NON-STATIONARY FLOOD FREQUENCY ANALYSIS IN SOUTERN GERMANY

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ABSTRACT

The paper proposes an implementation of an extended Gumbel distribution and Log-Pearson III (LP3) distribution respectively in a non-stationary extreme discharge study. Three types of time dependant functions are proposed for the statistical distribution parameters. Among the three types, a modified logistic regression function is applied to the Gumbel scale parameter as well as the scale and shape parameters of the LP3. Simulated Annealing is employed as optimization algorithm for parameter estimation towards an exploration of the maximum likelihood. Based on the extended Gumbel and LP3 distributions, significance tests and trend analysis are carried out through bootstrap re-sampling. Discharge data, made up of annual maxima obtained from ten gauging stations located in southern Germany, is used as a case study.

The results demonstrate satisfactory non-stationary parameter fitting and flood estimation using both extended distribution functions. The study is an attempt to provide an alternative approach for a more reliable estimation of the design return flood for engineering purposes. Through the extended non-stationary setting, the study gives an impression of the impact of climate/landuse change on flood occurrences and magnitudes. It can also serve as a useful tool for studying climate change scenarios along with climate model simulations.

1. INTRODUCTION

Extreme events assume different dimensions when one considers worldwide millions of people are negatively affected by natural disasters resulting in high loss of live and billions of dollars in property damage. Be it floods, hurricanes and earthquakes, extreme events will have to be properly dealt with to protect human life and property, especially in the context of climate change.

Design discharge for hydraulic structures can be computed from extreme value statistics with the conventional assumption that flood frequency is constant and stationary in the past and in the future. Unfortunately, this is not the case in reality. Changes such as landuse change are inevitable. The climate is also suspected to be undergoing change particularly in the recent decades.

A common practice in flood frequency analysis is to select a statistical distribution function for the time series of annual extreme discharges. The distribution parameters can be estimated through procedures such as method of moments, probability weighted moments (Greenwood et al.,

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1979) and maximum likelihood method. Finally, the design discharges for a certain return period can be computed based on the selected statistical distribution function and its estimated parameter values. Unavoidably, the prediction of an ***n***-year return flood is always beyond the range of data used for parameter estimation (Khaliq et al., 2006). This is the business-as-usual approach. The implicit premises in this approach are that the extreme events are independent from each other and the climate remains the same in the past and the future, in particular, during the design life of a hydraulic structure.

The premise that floods are independent and identically distributed in time is at odds with the recognition that climate naturally varies at all scales and may be responding to human activities, which might have changed the climate forcing and perhaps the hydroclimatic response on regional scales in recent decades (NRC 1999). In addition, some apparent hydro-meteorological regime changes may be due to changes in instrument type or faulty instrumentation or changes in exposure (Smith, 1981; Palutikof et al., 1984; Dahman and Hall, 1990, Khaliq et al., 2006). Change in the flood process comes mainly from naturally structured, low frequency climate variability and from human changes to the watershed (NRC 1999). The structured, oscillatory, interannual- to millennia-scale climate variability (NRC 1999) is also referred to as long-term persistence or memory in the system (Bunde 2005). The body of literature on climate change is on the increase. An example is the IPCC report (2001) which demonstrated that hydro-meteorological regimes will likely be modified significantly over the next 50-100 years. Over the course of the 20th century the average surface air temperature increased by almost 1 C in Europe (EC 2005). Milly et al. (2005) demonstrated 10–40% increases in runoff in eastern equatorial Africa with an ensemble of 12 climate models. Regardless of the direction in which the change is heading towards to and/or for how long this change is expected to persist, the climate system would have stationary dynamics in the long term, but a finite period may exhibit apparent non-stationarity in terms of the statistics (NRC 1999).

Non-stationarity has been taken into consideration since almost a century ago. The open questions that remain are whether these changes affect extreme discharges and if so, how one takes the changes into consideration in the estimation of design discharges. Enormous work has been undertaken in this field. Motivation is ever being heated to diagnose the underlining questions and replace the existing static flood risk framework with a dynamic one (NRC 1999).

This paper studies the temporal change within the time series of extreme discharges. Based on the assumption of non-stationarity in the extreme discharges, the paper proposes an implementation of an extended time-dependant Gumbel and Log-Pearson III (LP3) distribution respectively.

2. STUDY AREA AND METHODOLOGY

2.1 Study area

Discharge data made up of annual maxima obtained from ten gauging stations located in southern Germany (see Figure 1) is used as a case study. The average length of the data records is 95.9 years. All records are available till 2004 (refer to Table 1 for details).

The study area lies in a temperate climatic zone that is predominantly continental. Annual precipitation ranges from 500 mm to about 1500 mm. In the south of Bavaria in the alpine regions precipitation may exceed 2000 mm. Of the ten gauges used in the study, nine show peak flood occurrence in winter. Wet or/and frozen ground leads to a high discharge factor hence long-duration precipitation events eventually lead to high discharges. An exception is the gauge Passau where peak discharges are caused by spring snow melt in the Alps.

Table 1 Description of the time series for the 10 gauging stations

Ind. Nr.	Federal state	Gauges	Tributaries	Start Year	End Year	NAHQ[3]
01	BW[1]	Bad Rotenfels	Murg	1883	2004	121
02	BW	Gerbertshaus	Schussen	1920	2004	85
03	BW	Oberlauchringen	Wutach	1913	2004	92
04	BW	Schwaibach	Kinzig	1882	2004	123
05	BW	Stein	Kocher	1885	2004	120
06	BY[2]	Donauwörth	Donau	1924	2004	81
07	BY	Hofkirchen	Donau	1901	2004	104
08	BY	Kemmern	Main	1931	2004	70
09	BY	Passau (Ingling)	Inn	1921	2004	84
10	BY	Wolfsmünster	Fränkische-Saale	1931	2004	69

1. BW – the state of Baden-Wuerttemberg, Federal Republic of Germany
2. BY – the state of Bayern, Federal Republic of Germany
3. NAHQ – the number of annual maximum discharges

