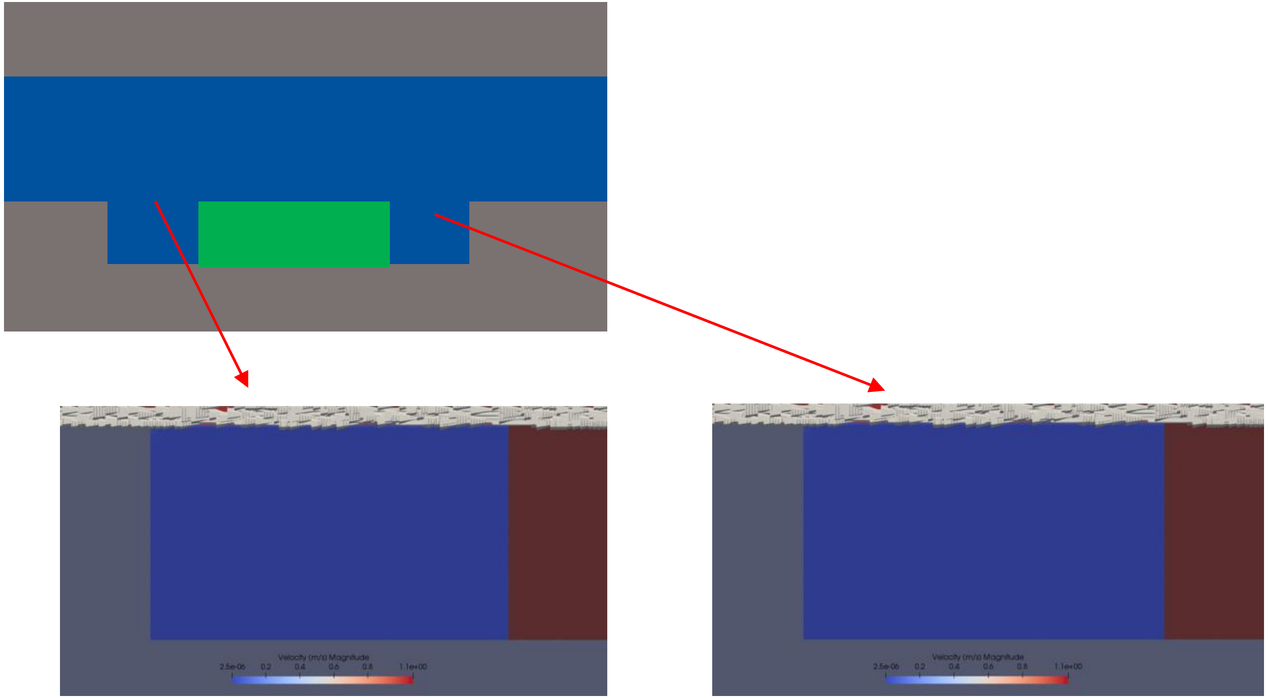
Complex Flow Field

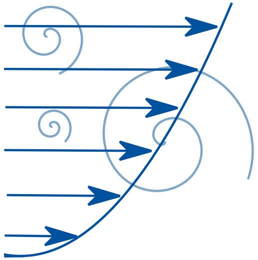


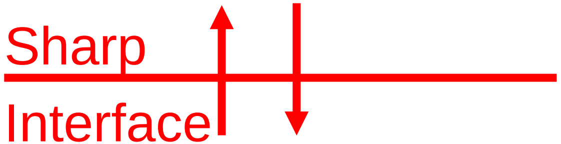
[Coltman, E., Lipp, M., Vescovini, A., & Helmig, R. (2020). *Transport in Porous Media*, *134*(2), 275--301.]

Model

Model

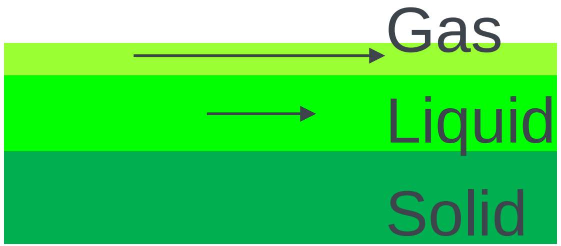
Free Flow





Porous Medium Flow

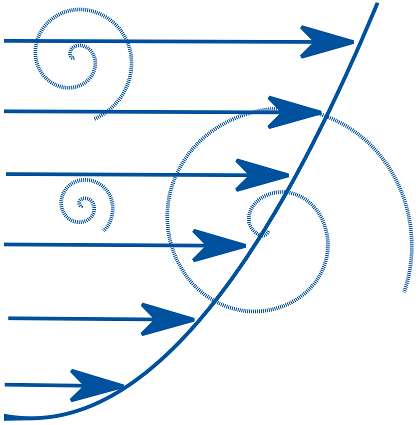
Pore network model



Representative Elementary Volume -scale model

[Weishaupt et al. 2019, J Comput Phys X][Baber et al. 2012, IMA J Math]

Free Flow – Navier Stokes Equations



Mass Balance

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \boldsymbol{u}) - q_{\mathrm{p}} = 0$$

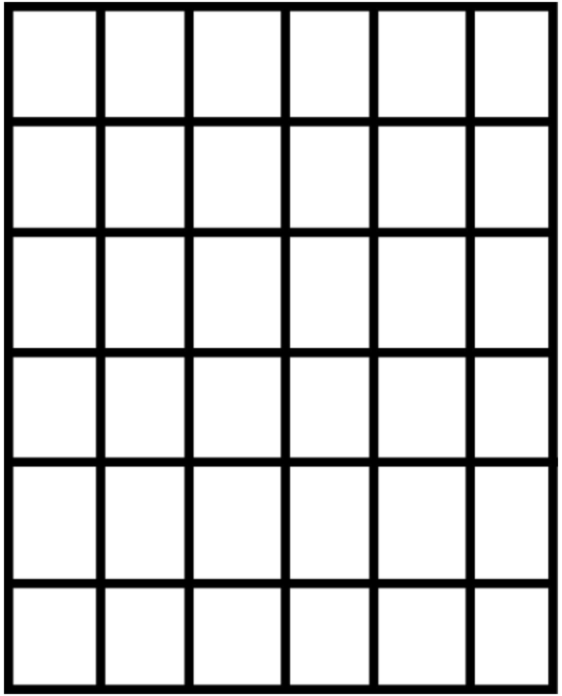
Momentum Balance

$$\frac{\partial (\varrho \boldsymbol{u})}{\partial t} + \nabla \cdot (\varrho \boldsymbol{u} \boldsymbol{u}^{\mathrm{T}}) - \nabla \cdot (\mu (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}})) + \nabla p - \varrho \boldsymbol{g} - q_{\mathrm{u}} = 0$$

