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Non-stationary flood frequency analysis in Southern Germany

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Motivation

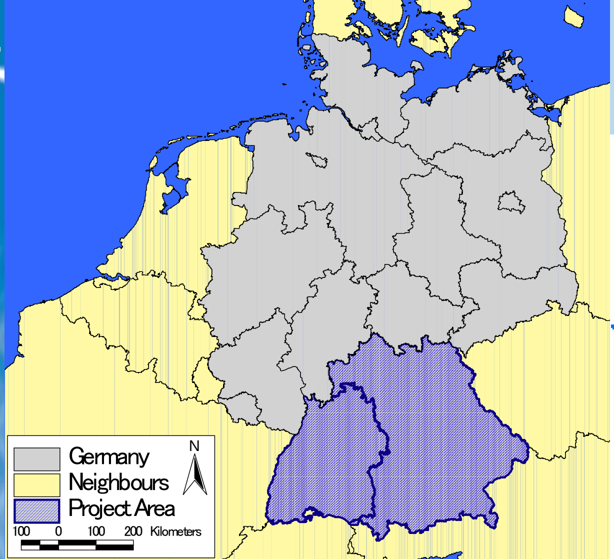
- **Design discharge for hydraulic structures is calculated from extreme value statistics:**
  - Basic assumption: stationarity
- **Several climate variables show a change**

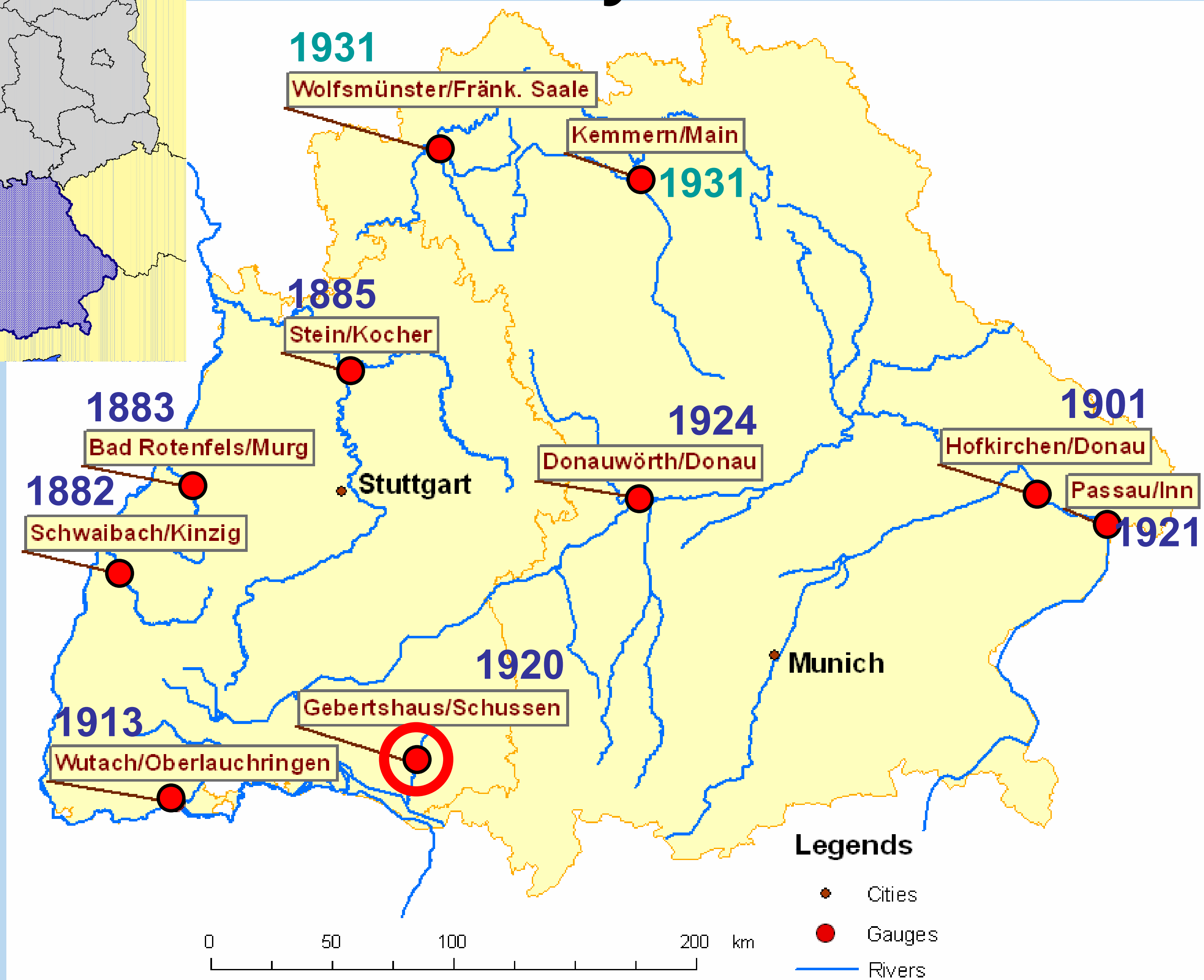
- **Do extreme discharges change?**
- **How can the change be considered in the estimation of design discharges?**

Approach

- **Stationary Gumbel / Pearson Distribution**
- **Non-Stationary Gumbel / Pearson Distribution**
- **Trend Significance Test → Bootstrapping**

