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Stochastic Generation of Synthetic Precipitation Time Series with High Temporal and Spatial Resolution for Engineering Practice

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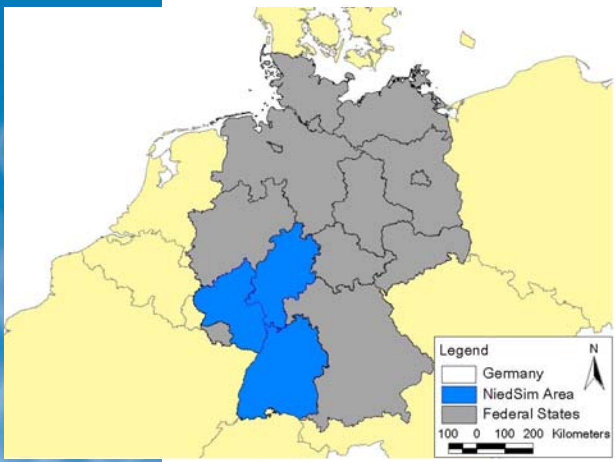


Introduction

The stochastic precipitation time series generator, NiedSim, has been developed and installed in the state of Baden-Wuerttemberg between 1996 and 2000. Since February 2000 more than 700 time series have been generated. In the year 2004 NiedSim was set up for Hessen and Rheinland-Pfalz. The total project area is now 72,901 km².

NiedSim can generate synthetic time series of any given length within the years 1958 to 2001. The maximum temporal resolution is 5 minutes.

The system generates a time series on the basis of statistical properties of the natural rainfall. The parameters are estimated at gauging stations and interpolated on a 1 km² grid using **External Drift Kriging**. The entire derived data is stored in a database.

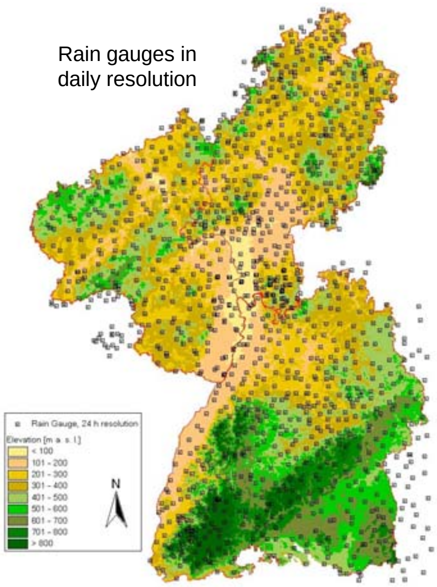
Time Series Generation

To create an annual precipitation time series for an arbitrarily chosen point in the project area the related properties are culled from the database. A time series for **hourly values** is randomly generated using a fitted gamma- and an exponential distribution. Annual rainfall for the given year is considered in this generation .

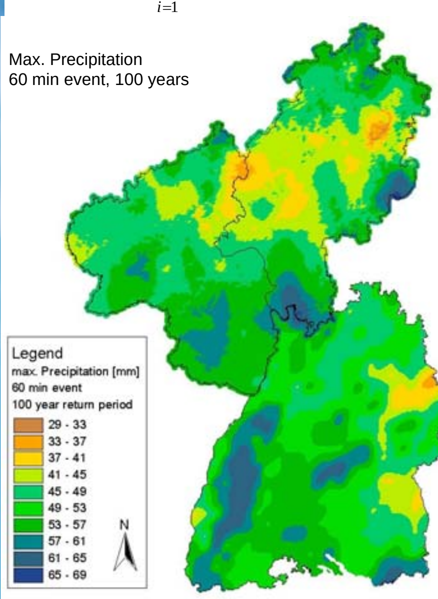
In the next step, statistical properties of the obtained time series are estimated. The weighted deviation between these properties, $\rho_i(Z)$, and those provided for the simulation point by the database, $\rho_i^*(Z)$, is formulated in an objective function, $O(Z)$:

$$O(Z) = \sum_{i=1}^{n} w_i \cdot \left(\rho_i(Z) - \rho_i^*(Z)\right)^2$$

Rain gauges in daily resolution



Max. Precipitation
60 min event, 100 years



Simulated Annealing is used to adapt the time series' properties to the target properties i.e., minimizing the objective function's value:

Two values of the time series are exchanged randomly. The change in the objective function, $O(Z)$ - $O(Z')$, is computed. If it decreases, the exchange is accepted. Otherwise, the probability, P, is calculated as follows.

$$P = \exp\left(\frac{O(Z) - O(Z')}{T}\right)$$

T is the temperature of the annealing algorithm. A random number R is drawn and compared to P. If $R < P$ the exchange is accepted, otherwise it is discarded. These exchanges are repeated N times. Afterwards the annealing temperate, T, is reduced in M steps, each followed by N repetitions of the procedure described above.

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Disaggregation

Following the same scheme the hourly time series can be **disaggregated** to a series with 5 minute resolution: First the hourly precipitation amounts are equally distributed to the appropriate 5 minute intervals. Then another objective function is formulated. In this case, a precipitation increment is randomly subtracted from one time step and added to another. Stepwise the properties of the time series are fitted to the target properties.

Time Series Properties

Apart from the parameters previously mentioned, the properties considered in generation and disaggregation are:

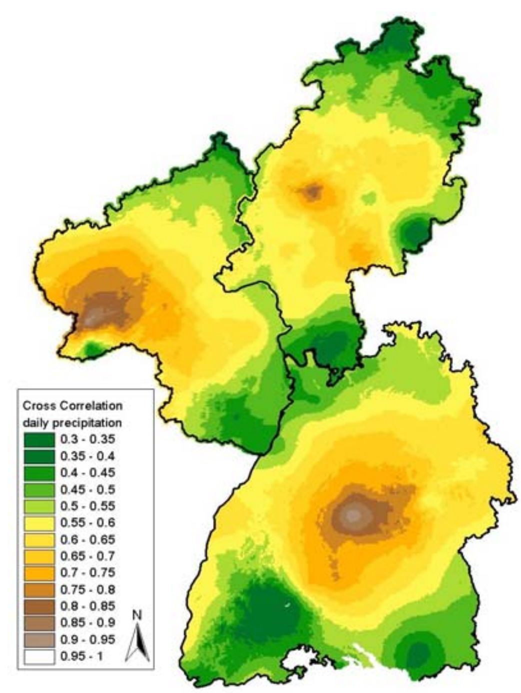
- ✓ Monthly sum of precipitation
- ✓ 1st to 3rd moment, scaling exponents
- ✓ Autocorrelation and weighted Autocorrelation with Lag 1 to 10
- ✓ Exceedence probabilities of certain precipitation totals
- ✓ Average, standard deviation, rain probability for 3 different Circulation Patterns (CPs)
- ✓ Cross correlation

These parameters are evaluated on different time scales, ranging from 5 minutes to 24 hours.

Link to Other Time Series

To imitate heteroscedasticity and persistence of natural time series in the generated series, a reference station with complete precipitation time series and ambient CPs are taken into account.

The interdependency between reference station and simulated station is measured by the cross correlation, which has been calculated and regionalized for each state to its reference station. CP related average, standard deviation and rainfall probability allows generation of wet and dry spells in the series, which refer to the natural conditions.