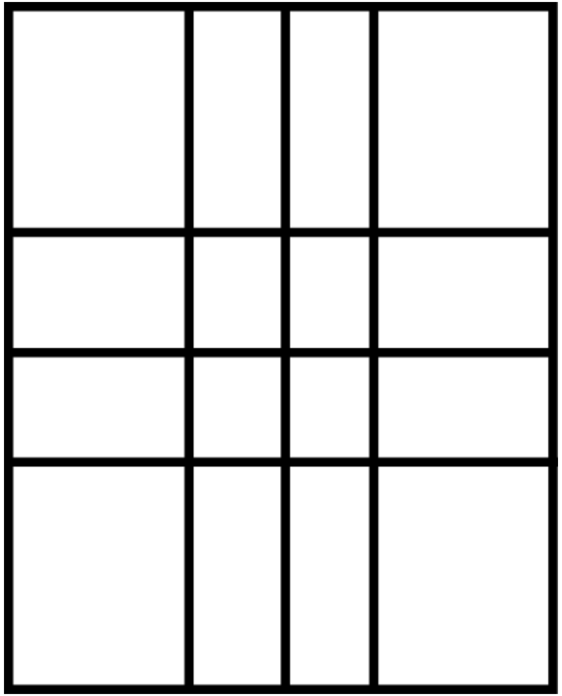
Finite-Volume Staggered Grid Discretization