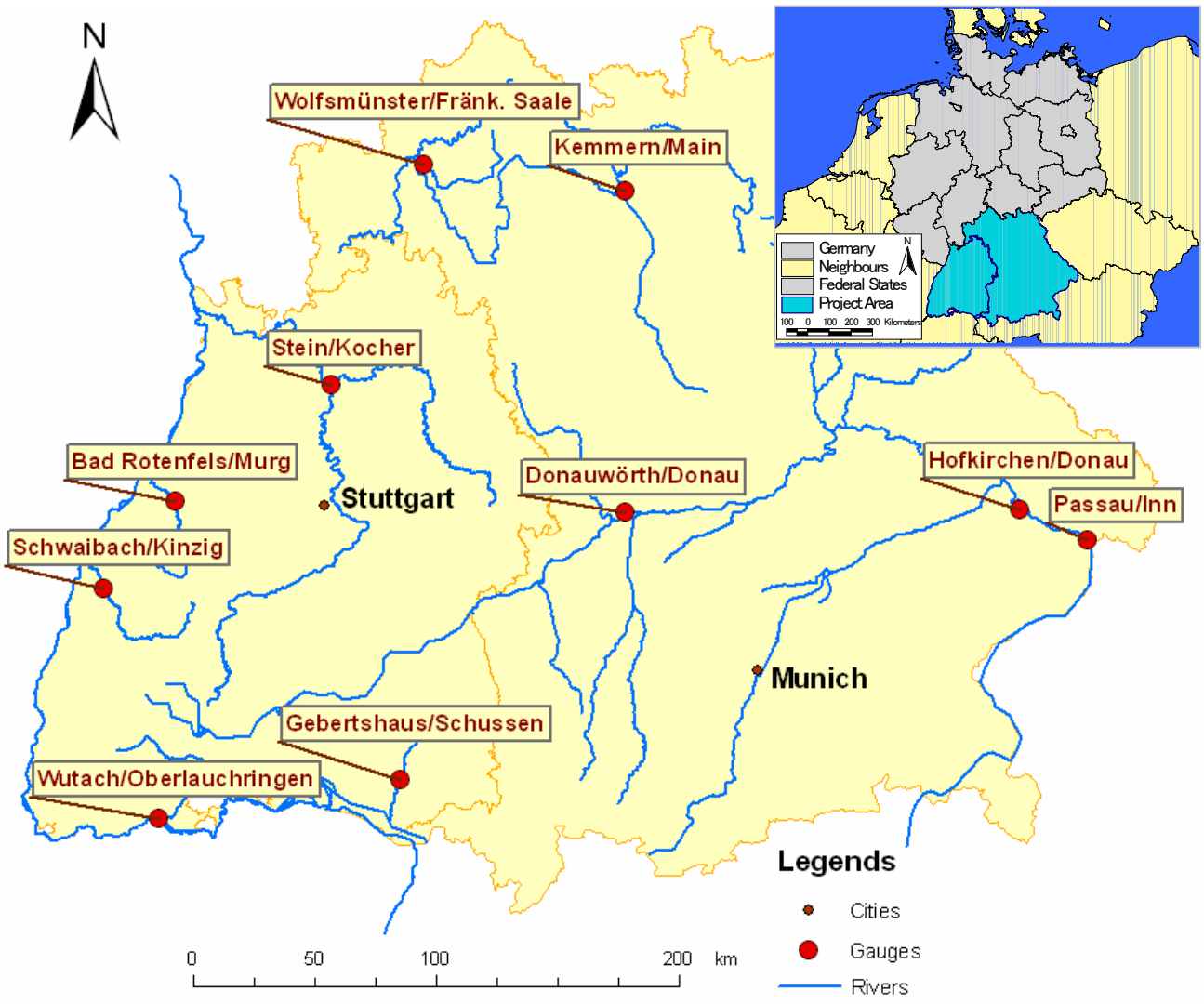


Figure 1 The 10 gauging stations in Southern Germany

2.2 Methodology

Conventional technique of flood frequency analysis is in principal flawed or imperfect with respect to its two basic assumptions. First, it ignores the presence of non-stationarity in the time series when estimating design values for future time horizons (Cunderlik and Burn 2003). Secondly, it assumes dependence among sample observations which shall not be rigorously applied. Khaliq et al. (2006) argued that a sample of n correlated observations gives less information than a sample of n independent observations. Eichner et al. (2006) showed that the observations are not independent, but the correlations in real river runoff data are rather weak and Q_{100} is still a good estimator for centennial floods. This paper assumes independence and non-stationarity in the annual maximum discharge observations.

Khaliq et al. (2006) reviewed mostly available and documented methods up to the year 2005 on frequency analysis of a sequence of dependent and/or non-stationary hydro-meteorological observations. In the review, five schemes are categorized to incorporate non-stationarity in the conventional technique (Khaliq et al. 2006):

1. Extremal, r-largest, peak-over-threshold (POT) and point process models with covariates. In principal, these Extremal, r-largest, peak-over-threshold (POT) and point process models are different only in the way the extreme records are taken from the

entire time series. For example, the POT takes all records above a certain threshold. While the extremal model (Fisher and Tippett, 1928) takes the maximum/minimum from a certain time sequence (e.g. year) and composes a new time series. The latter is very much limited by the length of the new time series since only one value is taken from one time sequence and records of various time sequences are not always available in many locations. The other methods allow further information provided by the records in addition to the maxima/minima. A vector of covariates is assumed for the parameters of the Generalized Extreme Value (GEV) distribution or other extreme value distribution functions. Instead of estimating the value of the parameters of the distribution functions, the variables associated with the covariates are to be estimated using the maximum likelihood method.