Study Domain





Annual
Winter
Summer
Q_{max}

until 2004

Gumbel Distribution Function

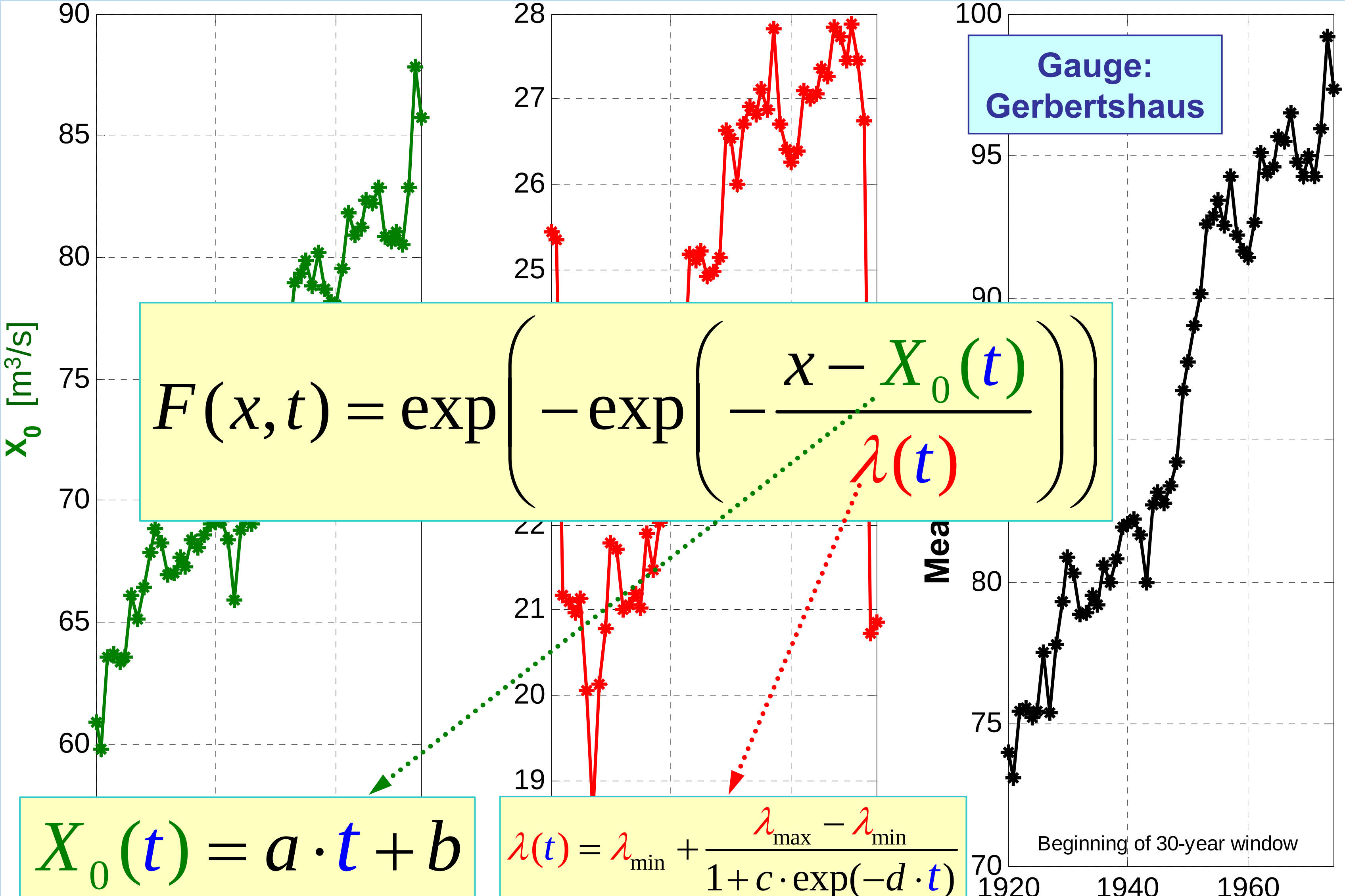
$$F(x) = \exp\left(-\exp\left(-\frac{x - X_0}{\lambda} \right) \right)$$

Two Parameters:

Location X_0

Scale λ

Stationary Parameters in a 30-year Moving Window



$$F(x,t) = \exp\left(-\exp\left(-\frac{x - X_0(t)}{\lambda(t)}\right)\right)$$

$$X_0(t) = a \cdot t + b$$

$$\lambda(t) = \lambda_{min} + \frac{\lambda_{max} - \lambda_{min}}{1 + c \cdot \exp(-d \cdot t)}$$

Non-Stationary Gumbel Distribution

Gumbel Distribution Type		**Parameters to estimate**
GD I	$\lambda = \text{constant};\ X_0(t) = at + b$	$\lambda \quad a \quad b$
GD II	$\lambda(t) = \lambda_{\min} + \dfrac{\lambda_{\max} - \lambda_{\min}}{1 + c \cdot \exp(-d \cdot t)}$	X_0
GD III	$\lambda(t) = \lambda_{\min} + \dfrac{\lambda_{\max} - \lambda_{\min}}{1 + c \cdot \exp(-d \cdot t)}$ $(\lambda(t) > 0)$ $X_0(t) = at + b$	$a \quad b \quad c \quad d$