Refinement Types

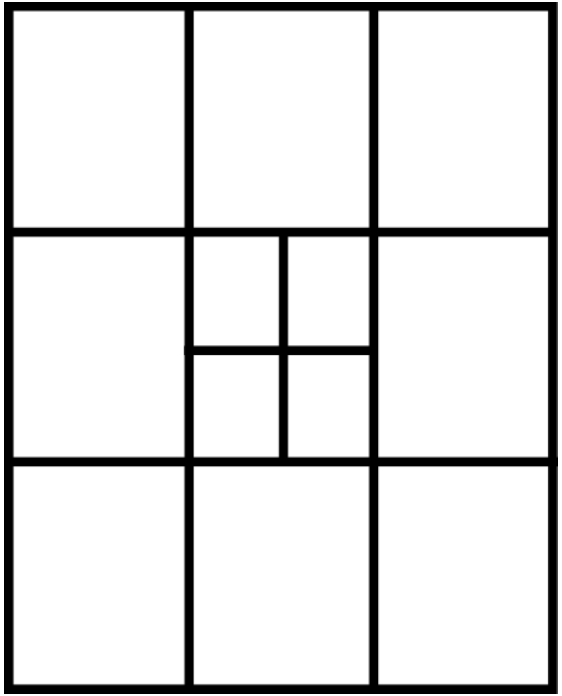
Everything Fine



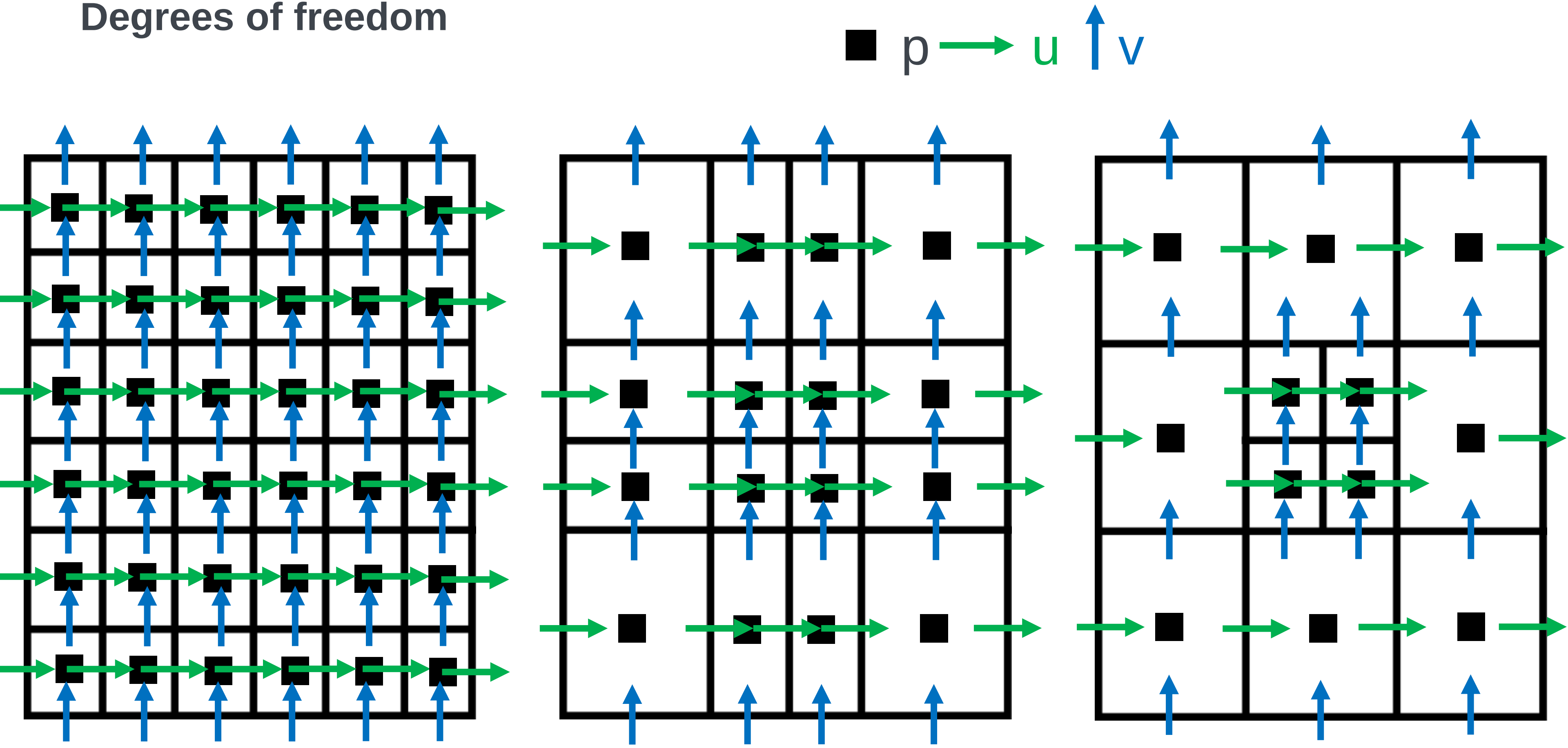
Global Refinement



Local Refinement

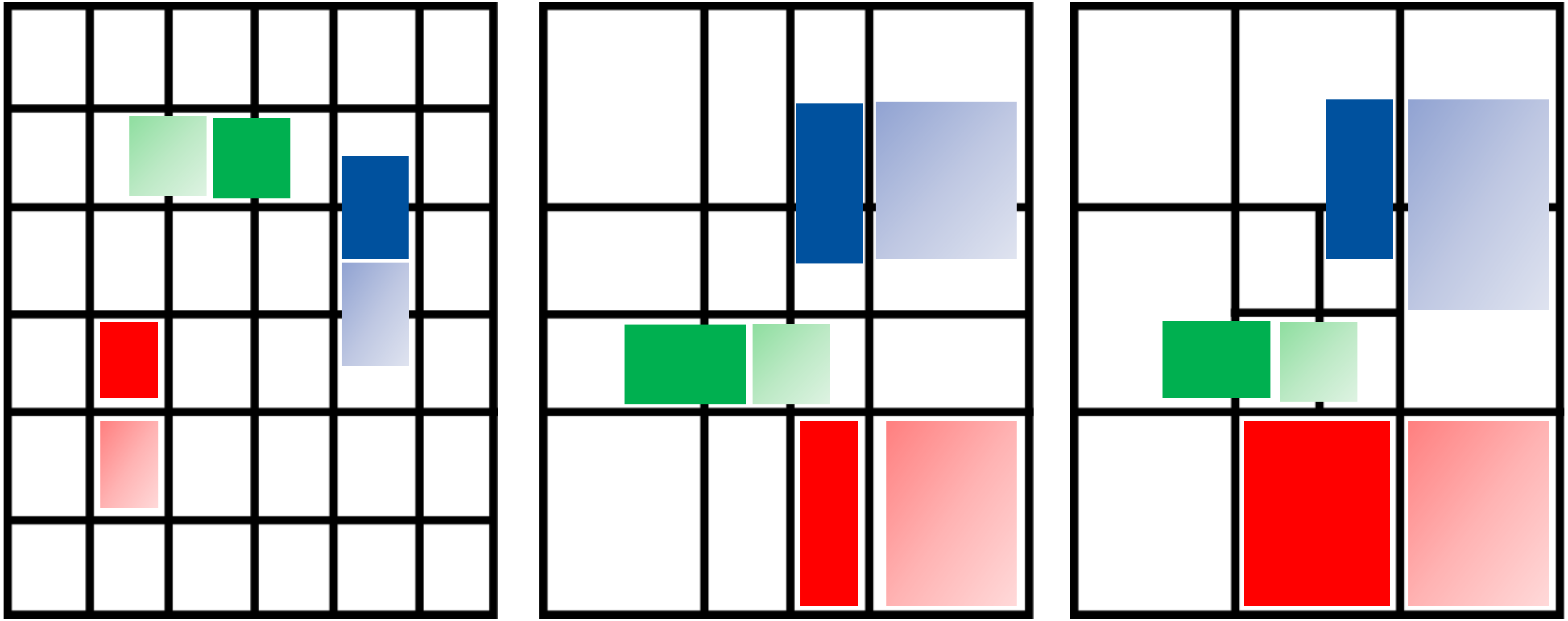


Degrees of freedom



Control volumes