2. Time-varying moments. Strupczewski et al. (2001) applied linear and square trinomial (parabolic) regression functions to the first two moments (mean and variance) for six distribution functions. 56 combinations of regression settings were tested and compared. Finally, the Akaike Information Criterion (AIC) was used to identify the optimum distribution and regression function. The difference between the time-varying moments and the first scheme lies in the means the trend is accounted for. The latter associates the trend with the distribution parameters while the former with the first two moments. Both schemes recognize the existence of non-stationarity and demonstrate the possibility to take a range of forms of trend into consideration. Khaliq et al. (2006) placed them as covariate-based methods and recommended the appropriateness of the governing functions be investigated further.
3. Non-stationary pooled flood frequency analysis. Cunderlik and Burn (2003) outlined a second order non-stationary approach to pooled flood frequency analysis. The method separates the non-stationary pooled quantile function into a local time-dependent component (the location and scale parameters) and a regional component (time-dependent second-order non-stationarity). This method was applied to annual maximum floods of a group of catchments from the South British Columbia Mountains Climate region which is a roughly homogeneous region.
4. Local likelihood approach. Ramesh and Davison (2002) proposed a local likelihood approach which is classified as a semi-parametric approach. The distribution parameters are assumed a linear polynomial function of time $\boldsymbol{t}$ with close time points being given more weights. The weight takes a symmetric function known as a kernel. The maximum likelihood method is used for parameter estimation. This approach can be further extended if covariates other than time $\boldsymbol{t}$ are introduced into the parameter function. New covariates that can be introduced are climatic indices (Sankarasubramanian and Lall, 2003).
5. Quantile regression method. Koenkar and Basset (1978) defined the ***pth*** conditional quantile through a linear/non-linear function of a vector of covariates. In addition to the first term, an error term $\boldsymbol{\varepsilon_p(V)}$ is added to the quantile. The usefulness and philosophy of the quantile regression and/or censored quantile regression model were summarized by Buchinsky (1998). Sankarasubramanian and Lall (2003) noted that the local likelihood approach with covariates is relatively superior to the quantile regression approach with respect to predictability of various conditional quantiles.

In addition to the five schemes, extensive efforts have also been made to relate hydro-meteorological extremes with low frequency climatic indices (Khaliq et al., 2006) to improve the efficacy of the governing regression function.

This paper aims to contribute to the first non-stationary scheme and furthers exploration of the possible regression functions to incorporate non-stationarity into the distribution parameters. Three extended Gumbel distribution function settings are introduced to account for the observed non-

stationarity. The three-parameter Log-Pearson 3 (LP3) distribution function is then implemented as a more flexible distribution function to achieve a better fitting to the time series. Similarly, three extended LP3 distribution function settings are formulated to incorporate the temporal changes of extreme floods.

The primary source of the trend information is the time series of the data described in Section 2.1. To get a first impression of the mean trend, a 30-year moving average is taken for the Gumbel location parameter X_0 and scale parameter λ (their values are estimated through the moment method) and the mean annual maximum discharges. The result obtained from the gauge Gerbertshaus is shown in Figure 2. The Gumbel location parameter X_0 shows a fairly constant increase over the entire record length. On the contrary the scale parameter λ does not show a constant increase but rather a kind of oscillation within a certain range. The annual maximum discharge follows a very similar trend as the X_0. The other gauges have demonstrated similar behavior as well. A straightforward way is to apply a linear function of time t to X_0 (see Equation 1) and a logistic function to λ (see Equation 2). The logistic function actually restrains the estimated value within a range that is based on the moving average or can be artificially defined. It is worth noting that the range is very subjective depending on the length of the moving window and the trend expectation into the near future.

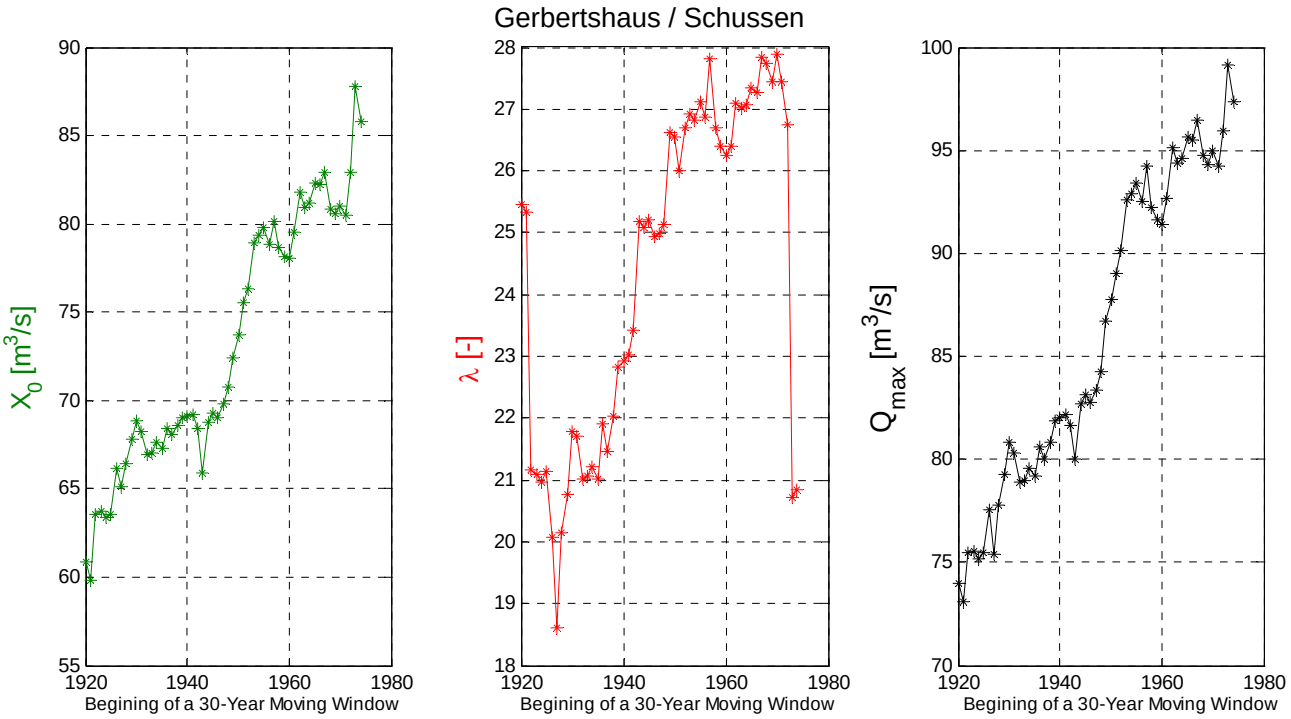


Figure 2 Extreme discharges and stationary Gumbel parameters in a 30-year moving window

