Non-stationary flood frequency analysis in Southern Germany

Yi He, András Bárdossy, Jürgen Brommundt

Institute of Hydraulic Engineering, Universität Stuttgart
Motivation

• Design discharge for hydraulic structures is calculated from extreme value statistics:
  – Basic assumption: stationarity

• Several climate variables show a change

  • Do extreme discharges change?
  • How can the change be considered in the estimation of design discharges?
Approach

• Stationary Gumbel / Pearson Distribution

• Non-Stationary Gumbel / Pearson Distribution

• Trend Significance Test → Bootstrapping
Study Domain

- Annual
- Winter
- Summer
- $Q_{\text{max}}$
- until 2004

Legend:
- Cities
- Gauges
- Rivers

Germany Neighbours

Project Area

0 50 100 150 200 km

Rivers
Gumbel Distribution Function

\[ F(x) = \exp \left( -\exp \left( -\frac{x - X_0}{\lambda} \right) \right) \]

Two Parameters:

Location \( X_0 \)
Scale \( \lambda \)
Stationary Parameters in a 30-year Moving Window

\[ F(x, t) = \exp \left( -\exp \left( -\frac{x - X_0(t)}{\lambda(t)} \right) \right) \]

\[ X_0(t) = a \cdot t + b \]

\[ \lambda(t) = \lambda_{min} + \frac{\lambda_{max} - \lambda_{min}}{1 + c \cdot \exp(-d \cdot t)} \]
Non-Stationary Gumbel Distribution

Gumbel Distribution Type

<table>
<thead>
<tr>
<th>GD I</th>
<th>Parameters to estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = \text{constant}; \ X_0(t) = at + b )</td>
<td>( \lambda \quad a \quad b )</td>
</tr>
</tbody>
</table>

\[
\lambda(t) = \lambda_{\min} + \frac{\lambda_{\max} - \lambda_{\min}}{1 + c \cdot \exp(-d \cdot t)}
\]

GD II

Maximum Likelihood Method

Simulated Annealing Optimization

<table>
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<th>Parameters to estimate</th>
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<td>( \lambda(t) = \lambda_{\min} + \frac{\lambda_{\max} - \lambda_{\min}}{1 + c \cdot \exp(-d \cdot t)} )</td>
<td>( \lambda(t) &gt; 0 )</td>
</tr>
<tr>
<td>( X_0(t) = at + b )</td>
<td>( a \quad b \quad c \quad d )</td>
</tr>
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<table>
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<th>GD III</th>
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<tr>
<td>( \lambda(t) = \lambda_{\min} + \frac{\lambda_{\max} - \lambda_{\min}}{1 + c \cdot \exp(-d \cdot t)} )</td>
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<tr>
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Non-Stationary Gumbel Distribution

\[ F(x, t) = \exp \left( -\exp \left( -\frac{x - X_0(t)}{\lambda(t)} \right) \right) \]

GD I

GD II

GD III

\[ \lambda(t) = \lambda_{\text{min}} + \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{1 + c \cdot \exp(-d \cdot t)} \]  
\[ (\lambda(t) > 0) \]
\[ X_0(t) = at + b \]
The diagram illustrates the estimated cumulative distribution function (CDF) for the maximum discharge, denoted as $Q_{\text{max}}$, for different years: 1940, 1970, 2000, and 2030. The CDF is shown for different time periods:

- **Estimated CDF (Stationary)**
- **Estimated CDF (1940)**
- **Estimated CDF (1970)**
- **Estimated CDF (2000)**
- **Estimated CDF (2030)**

The discharge values are indicated on the x-axis in m$^3$/s, ranging from 140 to 300. The CDF values are indicated on the y-axis ranging from 0.95 to 1.0. The diagram also highlights the maximum discharge for a recurrence interval of 100 years, labeled as $Q_{\text{max}} (R = 100 \text{ years})$. The location of Gerbertshaus is marked on the diagram.
Pearson Distribution Function

\[ F(x) = \int_{X_0}^{x} \frac{(\ln u - X_0)^{r-1}}{u \cdot \lambda^r \cdot \Gamma(r)} \exp \left( -\frac{\ln u - X_0}{\lambda} \right) du \]

Three Parameters:

Location \( X_0 \)

Scale \( \lambda \) \((\lambda > 0)\)

