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Application of Correlation Calculated from Radar Data in a Time Series Generator for Precipitation on the Ground

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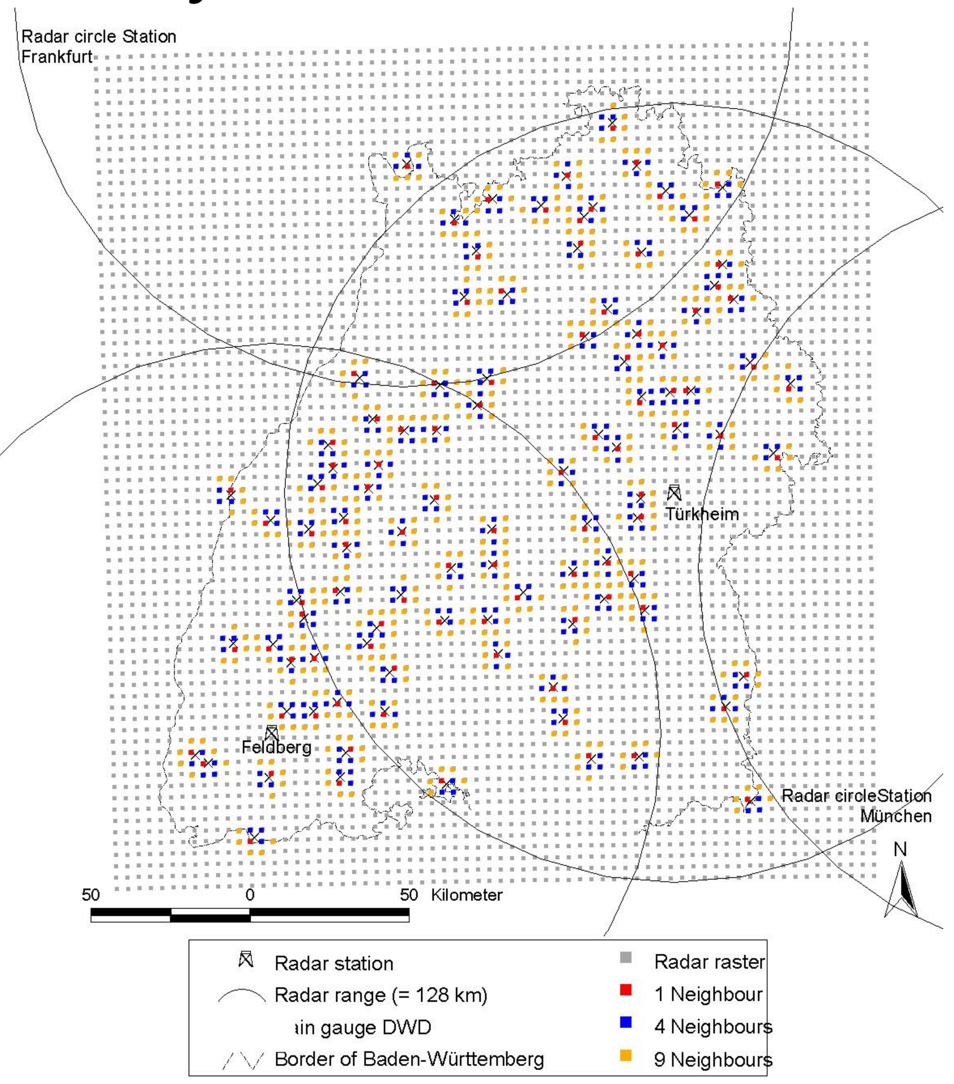
Introduction

The stochastic time series generator NiedSIm for hourly rainfall at single points should be expanded to generate simultaneous time series for multiple locations. Therefore Pearson correlation should be used to describe the spatial relationship, since it is fast and easy to calculate.

Correlation is known to be heterogeneous and anisotropic due to various influences, for e.g. advective transport or topography. This has been proved by preliminary tests in the project area (not shown).