As described in Equation (1)-(3), a linear function is implemented to the location parameter X_0. A logistic regression function is modified and applied to the Gumbel scale parameter λ as well as the scale parameter λ and shape parameter r of the LP3. Based on the linear and logistic regression functions, six non-stationary settings are established for Gumbel and LP3 distributions (Table 2). Simulated Annealing (Aarts & Korst 1989) is employed as the optimization algorithm for parameter estimation towards an exploration of the maximum likelihood.

$$X_0(t) = b + a \cdot t \tag{1}$$

$$\lambda(t) = \lambda_{\min} + \frac{\lambda_{\max} - \lambda_{\min}}{1 + c_\lambda \cdot \exp(-d_\lambda \cdot t)} \tag{2}$$

$$r(t) = r_{\min} + \frac{r_{\max} - r_{\min}}{1 + c_r \cdot \exp(-d_r \cdot t)} \tag{3}$$

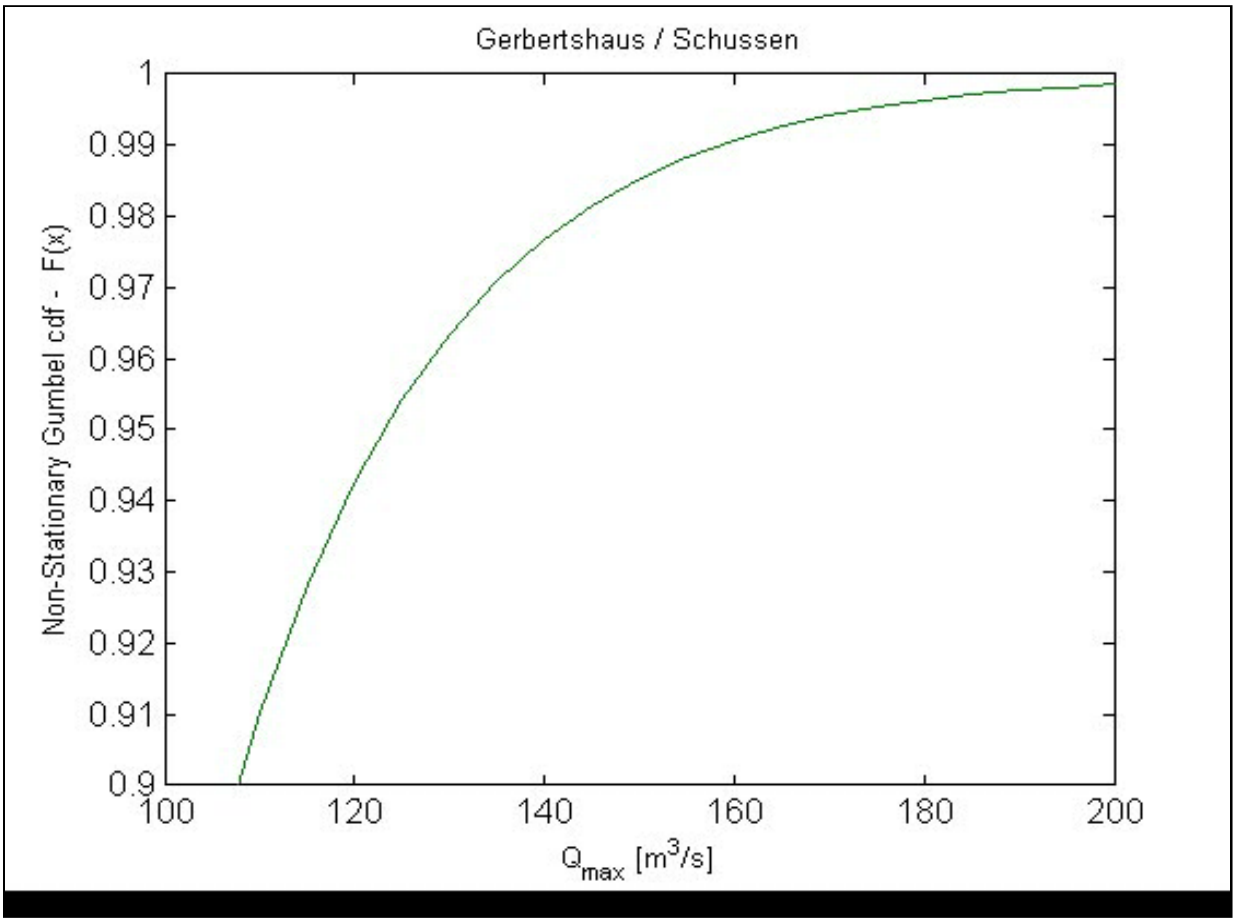
Animation 1 illustrates the variation of the Gumbel CDF based on the non-stationary GD I, GD II and GD III setting. The level of sensitivity of the location X_0, scale λ and shape r parameters of the LP3 distribution is shown in Animation 2. The shape parameter r shows the highest sensitivity, followed by the scale λ parameter. A same magnitude of variation of the r will have significant deviation with respect to the ***n***-year return flood which indicates that the shape parameter should be given special care when assigning a regression function to guarantee a mild change.