Maximum Likelihood Method

Simulated Annealing Optimization

Non-Stationary Gumbel Distribution

$$F(x,t) = \exp\left(-\exp\left(-\frac{x - X_0(t)}{\lambda(t)}\right)\right)$$

GD I

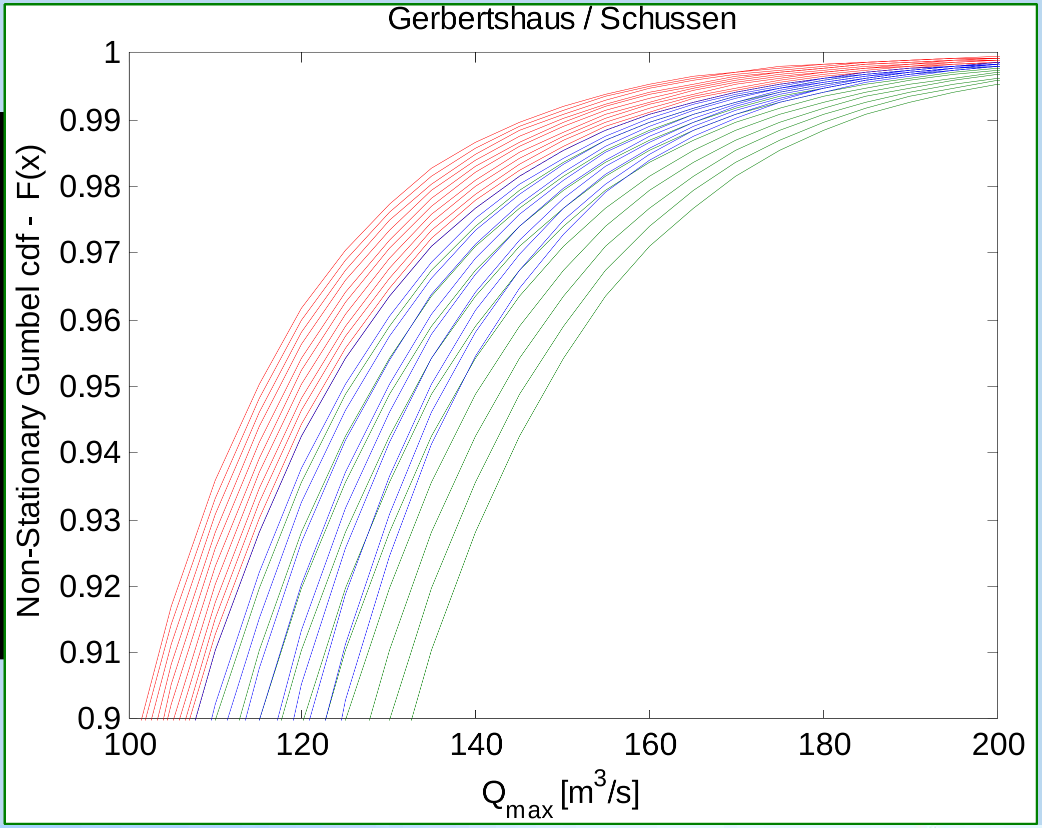
GD II

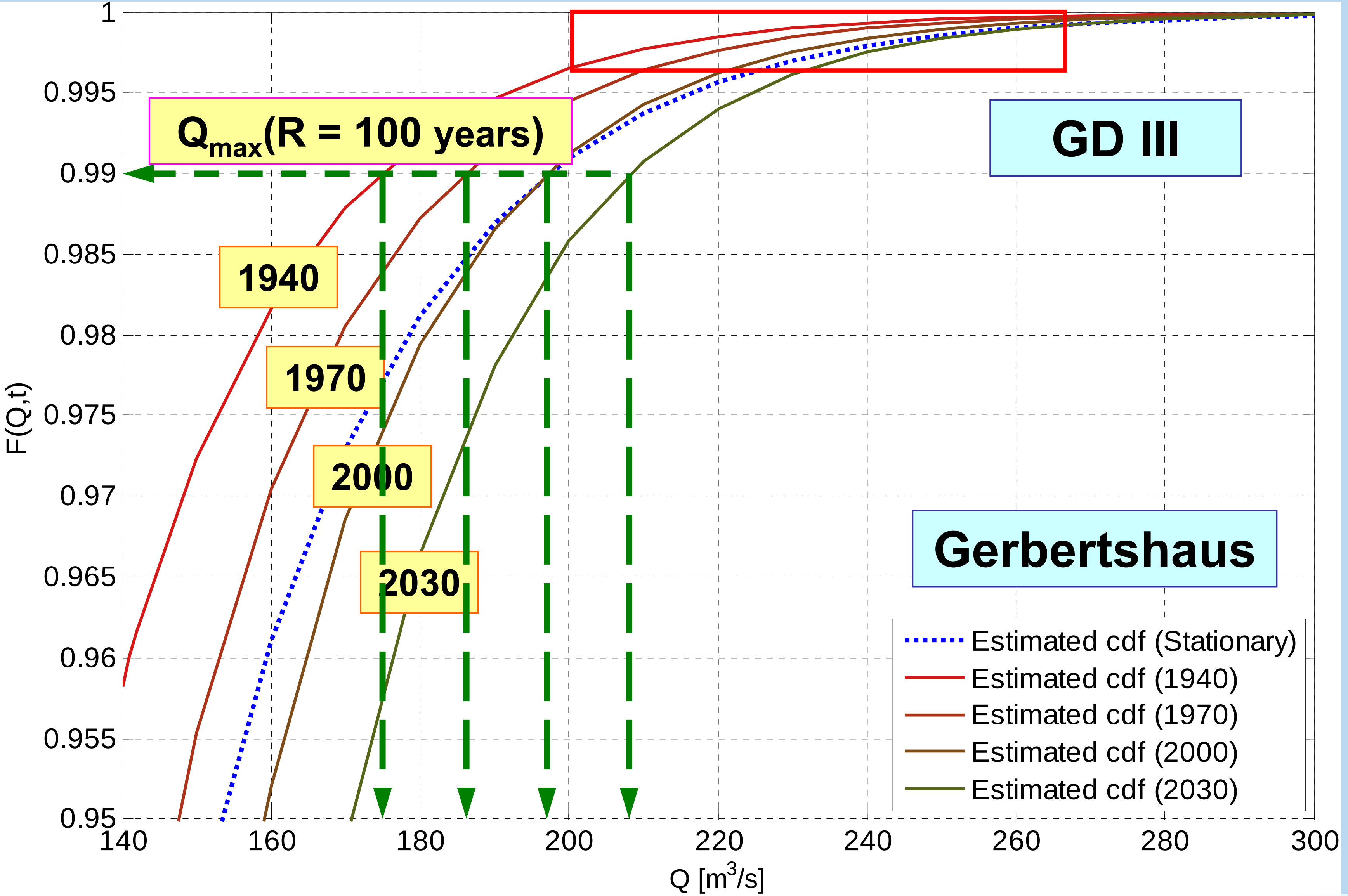
GD III

$$\lambda(t) = \lambda_{min} + \frac{\lambda_{max} - \lambda_{min}}{1 + c \cdot \exp(-d \cdot t)}$$