Cross Correlation to each reference station - daily precipitation

Experiences in engineering practice

- The time series generated have proved useful.
- NiedSim time series are commonly employed in sanitary engineering and urban hydrology
- One time series does not represent the aerial precipitation sufficiently, so

There is a need to generate time series simultaneously

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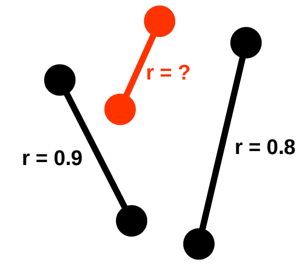
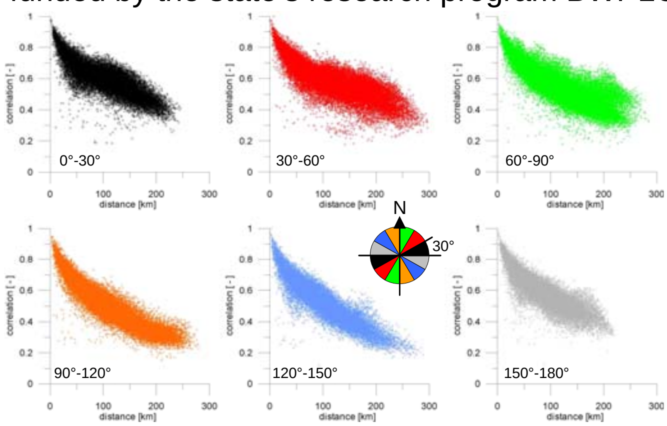
Enhancement – Simultaneous Generation

The long-term objective is to generate any given number of spatial correlated, simultaneous time series. In the first step NiedSim will be expanded to generate two simultaneous series. First, a station will be simulated the known way, then a second station will be generated using the first station as reference station. To realize this, the cross correlation (CC) between any two given points needs to be estimated. The enhancement of NiedSim is limited to Baden-Wuerttemberg, since it is funded by the state's research program BWPLUS.



Given (black) and unknown (red) correlation



Correlation according to direction and distance of station pairs

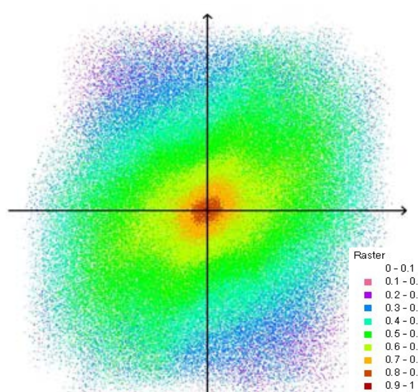
Data Basis

There are 575 rain gauges in and around the project area registering precipitation in daily resolution. Time series are ≤ 100 years. Evaluations are done for the period 1958 to 2003.

For each station the correlation to every other station has been calculated as a basis for the further studies.

Correlation Characteristics

The cross correlations depend on the location and orientation of the station pairs and are nonhomogeneous. Evaluations show that the correlation is strongest in southwest direction which is the prevailing wind direction in the project region. These nonhomogeneities should be considered in the estimation of unknown cross correlations between arbitrary points.



Original correlations as vectors

Repetitive External Drift Kriging

The first method tested consecutively used External Drift Kriging (EDK) twice to estimate the correlation, r^*, between any chosen points, x and y. The square root of the elevation above mean sea level served as external drift.

1. EDK: calculate correlations $r^*(x,x_j)$ from x to all gauging points x_j:

$$r^*(x,x_j) = \sum_{i=1}^{n} \lambda_i(x,x_j) \cdot r(x_i,x_j) \qquad x_i \quad i = 1,\ldots,n \text{ locations of rain gauges}$$

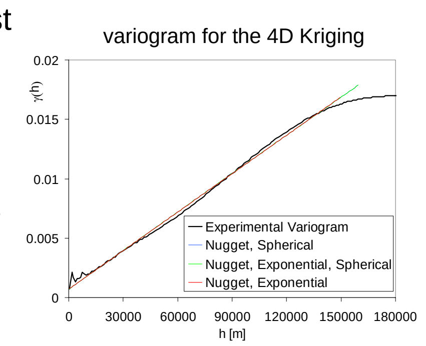
2. EDK: use all $r^*(x,x_j)$ to obtain $r^*(x,y)$:

$$r^*(x,y) = \sum_{i=1}^{n} \lambda_i(x,y) \cdot r^*(x_i,y) = \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i(x,y) \cdot \lambda_j(y,x_i) \cdot r(x_i,x_j)$$

This method works well, but is not very efficient because Kriging must be done twice with the entire data set.