Based on the extended distribution functions, significance tests and trend analysis are carried out through bootstrap re-shuffling. The steps are numerated as follows:

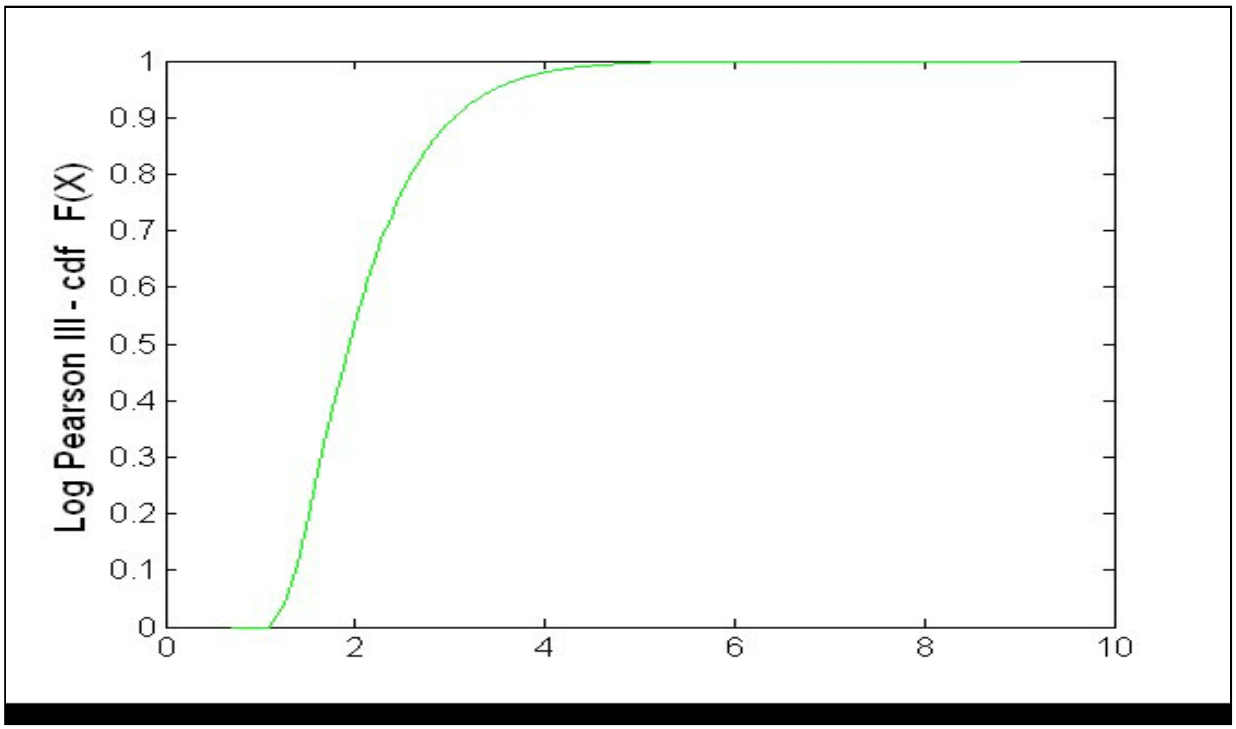
1. Randomly reshuffle time series from the original data;
2. Estimate the parameters and discharges $\mathbf{HQ100_R}$ for Type I through Type III for any year in the near future (the case study uses year 2030);
3. Compare them with the parameters and discharges $\mathbf{HQ100_O}$ estimated for the same year with the original data;
4. Repeat steps 1. to 3. for a minimum of 1000 times;
5. If more then 10 % of the $\mathbf{HQ100_R}$ are higher than $\mathbf{HQ100_O}$, there is no significant positive trend and vice versa.

Table 2 Six non-stationary settings established for Gumbel and LP3 distributions

Distribution Type	Parameter functions	Parameters to estimate
Gumbel Distribution: $F(x)=\exp\left(-\exp\left(-\frac{x-X_0}{\lambda}\right)\right)$		
GD I	$\lambda=$ constant; $X_0(t)=at+b$	λ a b
GD II	$\lambda(t)=\lambda_{\min}+\frac{\lambda_{\max}-\lambda_{\min}}{1+c_\lambda\cdot\exp(-d_\lambda\cdot t)}$ $(\lambda(t)>0)$ $X_0=$ constant	c_λ d_λ X_0
GD III	$\lambda(t)=\lambda_{\min}+\frac{\lambda_{\max}-\lambda_{\min}}{1+c_\lambda\cdot\exp(-d_\lambda\cdot t)}$ $(\lambda(t)>0)$ $X_0(t)=at+b$	a b c_λ d_λ
Log Pearson 3 Distribution: $F(x)=\int_{X_0}^{x}\frac{(\ln u-X_0)^{r-1}}{u\cdot\lambda^r\cdot\Gamma(r)}\exp\left(-\frac{\ln u-X_0}{\lambda}\right)du$		
PD I	$\lambda=$ constant; $r=$ constant; $X_0(t)=at+b$	λ a b
PD II	$\lambda(t)=\lambda_{\min}+\frac{\lambda_{\max}-\lambda_{\min}}{1+c_\lambda\cdot\exp(-d_\lambda\cdot t)}$ $(\lambda(t)>0)$ $X_0=$ constant, $r=$ constant	c_λ d_λ X_0 r
PD III	$r(t)=r_{\min}+\frac{r_{\max}-r_{\min}}{1+c_r\cdot\exp(-d_r\cdot t)}$ $(r(t)>0)$ $X_0=$ constant, $\lambda=$ constant	c_r d_r X_0 λ



Animation 1 Variation of the Gumbel CDF (upper tail) based on the non-stationary GD I (in green), GD II (in red) and GD III (in blue) setting (click image to start animation)



Animation 2 Parameter-driven variation of the LP3 CDF showing the sensitivity level of three parameters: constant increment of location X_0 (in green), constant increment of scale λ (in red) and constant increment of shape r (in blue) (click image to start animation)