$$(\lambda(t) > 0)$$

$$X_0(t) = at + b$$





1
0.995
0.99
0.985
0.98
0.975
0.97
0.965
0.96
0.955
0.95
F(Q,t)
140
160
180
200
220
240
260
280
300
Q [m3/s]
Qmax(R = 100 years)
1940
1970
2000
2030
GD III
Gerbertshaus
Estimated cdf (Stationary)
Estimated cdf (1940)
Estimated cdf (1970)
Estimated cdf (2000)
Estimated cdf (2030)

Pearson Distribution Function

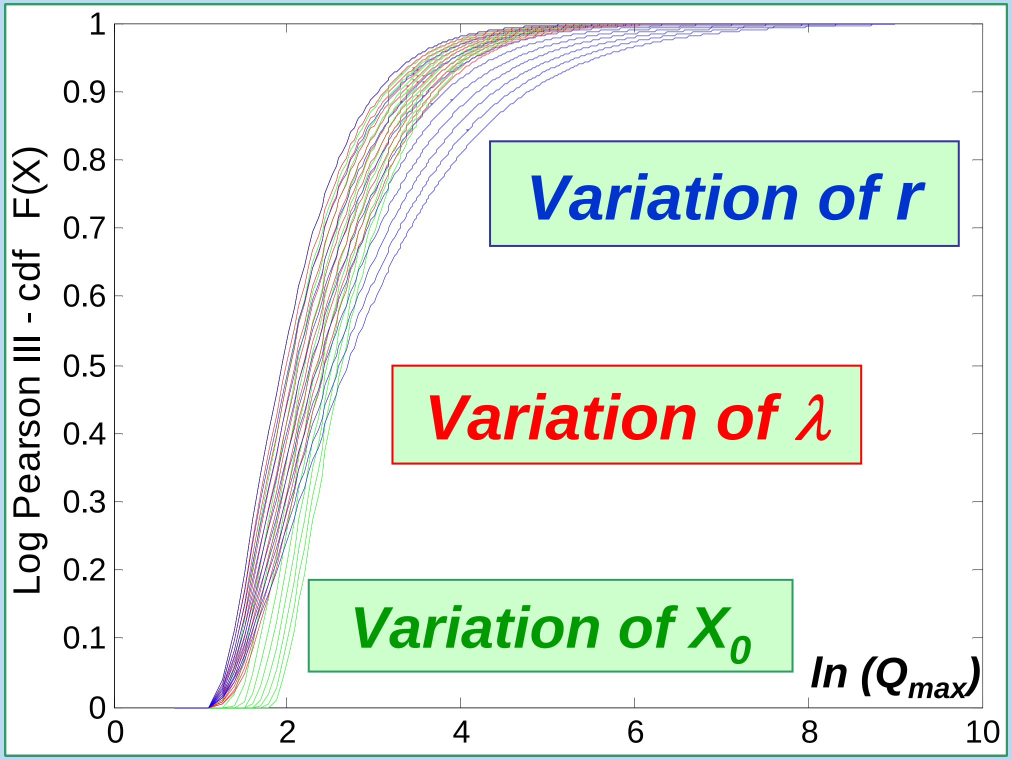
$$F(x) = \int_{X_0}^{x} \frac{(\ln u - X_0)^{r-1}}{u \cdot \lambda^r \cdot \Gamma(r)} \exp\left(-\frac{\ln u - X_0}{\lambda}\right) du$$

Three Parameters:

Location X_0

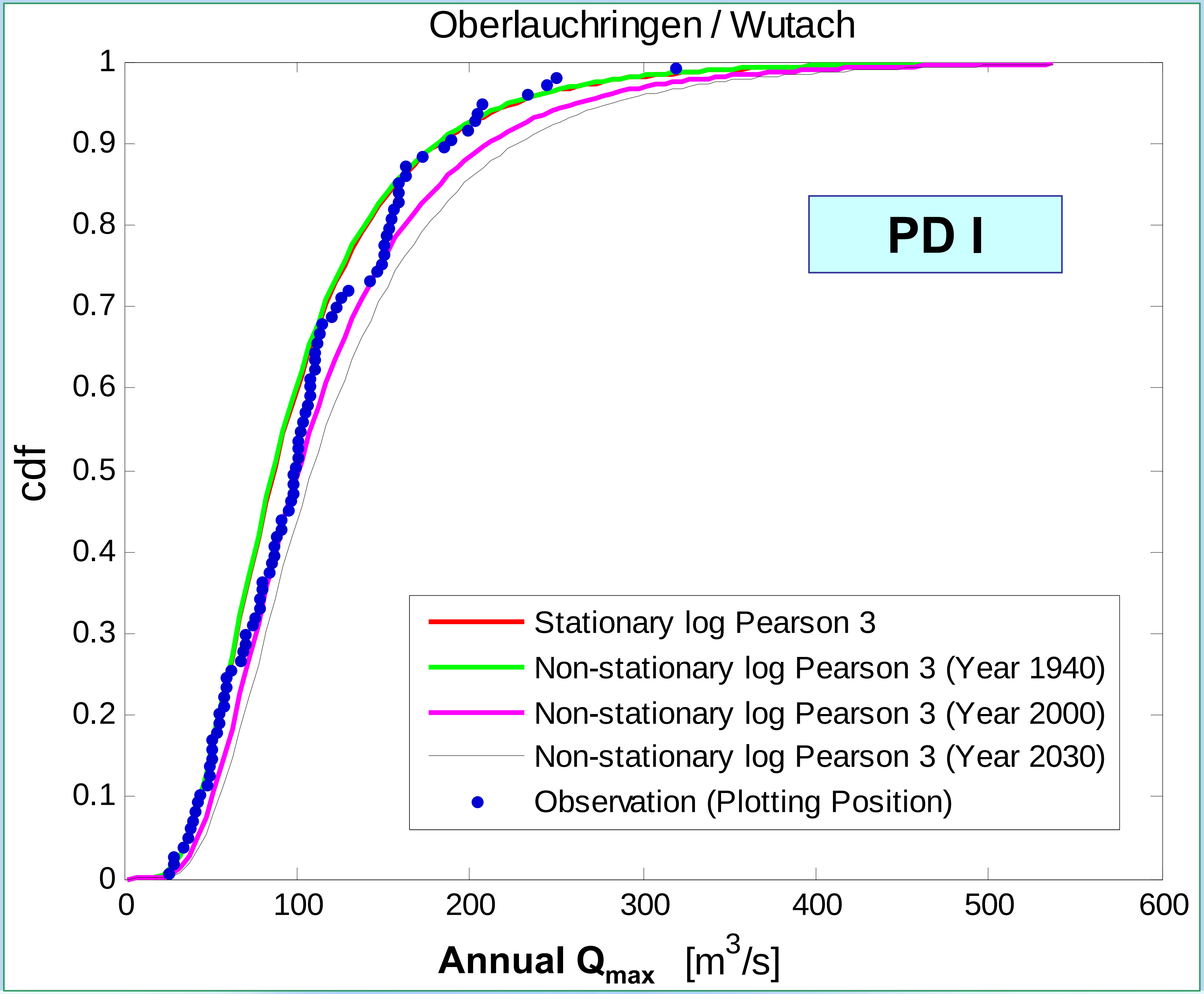
Scale λ $(\lambda > 0)$

Shape r $(r > 0)$

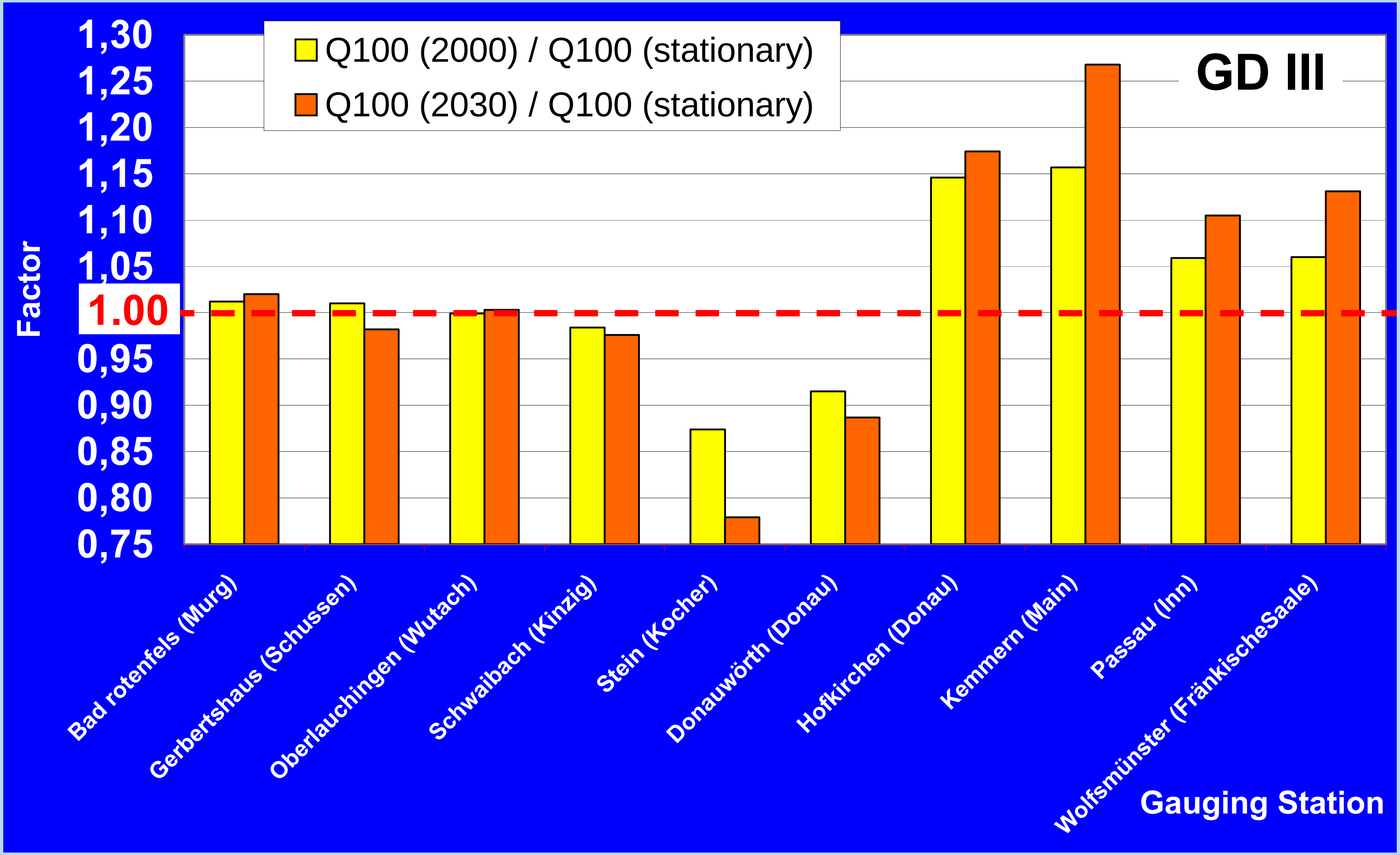


Non-Stationary **Log Pearson III** Distribution

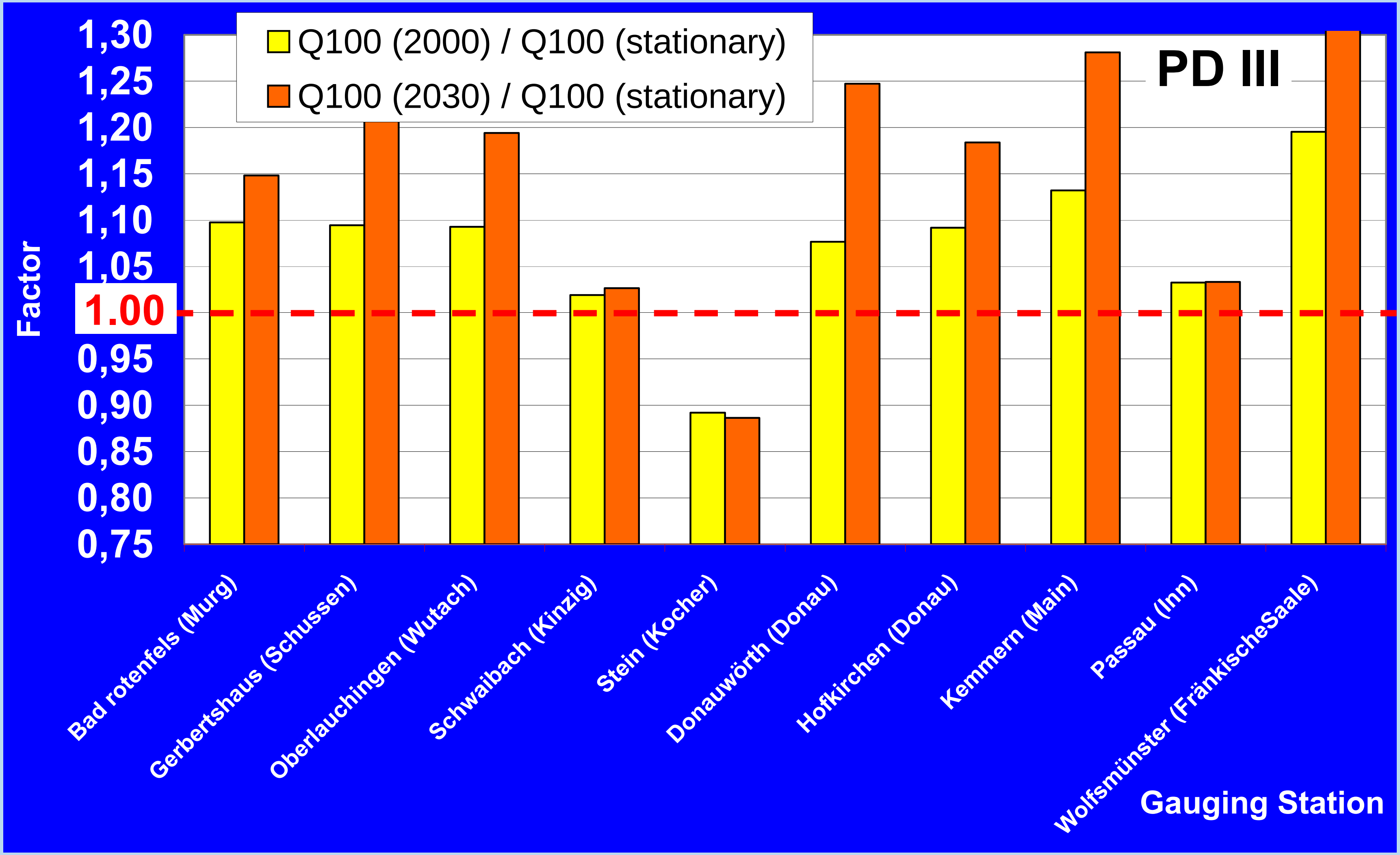
Pearson Distribution Type		**Parameters to estimate**
PD I	$\lambda = \text{constant};\ r = \text{constant};\ X_0(t) = at + b$	$\lambda \quad a \quad b$
PD II	$\lambda(t) = \lambda_{\min} + \dfrac{\lambda_{\max} - \lambda_{\min}}{1 + c_\lambda \cdot \exp(-d_\lambda \cdot t)}$ $(\lambda(t) > 0)$ $X_0 = \text{constant},\ r = \text{constant}$	$c_\lambda \quad d_\lambda \quad X_0 \quad r$
PD III	$r(t) = r_{\min} + \dfrac{r_{\max} - r_{\min}}{1 + c_r \cdot \exp(-d_r \cdot t)}$ $(r(t) > 0)$ $X_0 = \text{constant},\ \lambda = \text{constant}$	$c_r \quad d_r \quad X_0 \quad \lambda$


Oberlauchringen / Wutach
PD I
cdf
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
Annual Q_{max} [m^3/s]
Stationary log Pearson 3
Non-stationary log Pearson 3 (Year 1940)
Non-stationary log Pearson 3 (Year 2000)
Non-stationary log Pearson 3 (Year 2030)
Observation (Plotting Position)