Continuity equation, Momentum equation x-component, Momentum equation y-component



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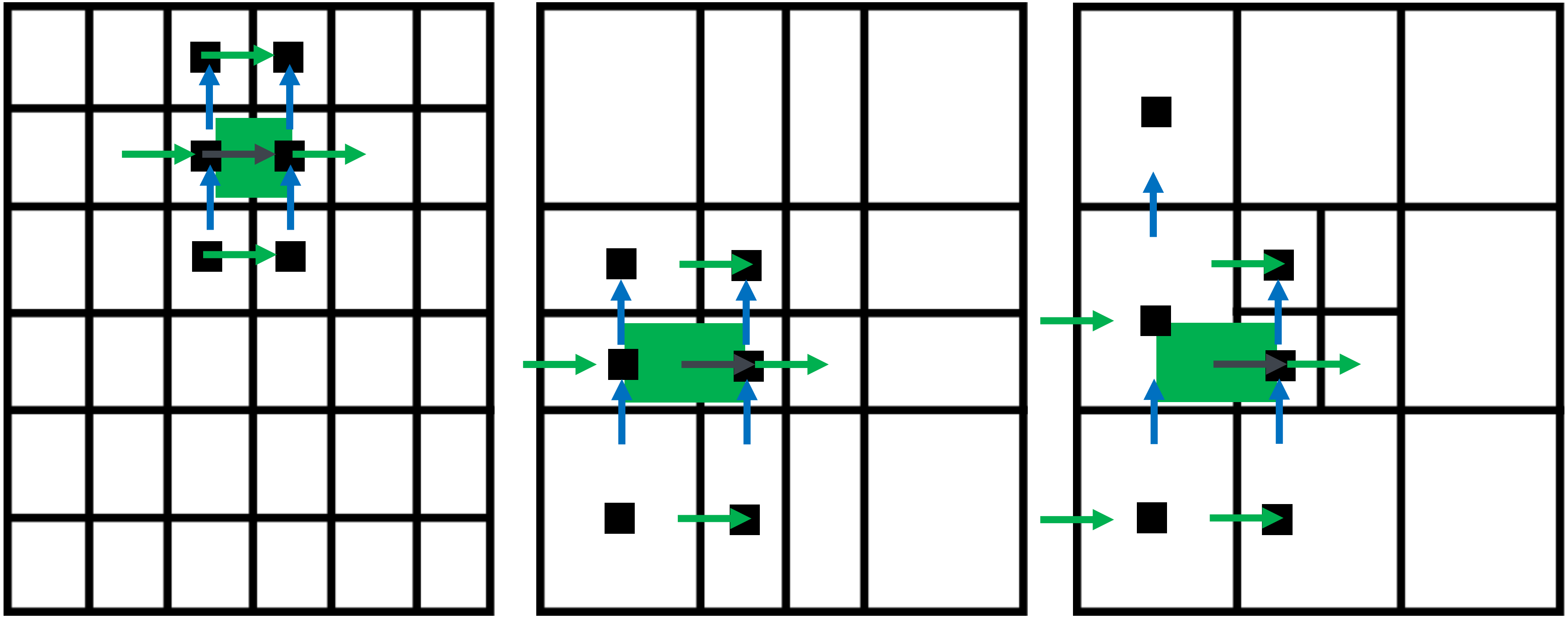
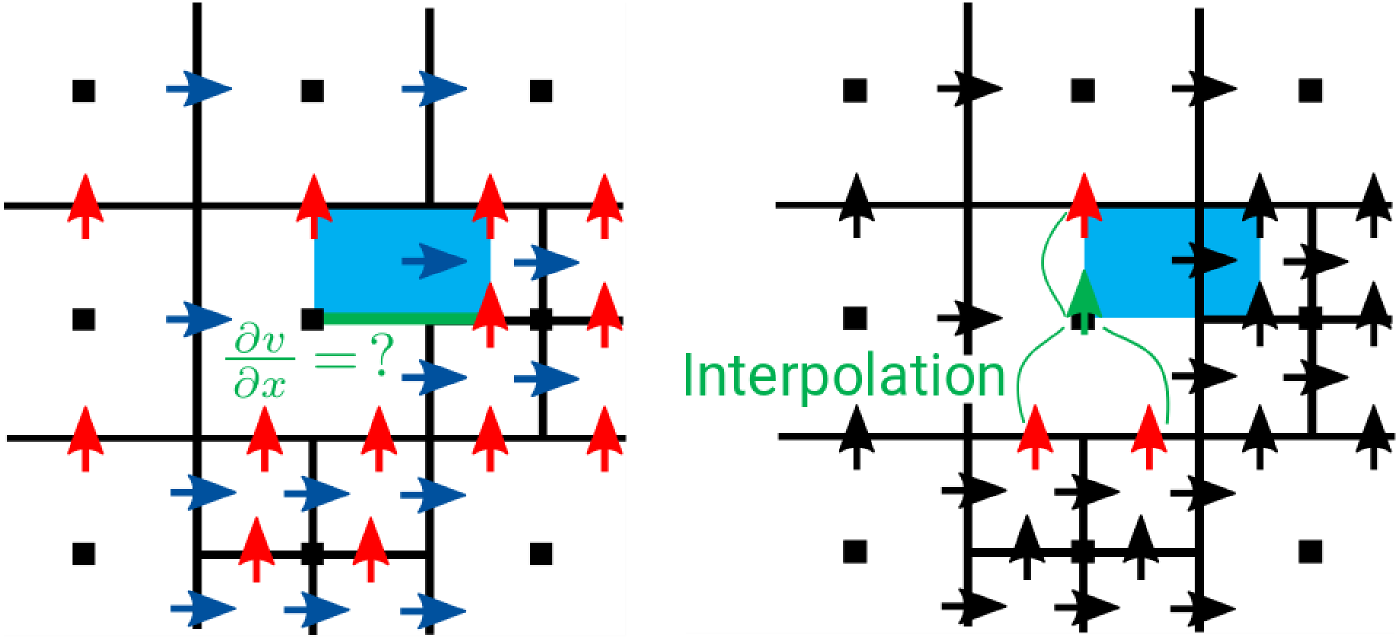
Stencils