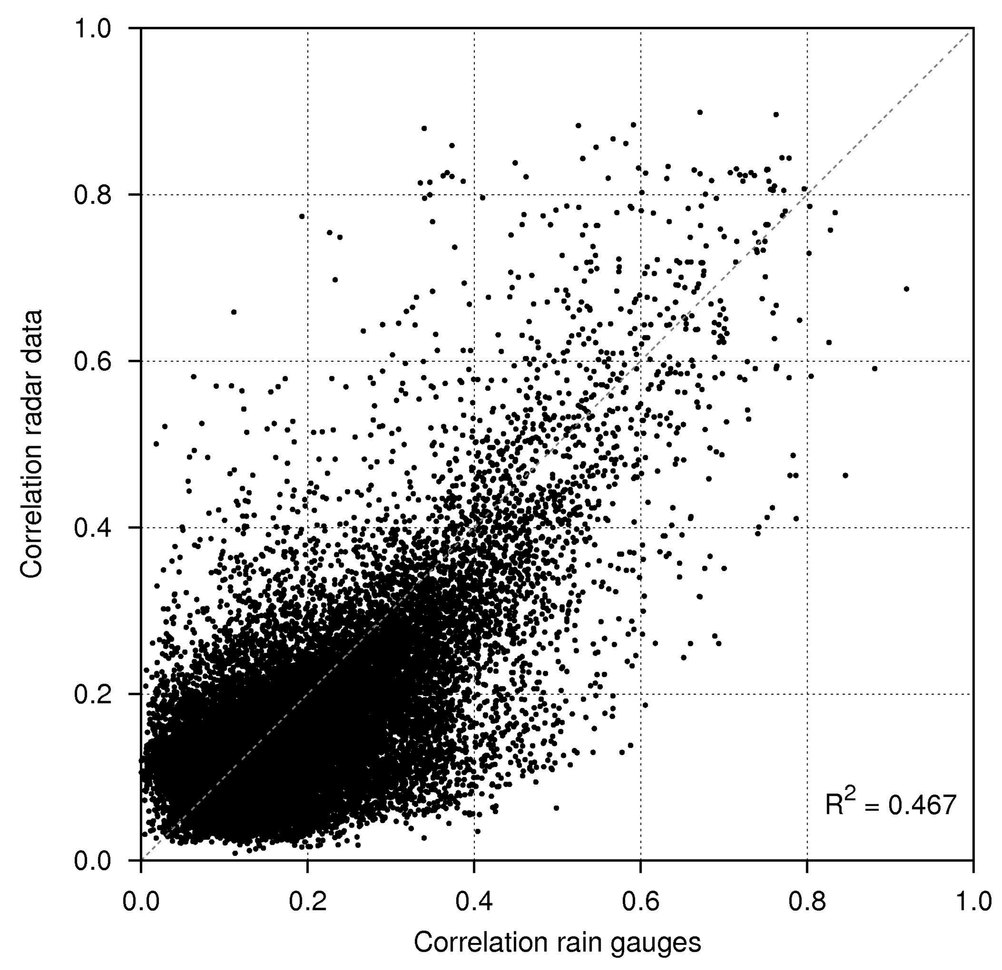


Project area with rain gauges and radar stations

The expanded generator should work in the whole project area, which is the state of Baden-Württemberg in southern Germany. Hence a method is needed to estimate exact correlation of hourly rain data for arbitrarily chosen (ungauged) pairs of locations.

To cover the shortcoming of an insufficient station density radar data is used. For this project, German Weather Service (DWD) provided 15 min radar images in 4 km² spatial resolution. These images contain censored reflectivity data in seven classes. Data is from July 1, 1997 to December 31, 2004. The overall failure rate is 1.55 %. A ZR-relationship is provided to convert this data into rainfall.

The easiest and most apparent approach to integrate radar information is to convert reflectivity into precipitation intensity and evaluate the spatial correlation of this data. This correlation has to be compared to the correlation calculated from rain time series measured at gauges.



Correlation of rain gauges and radar data

From the figure on the left it can be seen easily, that correlation of rain gauge and radar data are not directly comparable.

Correlation is influenced by outliers and the marginal distribution of data. The marginal distributions of radar and rain gauge data differ. This influence can be eliminated applying a transformation on the data before the calculation of correlation. Hence two transformations were tested.