3. RESULTS

The results demonstrate satisfactory non-stationary parameter fitting and return flood estimation using the extended distribution functions. An example is shown in Figure 3. A comparison is made between Gumbel and LP3 with respect to the magnitude of the estimated return flood. For most of the gauging stations, a comparison of Gumbel with LP3 shows a larger deviation of the return flood from the stationary to the non-stationary setting. Non-stationary settings usually lead to a considerable difference with respect to the ***n***-year return flood, which indeed proves the need to take non-stationarity into consideration. A striking point that is marked by Figure 4 is that the level of stationary estimation is almost equivalent as the level of non-stationary estimation for year 1940 (one can observe an overlap of the red line and green line). Figure 5 shows the change ratio of estimated ***n***-year return flood for 2030 between Gumbel non-stationary setting and stationary setting. The difference is calculated based on the Equation (4). The significance of the change is also shown in Figure 5.

$$change = \frac{(n-\text{year Return Flood}_{\,nonstationary}) - (n-\text{year Return Flood}_{\,stationary})}{(n-\text{year Return Flood}_{\,stationary})} \tag{4}$$

Figure 6 shows the ratio of ***100***-year return flood estimated for 2000 and 2030 between the non-stationary GD III and stationary setting. This figure has specific implications for designing purposes. A single safety factor can be assigned in order to allow a safety margin to be considered for designing hydraulic structures in a region or watershed. Such hydraulic structures would be placed in a better position as they are expected to prevent failure from extreme flooding caused by the ever changing climate and beyond.