- Momentum Equation, x-component



Local Refinement: Stencil - Interpolations



Lipp, M. and R. Helmig. A locally-refined locally-conservative quadtree finite-volume staggered-grid scheme. In G. Lamanna, S. Tonini, G.E. Cossali, and B. Weigand, editors, Droplet Interactions and Spray Processes, volume 121 of Fluid Mechanics and Its Applications, pages 149–159. Springer, 2020. ISBN 978-3-030-33337-9.

Our results

DuMu^X

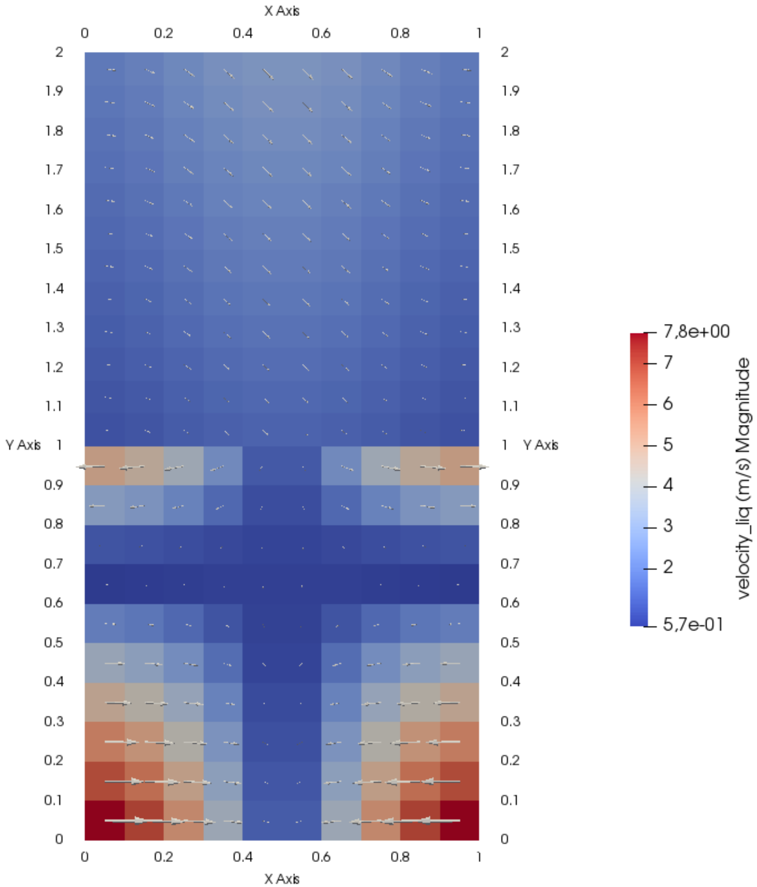
Dune

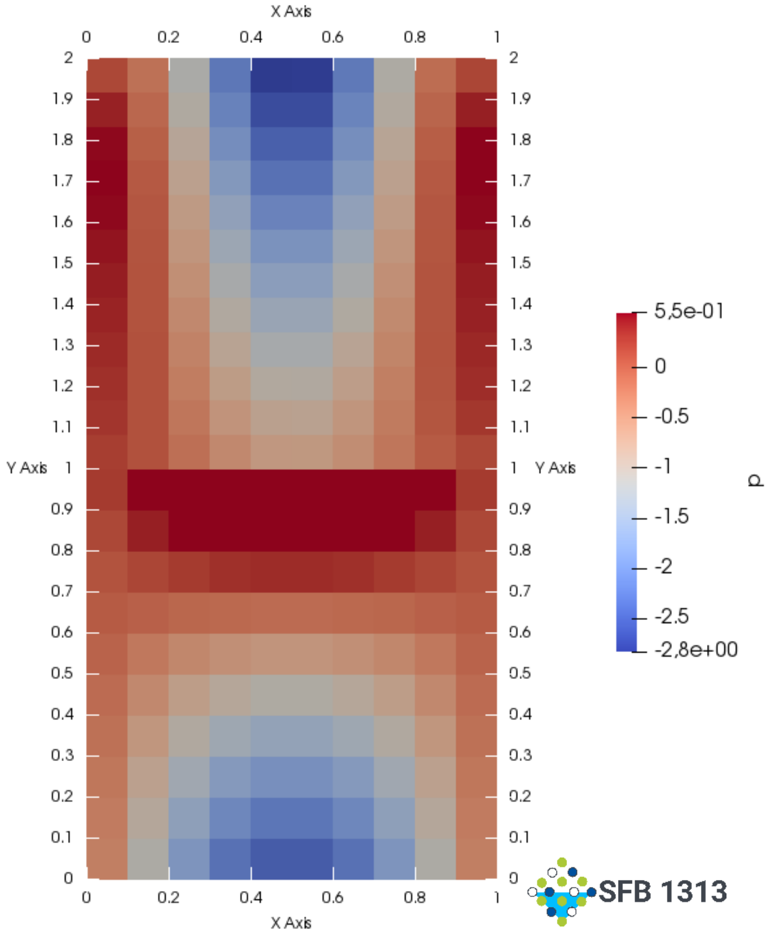
- Dune-Alugrid

Coupling to Porous Medium (Representative Elementary Volume Scale)
Setup

Manufatured solution

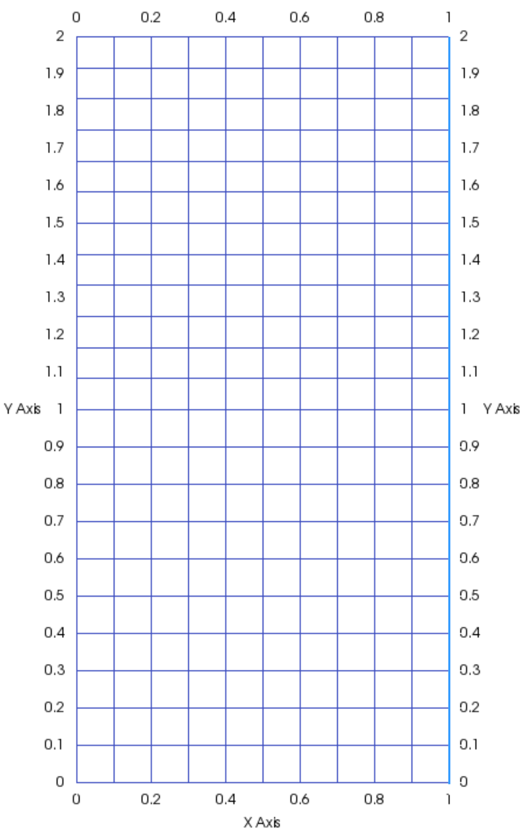
[Schneider, Martin, et al. "Coupling staggered-grid and MPFA finite volume methods for free flow/porous-medium flow problems." *Journal of Computational Physics* 401 (2020): 109012.]



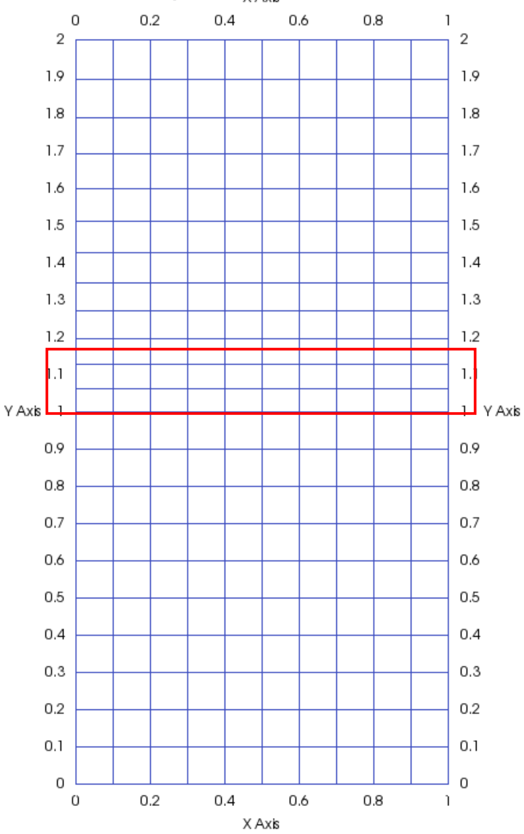


Coupling to Porous Medium (Representative Elementary Volume Scale)

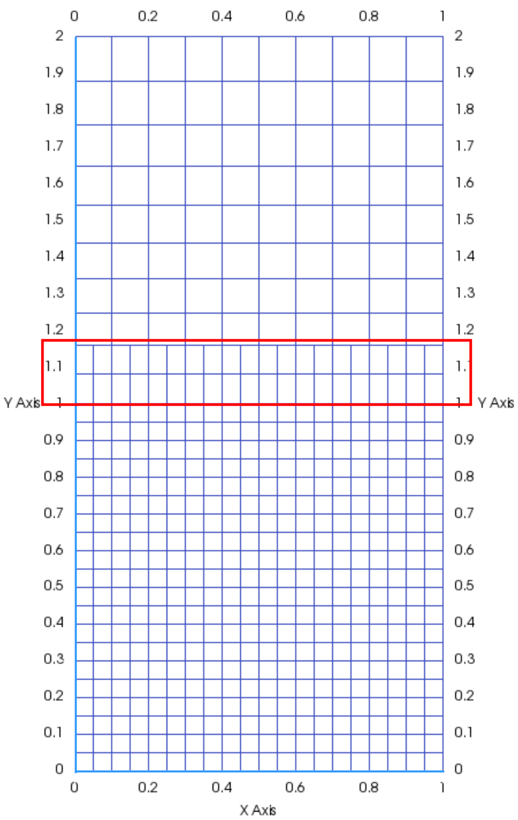
Uniform



Globally Refined



Globally +Locally Refined

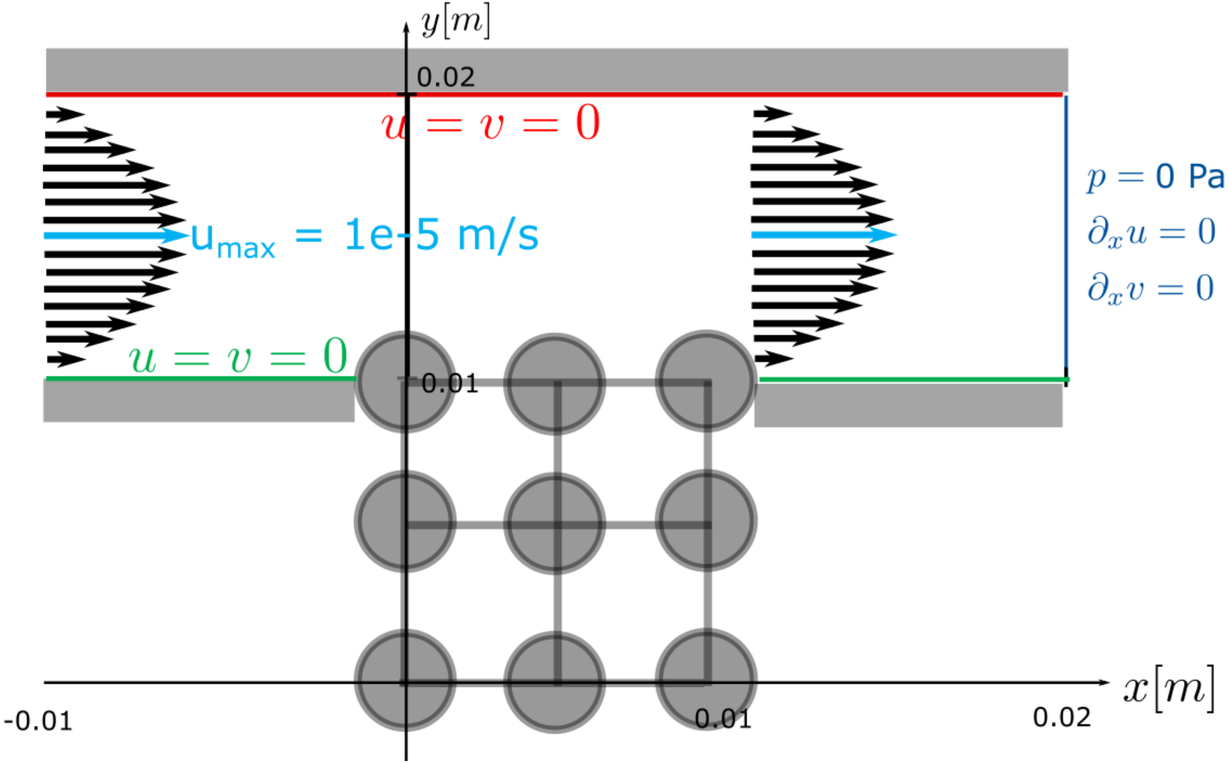


Coupling to Porous Medium (Representative Elementary Volume Scale)
Uni

For this example:

	L2(p),abs	L2(p),rel	L2(vx),abs	L2(vx),rel	L2(vy),abs	L2(vy,rel)
uniform	5,08E-01	3,33E-01	4,72E-03	3,09E-03	9,01E-03	8,34E-03
globally	4,32E-01	2,83E-01	4,25E-03	2,78E-03	8,67E-03	8,02E-03
	Better Than Uniform					
locally	4,92E-01	3,23E-01	4,79E-03	3,14E-03	7,95E-03	7,35E-03
	Better Than Uniform Worse Than Grading		Worse Than All		Better Than All	

Coupling to Porous Medium (Pore Network)
Setup



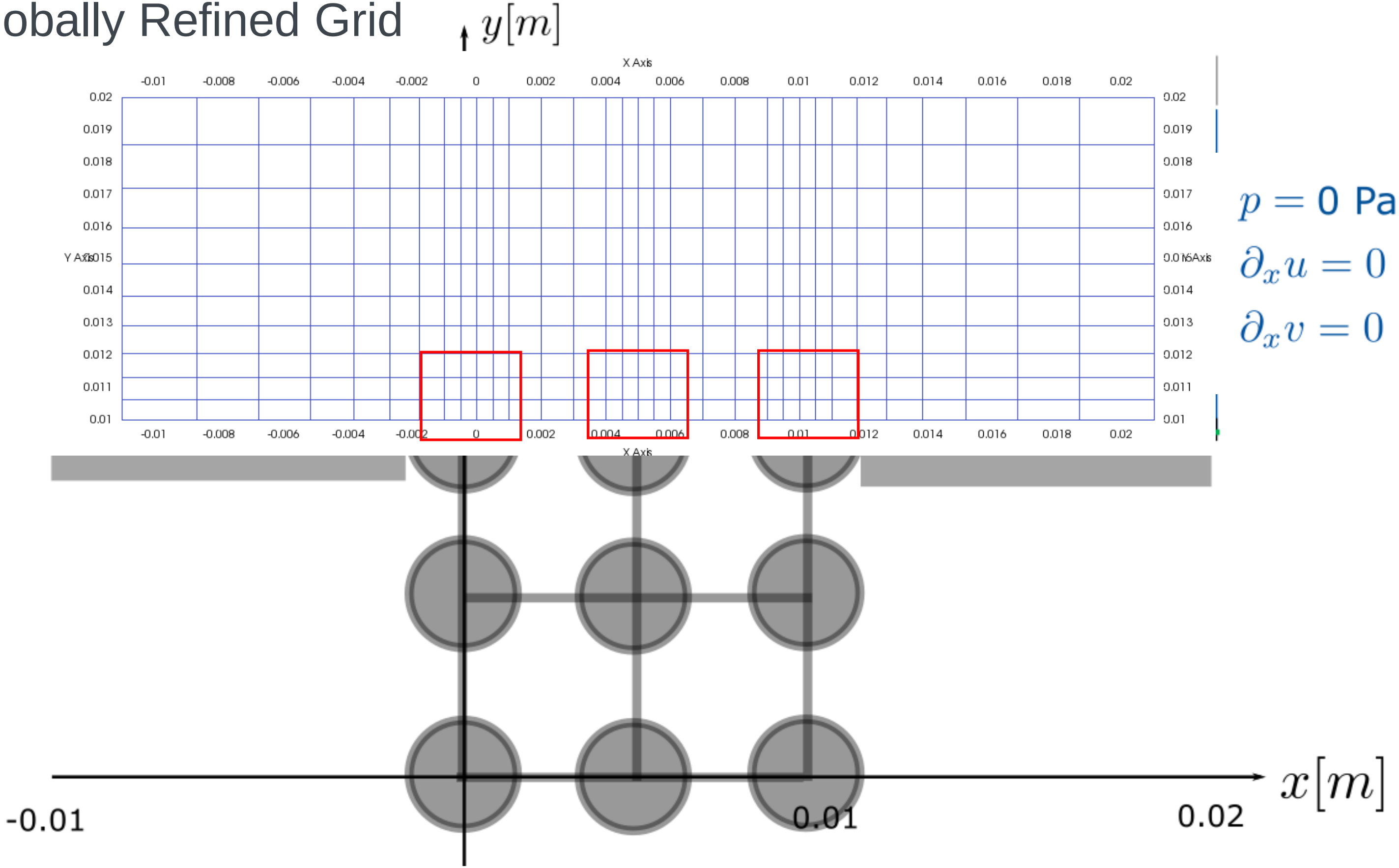
Freeflow:

- Stationary
- Laminar Stokes flow (Re = 0.1)
- 2D
- Single Phase (Gas)
- 1 component
- Isothermal
- no gravity
- Constant density (1e3)/

Kinematic viscosity(1e-6)

Coupling to Porous Medium (Pore Network)