Shape \( r \) \((r > 0)\)
## Non-Stationary Log Pearson III Distribution

### Pearson Distribution Type

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameters to estimate</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD I</td>
<td>$\lambda$, $a$, $b$</td>
<td>$\lambda(t) = \lambda_{\text{min}} + \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{1 + c_{\lambda} \cdot \exp(-d_{\lambda} \cdot t)}$</td>
</tr>
<tr>
<td>PD II</td>
<td>$c_{\lambda}$, $d_{\lambda}$, $X_0$, $r$</td>
<td>$(\lambda(t) &gt; 0)$</td>
</tr>
<tr>
<td>PD III</td>
<td>$c_r$, $d_r$, $X_0$, $\lambda$</td>
<td>$(r(t) &gt; 0)$</td>
</tr>
</tbody>
</table>

- $\lambda(t) = $ constant; $r = $ constant; $X_0(t) = at + b$
- $X_0 = $ constant, $r = $ constant
Discharge [m³/s]

Oberlauchringen / Wutach

Stationary log Pearson 3
Non-stationary log Pearson 3 (Year 1940)
Non-stationary log Pearson 3 (Year 2000)
Non-stationary log Pearson 3 (Year 2030)
Observation (Plotting Position)

PD I

Annual $Q_{\text{max}}$ [m³/s]
Non-Stationary / Stationary $Q_{100}$

- Bad rotenfels (Murg)
- Gerbertshaus (Schussen)
- Oberlauchingen (Wutach)
- Schwaibach (Kinzig)
- Stein (Kocher)
- Donauworth (Donau)
- Hofkirchen (Donau)
- Kemnern (Main)
- Passau (Inn)
- Wolsmünster (Frankische Saale)

Factor

$Q_{100}$ (2000) / $Q_{100}$ (stationary)
$Q_{100}$ (2030) / $Q_{100}$ (stationary)

Gauging Station

- GD III

Factors range from 0.75 to 1.30.
Non-Stationary / Stationary $Q_{100}$

The diagram compares the non-stationary and stationary Q100 factors for various locations. The y-axis represents the factor, ranging from 0.75 to 1.30, with a dashed red line at 1.00 indicating the stationary condition. The x-axis lists gauging stations such as Bad rotenfels (Murg), Gerberghaus (Schussen), Oberlaufingen (Wutach), Schwaibach (Kinzig), Stein (Kocher), Donauworth (Donau), Hofkirchen (Donau), Kemnern (Main), Passau (Inn), and Wolfsmünster (Fränkische Saale). The yellow bars represent Q100 (2000) / Q100 (stationary), and the orange bars represent Q100 (2030) / Q100 (stationary).
Trend Significance Test

- Randomly resample time series from the original data
- Estimate the parameters $p_r$ for Type I-III
- Compare them with the parameters of the original data $p_o$
- If less then 10% of the $p_r$'s are higher (lower) than $p_o$, the trend is significant
GD III

**WITHOUT** Significant Positive Trend

[Graph showing distribution of Q100-Yr2030 with reshuffled simulations]

- **Kemmern (Main)**
  - 96 times $Q_R > Q_O$
  - Q100-Yr2030 Original
  - Q100-Yr2030 Reshuffled

Reshuffled Simulations - 1000 times
Annual Maximum Discharge

**GD III**

- **Without**
  - Wolfsmünster Fränk. Saale
  - Kemmern Main
  - Stein Kocher
  - Bad Rotenfels Murg
  - Schwabach Kinzig
  - Wutach Oberlauchringen
  - Gebertshaus Schussen

  \[ \alpha = 9.0\% \]

- **With**
  - Donauwörth Donau
  - Hofkirchen Donau
  - Passau Inn

  \[ \alpha = 0.7\% \]

Significant positive trend
Annual Maximum Discharge

\[
\alpha = 0.3\% \\
\alpha = 2.7\% \\
\alpha = 3.4\% \quad \text{(Wolfsmünster Fränk. Saale, Kemmern Main)} \\
\alpha = 0.3\% \quad \text{(Stein Kocher)} \\
\alpha = 3.4\% \quad \text{(Bad Rotenfels Murg, Schwaibach Kinzig, Wutach Oberlauchlingen)} \\
\alpha = 0.3\% \quad \text{(Gebertshaus Schussen, Stuttgart, Donauwörth Donau, Hofkirchen Donau)} \\
\text{WITH significant positive trend}
\]
Summary and Outlook

• The method shown allows to estimate design discharges under non-stationary conditions

• Incorporate knowledge from climate change research to help determine functions to improve extreme value extrapolation

• Conduct a regional trend analysis with more gauging stations
Acknowledgment

Bayerisches Landesamt für Wasserwirtschaft
Landesanstalt für Umwelt, Messungen und Naturschutz Baden-Württemberg