Non-Stationary / Stationary Q_{100}



Non-Stationary / Stationary Q_{100}

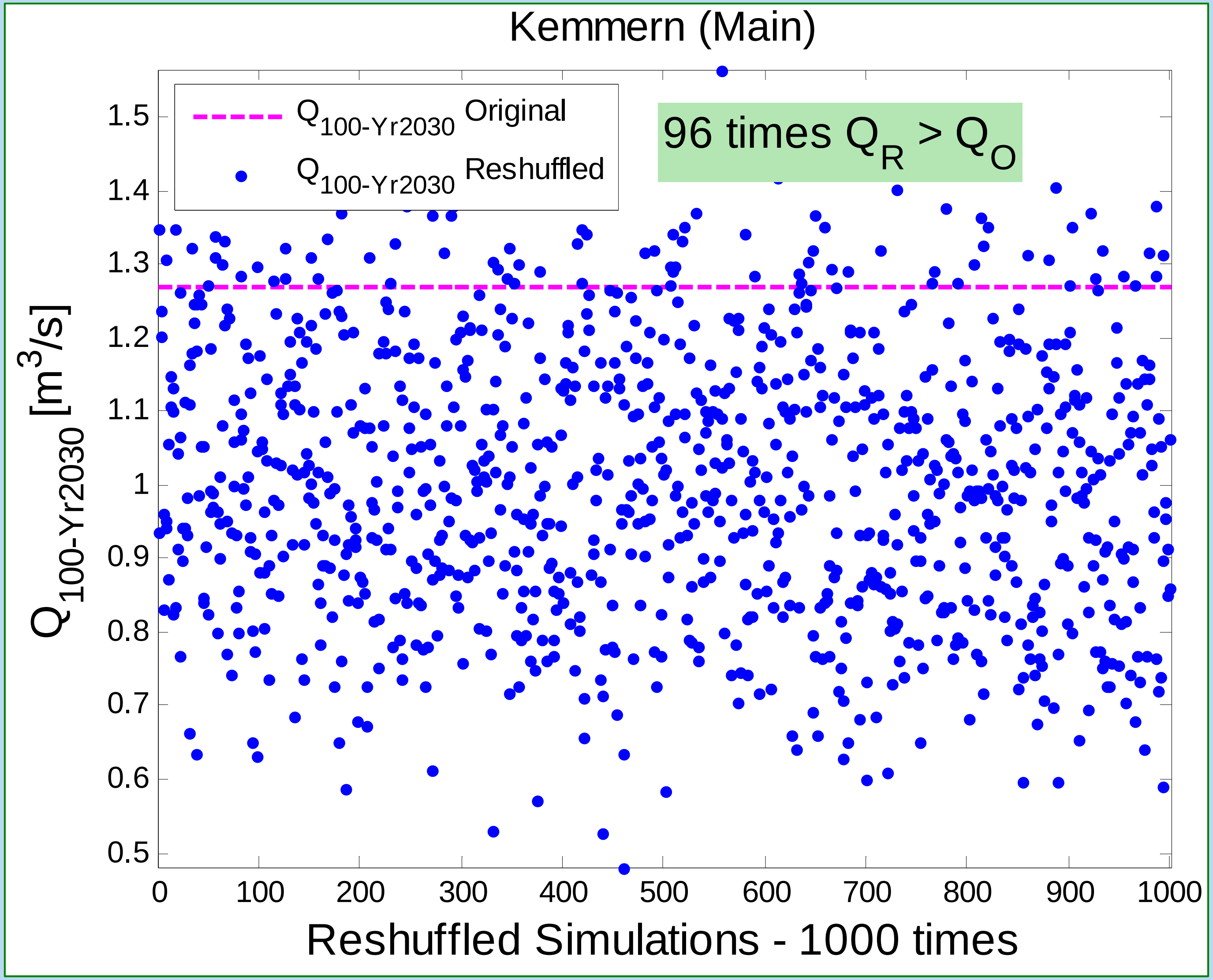


Trend Significance Test

- Randomly resample time series from the original data
- Estimate the parameters p_r for Type I-III
- Compare them with the parameters of the original data p_o
- If less then 10 % of the p_r‘s are higher (lower) than p_o, the trend is significant