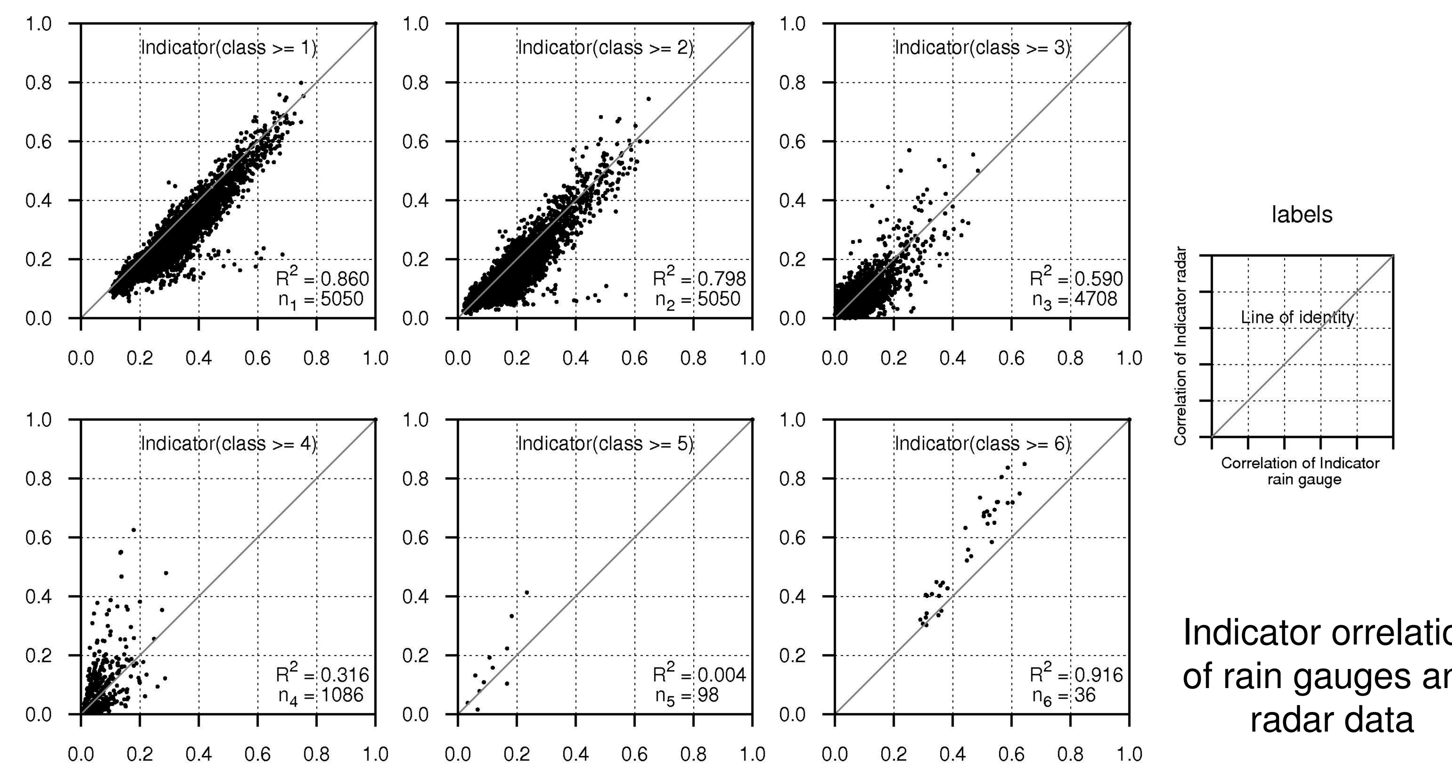
Indicator Correlation of Radar and Rain

To compare the spatial structure in both data, the rain measurements are divided into seven classes. This is done for each of the rain gauges according to the frequency of the radar classes at the closest radar raster point. The correlation for six indicator time series is subsequently calculated for the radar and the rain data.

$$I(t,c_{ref})=\begin{cases}1 & \text{if } c(t)\geq c_{ref}\\ 0 & \text{else}\end{cases},\quad c_{ref}=1,\ldots,6$$

$c(t)$ time series of classes (rain and radar)
c_{ref} class to calculate indicator for

The results show that correlation form radar and rain gauge data is similar (see figure below). It varies with the probability. The first class ≥ 1 is the binary time series with 1=wet and 0=dry. This indicator correlation could be used straight forward in the generation too.