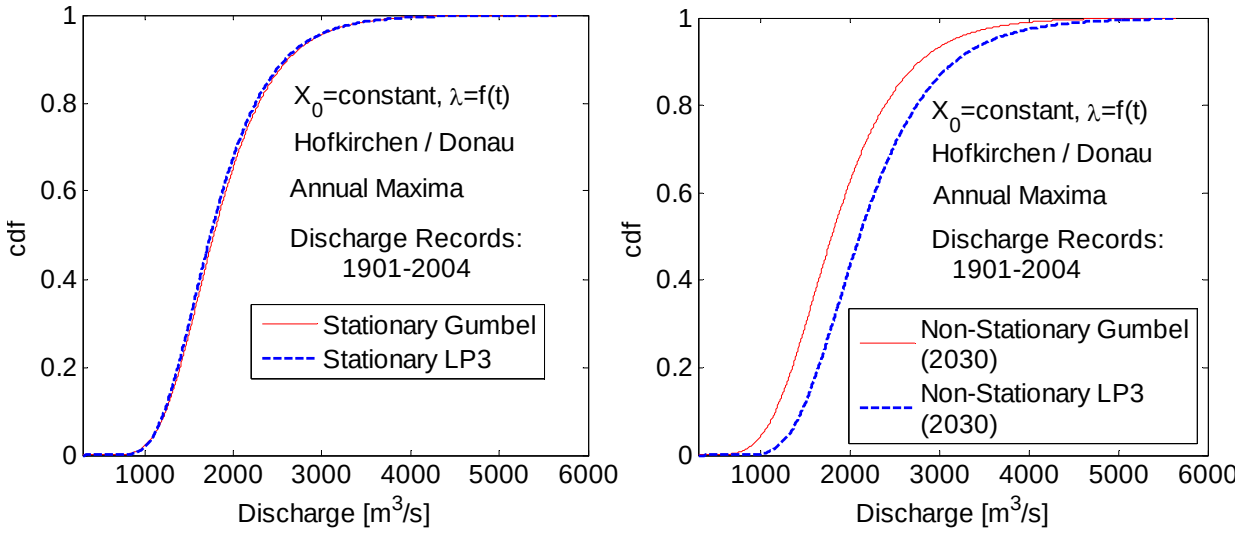


Figure 3 Comparison of the stationary and non-stationary distribution fitting

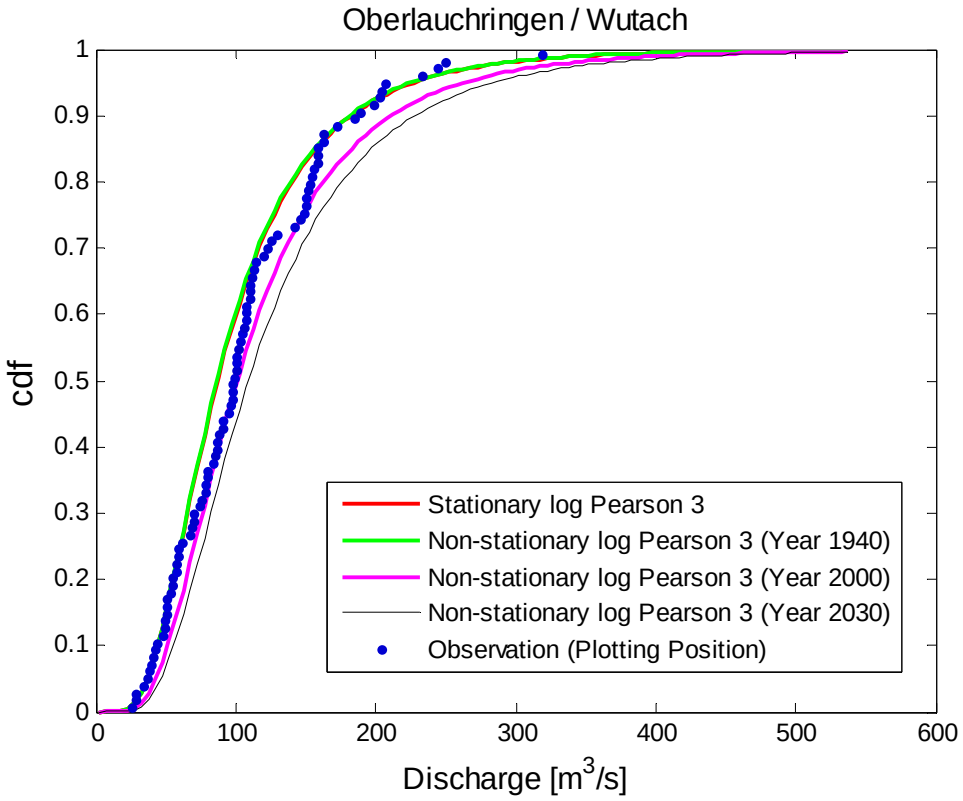


Figure 4 Comparison of stationary and PD I non-stationary LP3 CDF

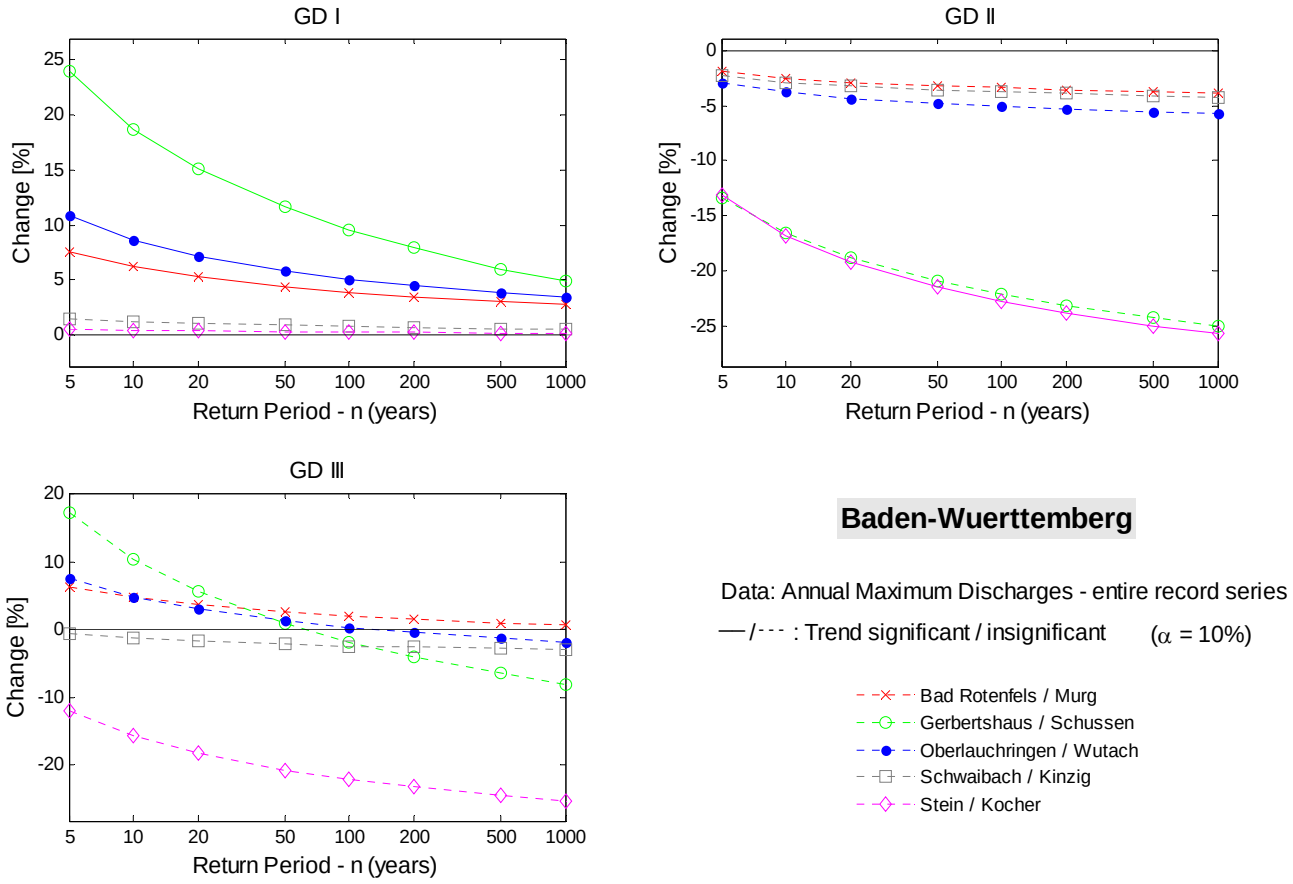


Figure 5 Change ratio between the non-stationary (2030) and stationary return flood and its significance

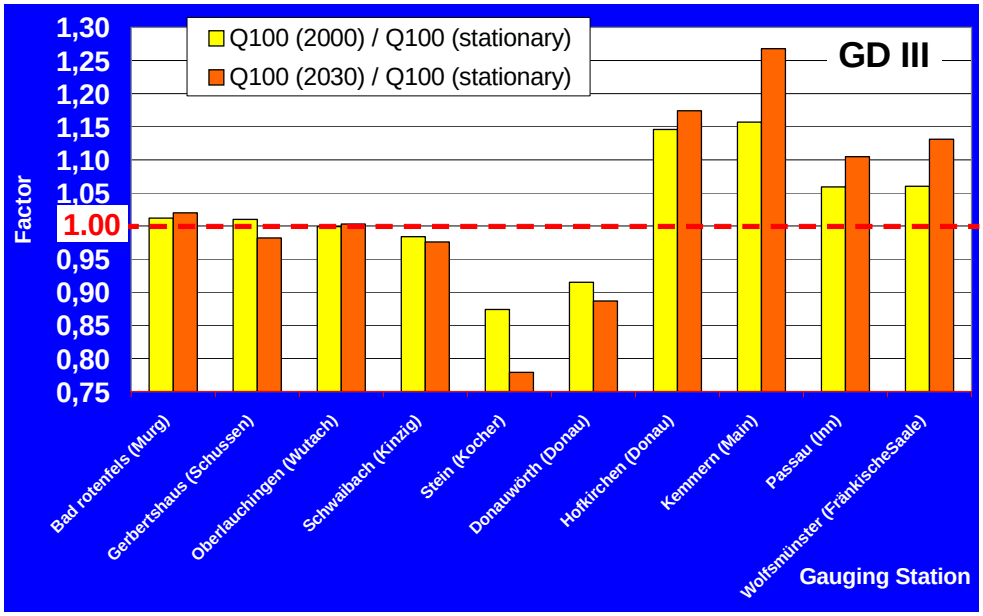


Figure 6 The ratio of ***100***-year return flood estimated for 2000 and 2030 between the non-stationary GD III and stationary setting