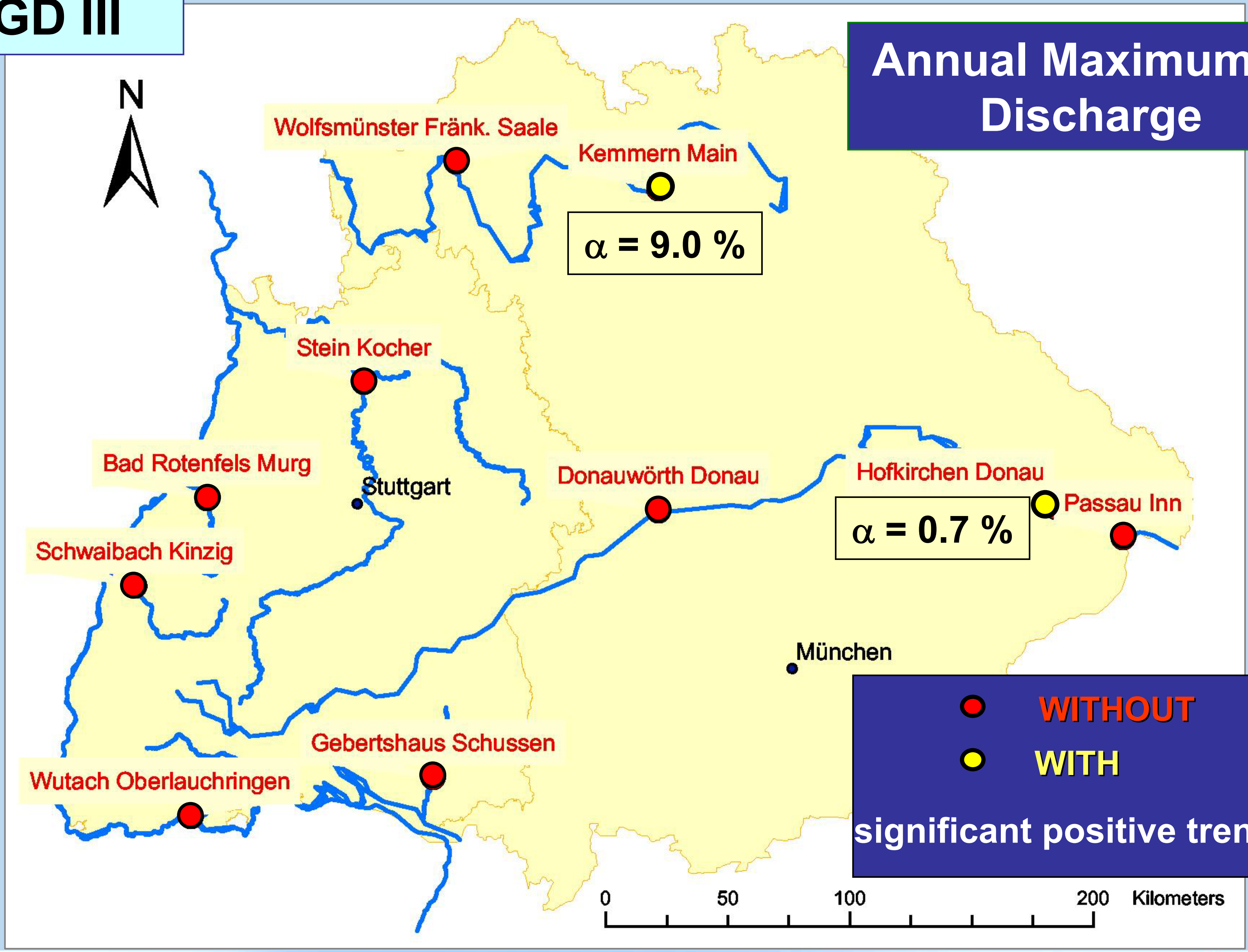
GD III

WITHOUT Significant Positive Trend



GD III

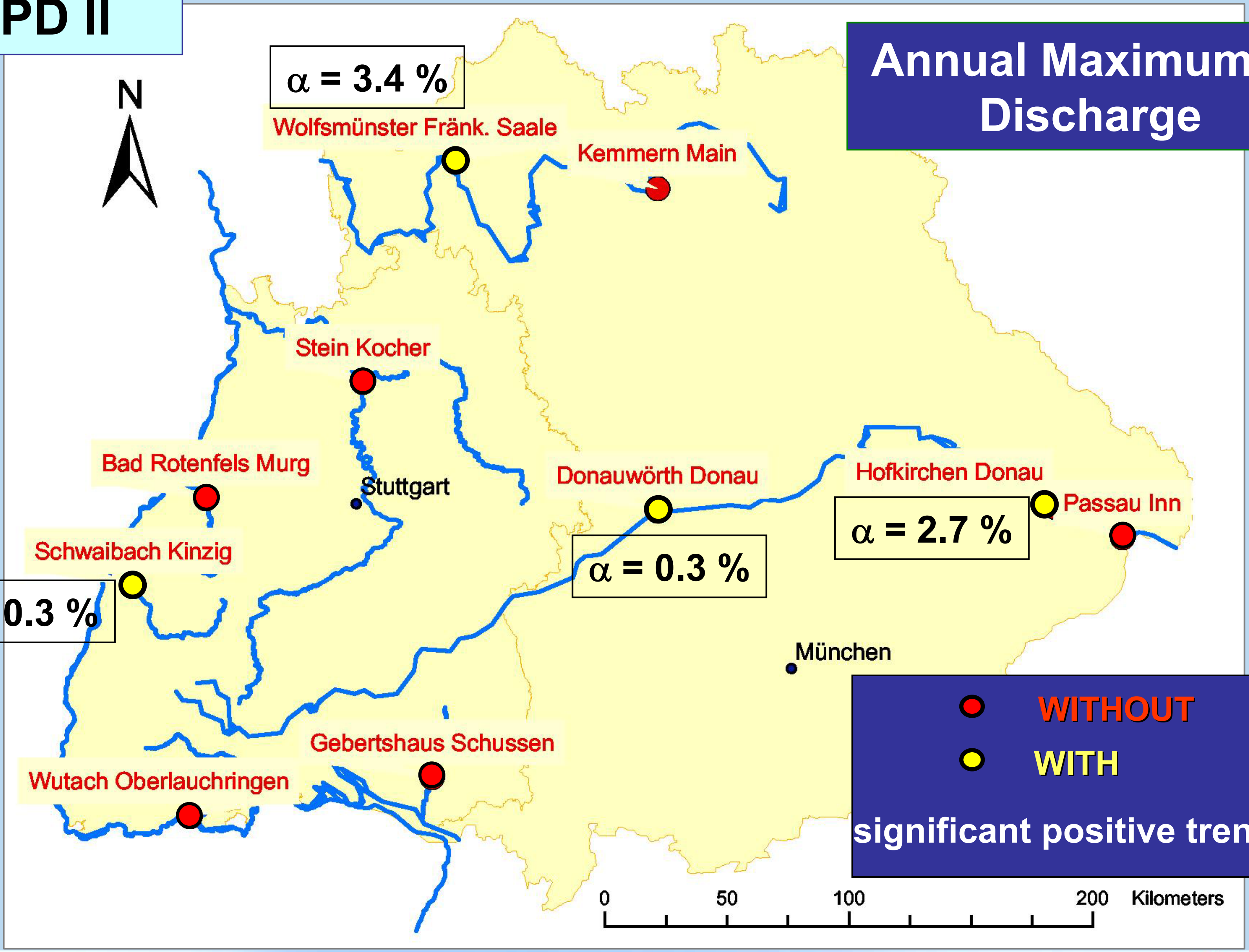
Annual Maximum Discharge



WITHOUT
WITH
significant positive trend

PD II

Annual Maximum Discharge



WITHOUT

WITH

significant positive trend

Summary and Outlook

- **The method shown allows to estimate design discharges under non-stationary conditions**
- **Incorporate knowledge from climate change research to help determine functions to improve extreme value extrapolation**
- **Conduct a regional trend analysis with more gauging stations**

Acknowledgment

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