Indicator orrelation of rain gauges and radar data

Correlation of normal transformed data

The second transformation tested is the normal transformation. From their empirical distribution radar and rain gauge are transformed into normal distribution. There are two problems doing this:

-Empirical distribution is discrete, i.e. gives just a range for certain data

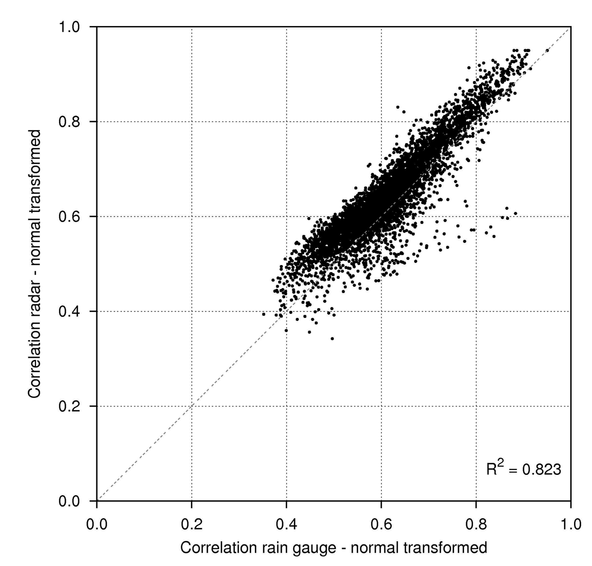
-Empirical distribution ends at 0 mm of precipitation

These two limits have to be considered. Therefore all values with a range and with 0 mm are marked. Then correlation ρ is estimated using maximum likelihood:

$$L(\rho)=\prod_{t=1}^{T} f\left(x_{nv}(t),y_{nv}(t),\rho\right)$$

One station	Other station	Term $f(.)$ in likelihood function
x_{nv}	y_{nv}	$\phi(x_{nv},y_{nv},\rho)$
$x_{nv},(x=0)$	$y_{nv},(y=0)$	$\Phi(x_{nv},y_{nv},\rho)$
$x_{nv},(x=0)$	y_{nv}	$\Phi(x_{nv}\mid y_{nv},\rho)\cdot\phi(y_{nv})$
$x_{1,nv},x_{2,nv}$	y_{nv}	$\Phi(x_{2,nv}\mid y_{nv},\rho)\cdot\phi(y_{nv})-\Phi(x_{1,nv}\mid y_{nv},\rho)\cdot\phi(y_{nv})$
$x_{1,nv},x_{2,nv}$	$y_{1,nv},y_{2,nv}$	$\Phi(x_{2,nv},y_{2,nv},\rho)-\Phi(x_{1,nv},y_{2,nv},\rho)$
		$-\Phi(x_{2,nv},y_{1,nv},\rho)+\Phi(x_{1,nv},y_{1,nv},\rho)$
$x_{1,nv},x_{2,nv}$	$y_{nv},(y=0)$	$\Phi(x_{2,nv},y_{nv},\rho)-\Phi(x_{1,nv},y_{nv},\rho)$

$_{nv}$: normal value, ϕ: df of $N(0,1)$., Φ: cdf of $N(0,1)$, x_{nv}, $(x=0)$: original value is zero, $x_{1,nv}$, $x_{2,nv}$: original value is range



Correlation of rain gauges and radar data, both normal transformed

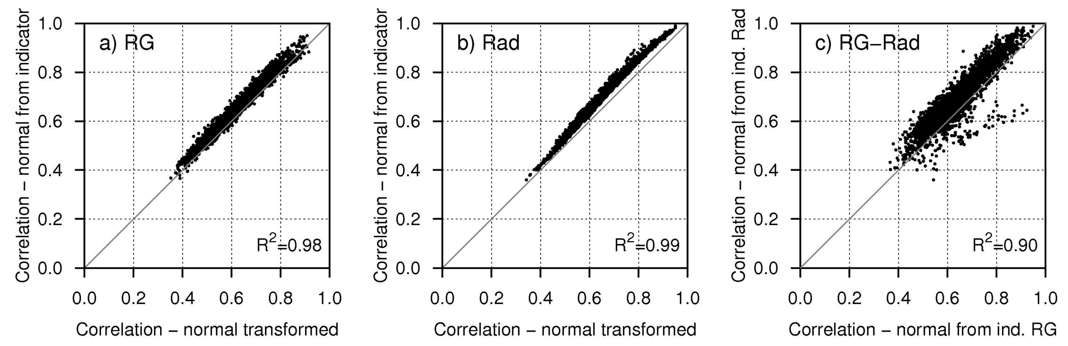
Results in the figure on the right show that correlation of normal transformed radar and rain gauge data can be transferred.