Many of the 10 gauging stations show increasing design discharges. An interesting question that needs to be answered is whether this increasing trend is persistent and significant. As described earlier, significant tests and trend analysis are carried out through bootstrap re-shuffling. Two examples of significant and insignificant positive trend are provided in Figure 7. Table 3 summaries the trend for all 10 gauging stations under the five non-stationary settings. PD III is excluded from this table due to the difficulties in the estimation of skewness and hence the shape parameter. The table shows various trend patterns with different non-stationary settings. This is due to the fact that any climate trend is not necessarily reflected in the location, scale, shape parameter or a combination of two/three of them. A trend or its extent could differ by study locations and the concerned time period. Therefore it is hard to use any single regression function unless a detailed investigation in the climatic trend is conducted. In addition, apart from the covariate time ***t***, climatic variables could be considered in the regression function to establish a link to the climate system.

Table 3 Summary of the trend for all 10 gauging stations under the 5 non-stationary settings (1 - significant trend; 0 - insignificant trend)

Gauging station / River	GD I	GD II	GD III	PD I	PD II
Bad Rotenfels / Murg	1	0	0	0	0
Gerbertshaus / Schussen	1	0	0	1	0
Oberlauchringen / Wutach	1	0	0	1	0
Schwaibach / Kinzig	0	0	0	0	1
Stein / Kocher	0	1	0	1	0
Donauwörth / Donau	1	0	0	1	1
Hofkirchen / Donau	1	0	1	1	1
Kemmern / Main	0	0	1	0	0
Passau / Inn	0	0	0	1	0
Wolfsmünster / FränkischeSaale	1	0	0	1	1

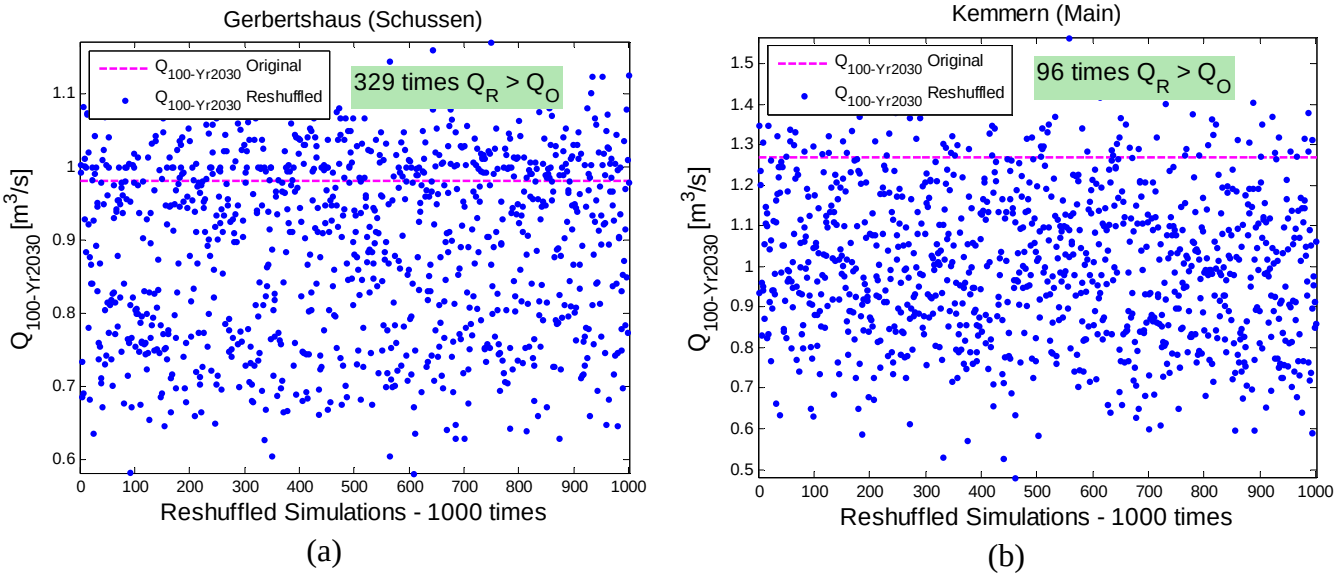


Figure 7 Insignificant (a) and significant (b) positive trends analyzed through bootstrapping test

4. DISCUSSION AND OUTLOOK

The study has clearly demonstrated that it is not only necessary but also feasible to introduce non-stationarity to extreme flood frequency analysis. The primary source of information about stochastic properties of hydrological processes is the time series of data (Strupczewski et al., 2001). Basically, the regression approach lets the data speak for itself. It would be more realistic and comprehensive if a linkage could be established between the climate system and the distribution parameters. Multivariate models could be considered in the future work.

Non-stationary Gumbel functions show a satisfactory fitting. A better fitting (i.e. a higher likelihood value) is achieved by implementing LP3 distribution functions. And it is no doubt that the non-stationary likelihood value can be further optimized if more free variables are introduced. But as to the validity of any extrapolation into our near future, the latter is not at all guaranteed. Assessment of uncertainty still proves to be an urgent and important issue if non-stationary flood frequency analysis is to be factored for designing purposes.

A longer time series into the past might not be useful if most changes began later. The near future is always assumed to behave similarly as the recent past. However, we are dealing with stochastic nature and one can never be absolutely certain that the previous weather condition is similar as the weather condition in the next moment. Randomness will always be a disturbing factor. It is close to a dilemma, unfortunately, without a complete understanding of climate change - local and seasonal, spatial and temporal - this question cannot be solved. To incorporate knowledge from climate change research will help to determine if there is a trend, when it will take place and when it might end.

A regional trend analysis is hoped to be conducted if more gauging stations are provided. The regional trend analysis can help stakeholders as well as designers in a region to make sound and informed decisions to prevent from failures of hydraulic structures and losses of live and property.

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