Globally Refined Grid



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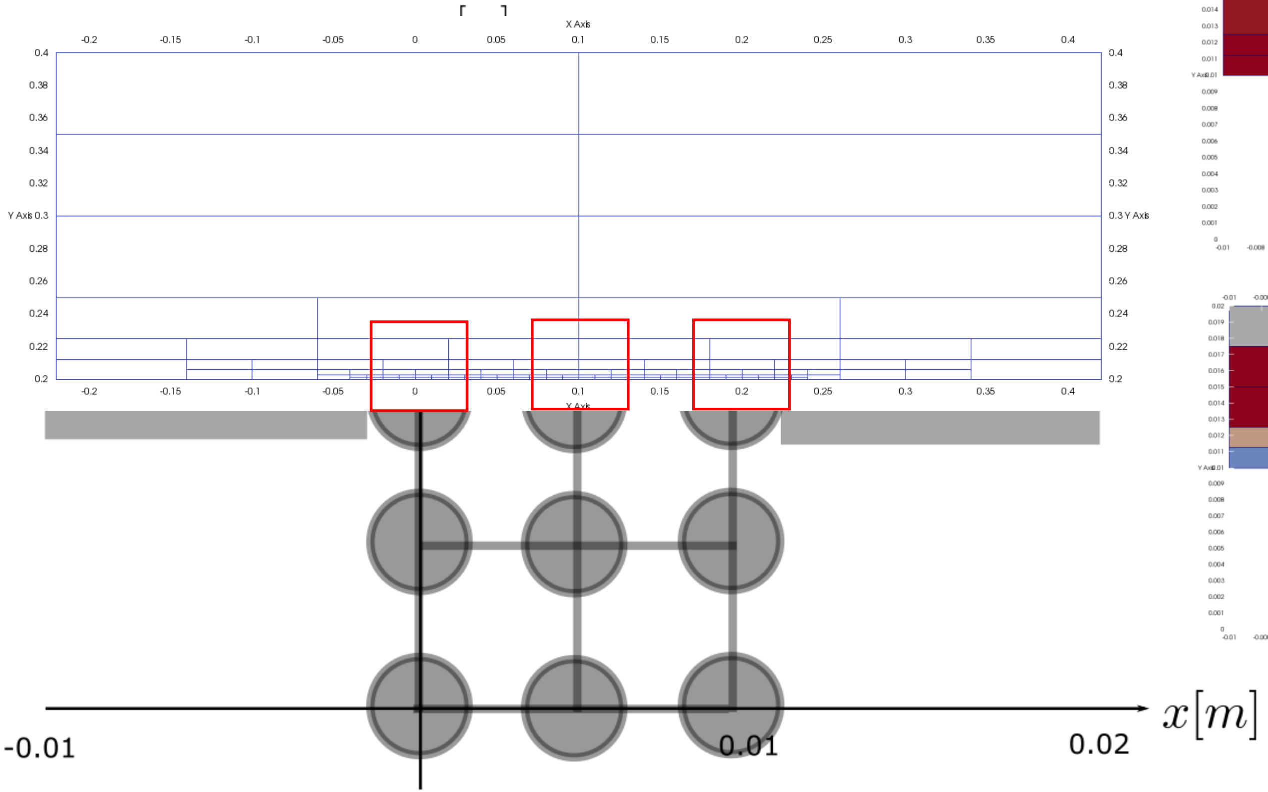
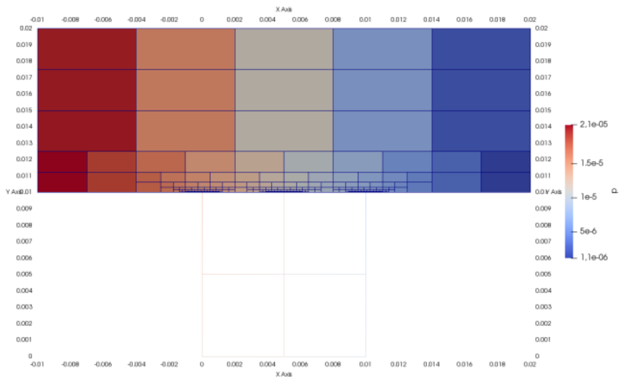
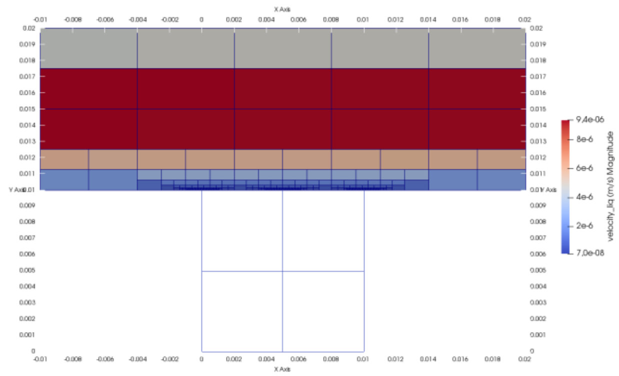
Coupling to Porous Medium (Pore Network)
Locally Refined Grid







Coupling to Porous Medium (Pore Network)

Results

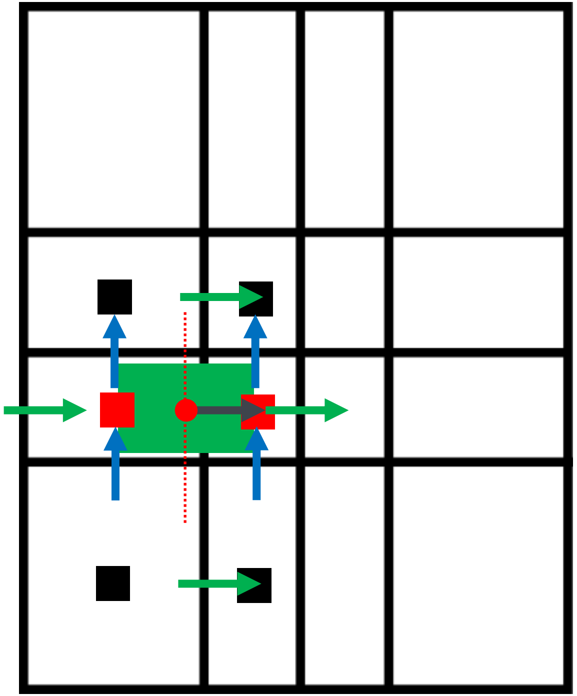
Sum over Pore Bodies

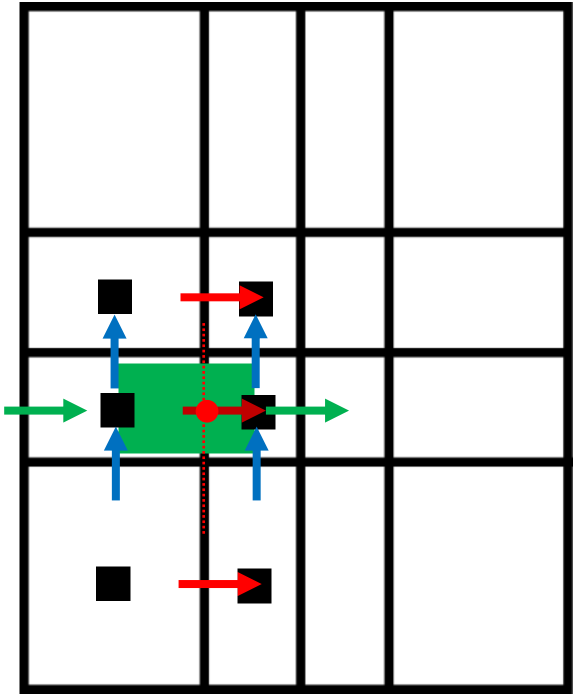
$$\sqrt{\sum_{i=0}^{N} \left(p_{i,\text{this grid}} - p_{i,120\times 40\text{ grid}}\right)^2}$$

	CC Dofs	Face Dofs	Deviation from 120x40 free-flow grid
Uniform 30x10	300	640	2.89e-6
Globally refined	300	640	3.01e-6
Locally refined	299	712	4.81e-6

Discussion

Distortion



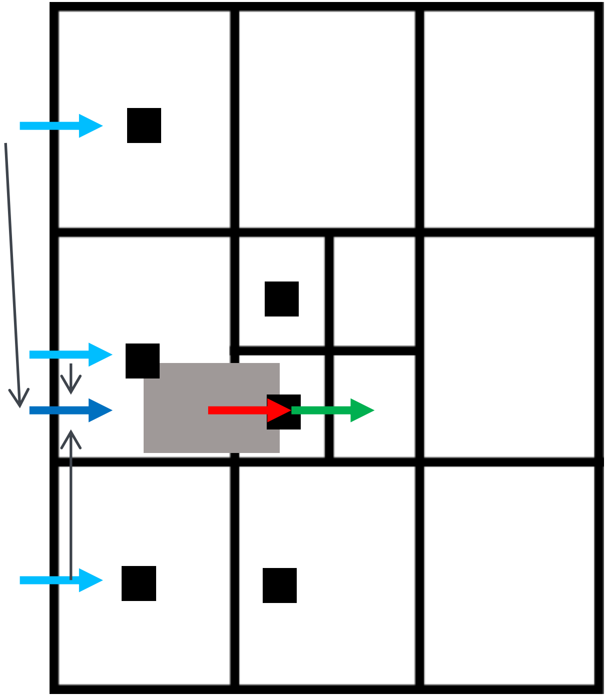


Local Truncation Errors

- Grading: Superconvergence
- Local Refinement:

$$\frac{\mu\left(x_r, y_c\right)\partial_x u\left(x_r, y_c\right) - \mu\left(x_l, y_c\right)\partial_x u\left(x_l, y_c\right)}{\Delta x} =$$