Comparison of methods

The indicator correlation ρ_I of the indicator with probability p between stations i and j can be estimated from the correlation of the normal transformed data ρ_N:

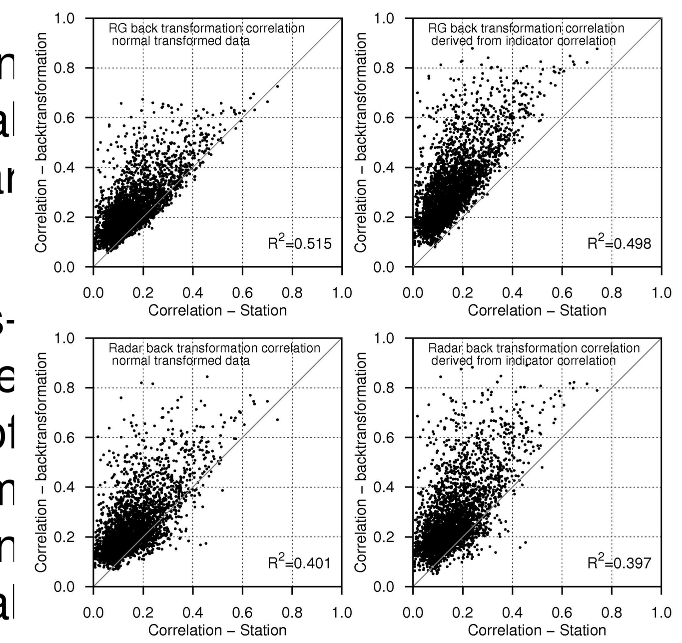
$$\rho_I(p,i,j)=\frac{1}{2\pi(1-p)}\int_0^{\arcsin\rho_N(i,j)}\exp\left(\frac{\Phi^{-1}(p)^2}{1+\sin t}\right)dt$$

This allows to prove the correct application of both transformations.



Back transformation of normal correlation

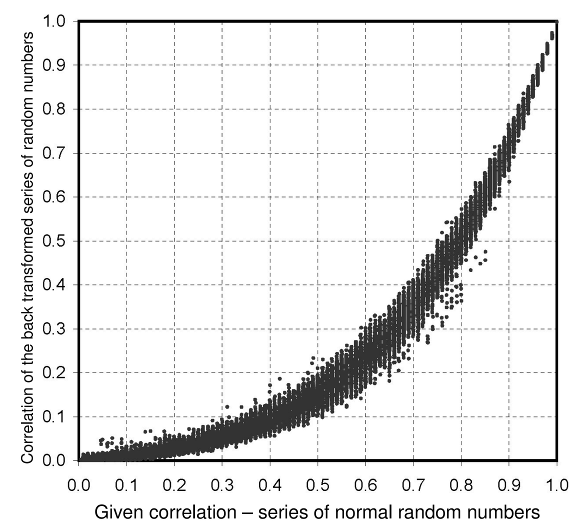
Both transformation showed that when removing the influence of the marginal distribution correlation calculated from radar and rain gauge data agree quite well.



Four back transformations

The planned application needs a back transformation, since correlation of rain gauge data is needed. Therefore two time series of bivariate normally distributed random numbers are generated having the given (normal) correlation. Using the two empirical marginal distributions of the original time series, these time series are transformed into "original" time series and correlation is calculated. As the figure on the right (top) shows, this method failed.

In a numerical experiment, time series of 100,000 elements were generated with correlation between 0.01 and 1.00. Each step was repeated 250 times. Results in the figure right (bottom) demonstrate that this back transformation is not stable.



Numerical experiment for back transformation

Results

- Indicator correlation of the wet/dry time series can be transferred straight from radar to rain gauge data.
- Normal is also applicable, but
- Back transformation of normal correlation to "original" correlation fails

References

Brommundt: Stochastische Generierung räumlich zusammenhängender Niederschlagszeitreihen. Dissertation, Stuttgart, 2008.

Acknowledgements

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Data supplier: DWD German Weather Service

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