4D Kriging

To avoid accessing data and kriging twice, two 4D Kriging approaches were applied. The idea is to define the distance between different station pairs, each having four coordinates, according to the Euclidean norm and krig with the corresponding variogram.



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Now the cross correlation, $r^*(x,y)$, can be estimated directly:

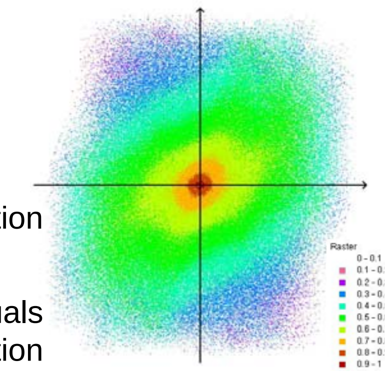
$$r^*(x,y) = \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{i,j}(x,y) \cdot r(x_i, x_j)$$

The approach commonly used in engineering practice to calculate cross correlations between unknown points is to fit an exponential function into the correlation data and evaluate the function for the given points:

$$f(x,y) = e^{-\frac{|x-y|}{a}}$$

This technique has been adapted in two following ways:

1. 4D External Drift Kriging (EDK) was completed using the function values as drift.
2. 4D Ordinary Kriging (OK) was applied interpolating only the residuals between the estimated cross correlations and the appropriate function values.



4D EDK correlations as vectors

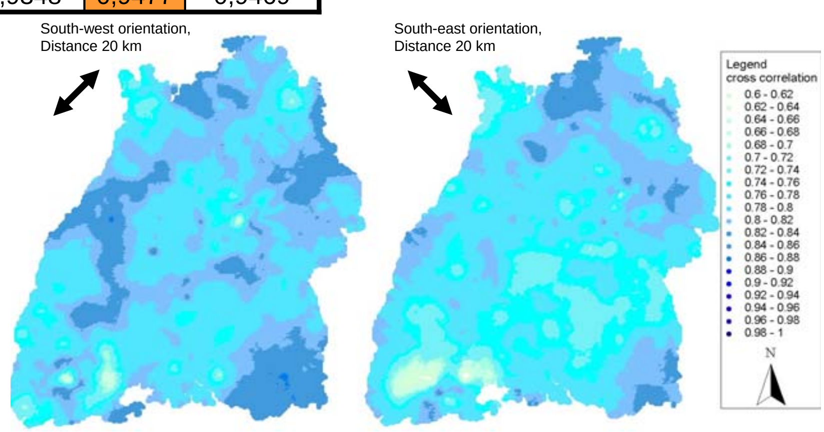
Both Kriging techniques were tested with different fitted theoretical variograms. Cross validations show that all applied Kriging methods produce satisfactory results.

Results of cross validations for different Kriging techniques

parameter	measured data	cross validation		
		2 x EDK	4D OK	4D EDK
Average CC	0,5514	0,5515	0,5521	0,5526
standard deviation of CC	0,1222	0,1163	0,1173	0,1176
standard deviation of errors		0,0436	0,0393	0,0396
correlation to measured data		0,9348	0,9477	0,9469

4D Kriging performs better than twice EDK. 4D OK obtains the strongest correlation in cross validation and the lowest standard deviation of errors.

4D Kriging of cross correlations will be used in Baden-Wuerttemberg in the future. Two maps show the variety of cross correlations in the project area. It depends on location and orientation of the station pairs. These maps were generated with 4D Ordinary Kriging of residuals.



Location and direction dependent cross correlations in south-west (left) and south-east (right) direction, 20 km distance, calculated on a 1 km² grid

Results and Outlook

- 4D Kriging is an efficient technique to estimate correlations of arbitrary station pairs reproducing nonhomogeneities
- 4D Ordinary Kriging of residuals obtains the best results (r = 0.9477)
- These results are to be validated with data in higher temporal resolution

References

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