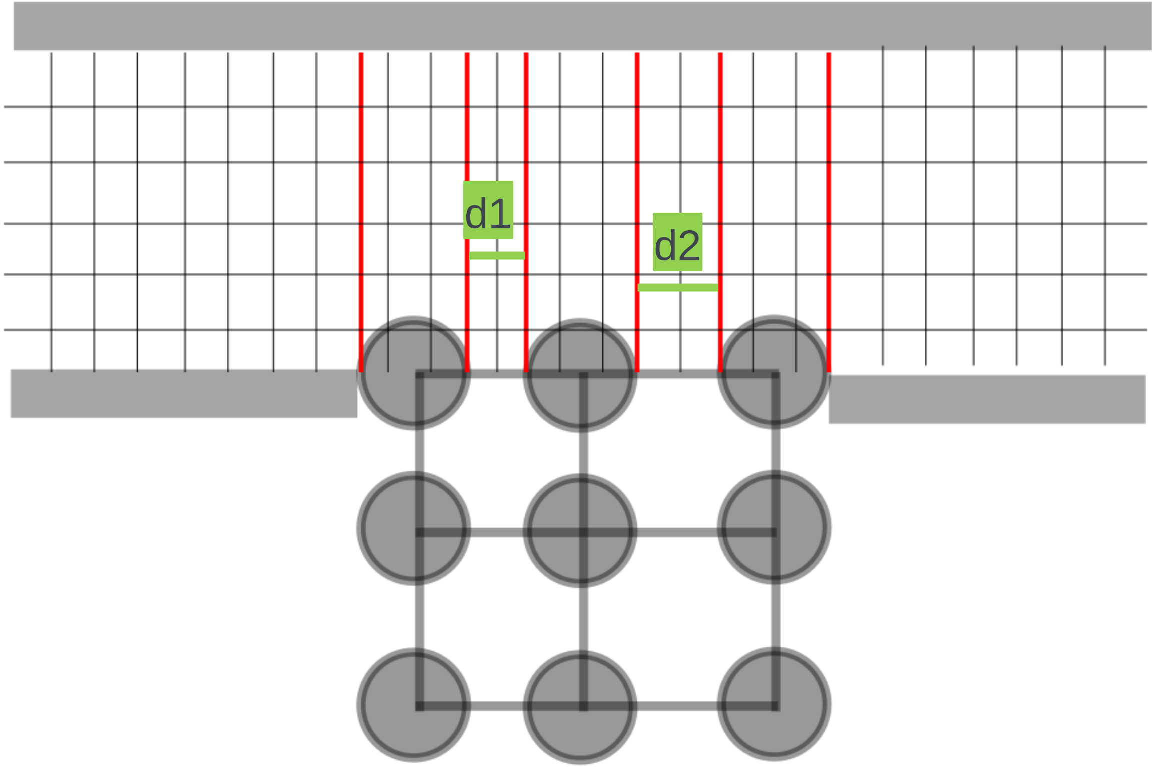
$$\frac{\mu_r}{\Delta x \Delta x_r} u_{rr} - \left(\frac{\mu_r}{\Delta x \Delta x_r} + \frac{\mu_l}{\Delta x \Delta x_l}\right) u_c + \frac{\mu_l}{\Delta x \Delta x_l} u_{ll} + \mathcal{O}\left(\Delta\right)$$



See also [van der Plas, P. (2017). Local grid refinement for free-surface flow simulations. [Groningen]: Rijksuniversiteit Groningen.]

Occasionally Graded Grid Needed

Pore-Network Model – Free-Flow



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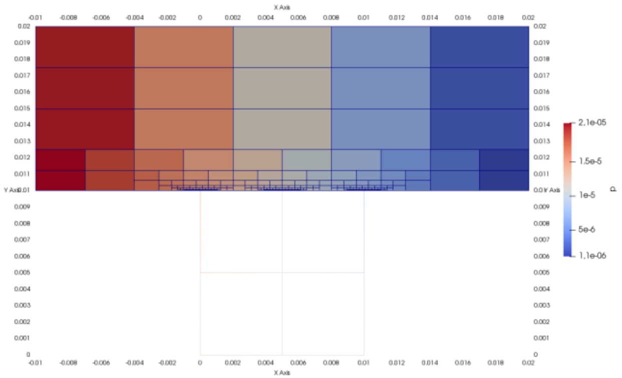
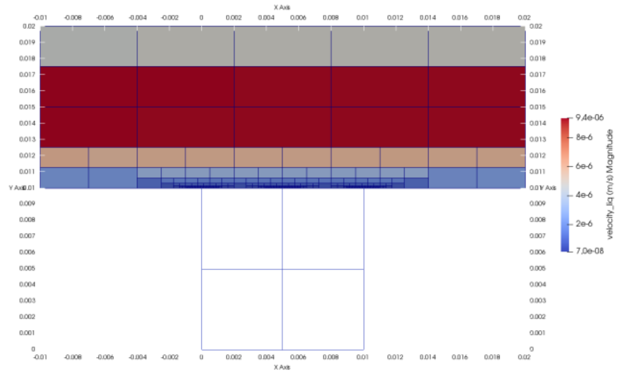
Summary

Summary





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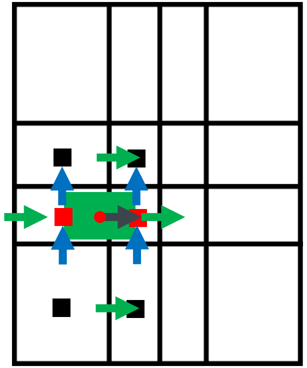
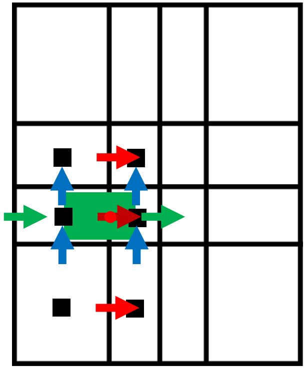
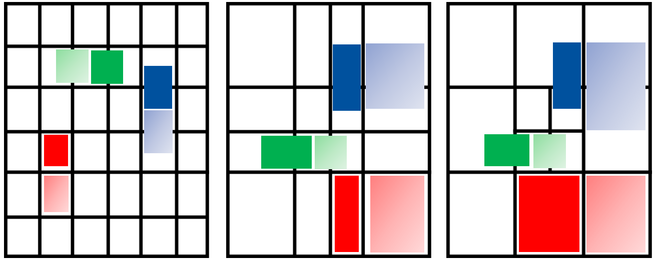




Thank you!



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