Comparison of Lower- and Equi-Dimensional, Discrete and Embedded Models for Fractured Porous Media Flow

International Research Training Group nupus

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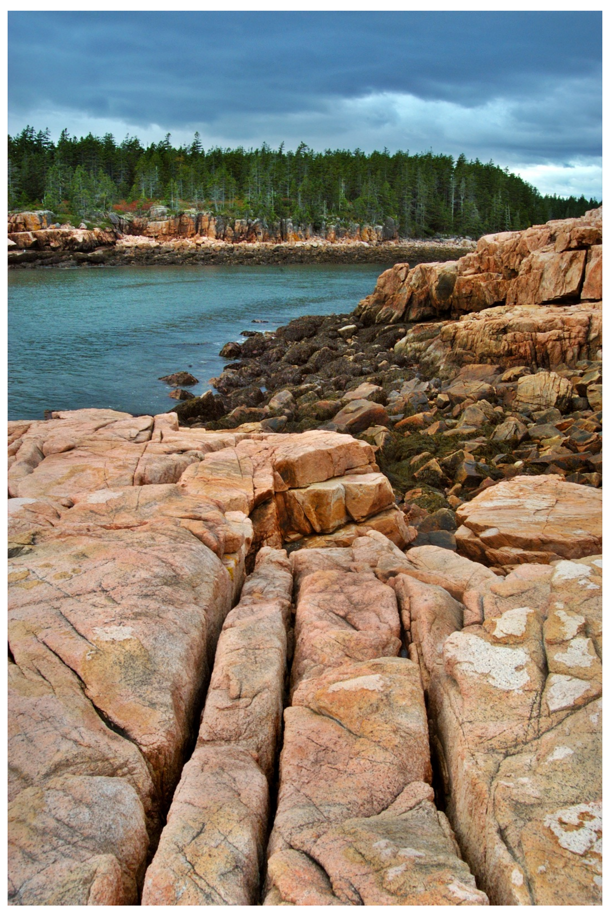
Motivation

- Flow in fractured porous-media systems is often dominated by their heterogeneities and discontinuities. Such systems characterise many applications, e.g.
  - CO_2 capture and storage,
  - reservoir engineering,
  - groundwater resource management.
- At the scale of interest the material properties differ in orders of magnitude for the fracture network and the surrounding rock matrix. Furthermore, the characteristic flow behaviour of the whole system depends crucially on both the fractures and the rock matrix.
- The exact fracture structure on the field scale cannot be determined. Thus the fracture-network model has to be stochastically generated. To get meaningful results several (> 100) realisations have to be simulated.

⇒ *The discrete fractured porous-medium model has to be meshed fast and produce accurate results.*



Granite, Maine, USA

Limestone, Adriatic Sea

Results

Regular Network

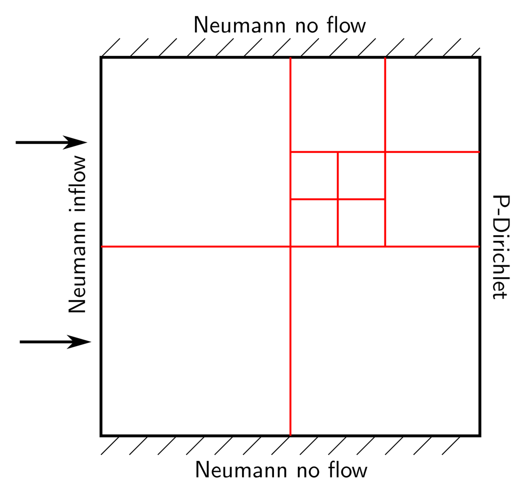


Figure 1: regular network: domain and boundary conditions

high contrast between permeabilities:
$\mathbf{K}_\mathrm{m} = 1.0 \cdot \mathbf{I}\,\mathrm{mD}$,
$\mathrm{k}_{\mathrm{f},\mathbf{n}} = \mathrm{k}_{\mathrm{f},\mathbf{t}} = 8.3 \cdot 10^7\,\mathrm{mD}$, $\mathrm{d} = 1\mathrm{mm}$
domain: $1 \times 1\,\mathrm{m}^2$ unit box

Test case based on: Geiger et al. – "A novel multi-rate dual-porosity model for improved simulation of fractured and multi-porosity reservoirs", SPE Reservoir Characterisation and Simulation Conference and Exhibition, 2011

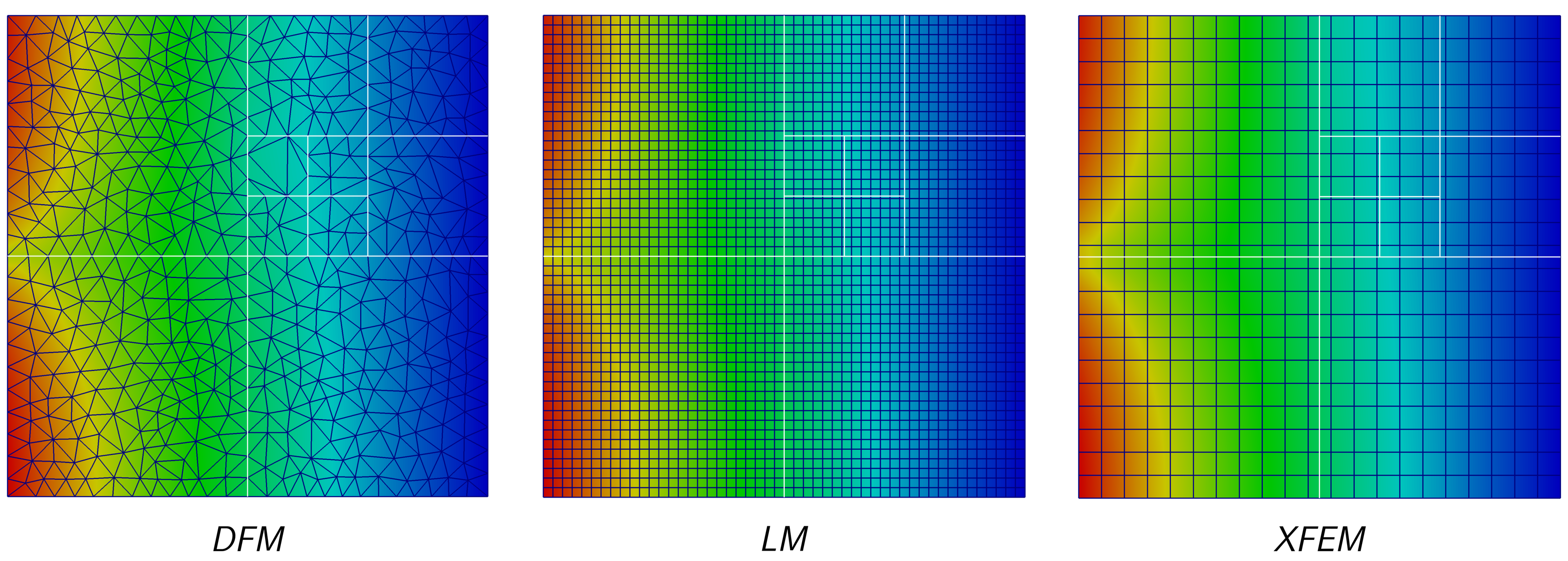


Figure 2: pressure distribution for the three models for the regular network test case

Irregular Network

dimensionless model:
domain: unit box, $\mathbf{K}_\mathrm{m} = \mathbf{I}$,
$\mathrm{k}_{\mathrm{f},\mathbf{n}} = \mathrm{k}_{\mathrm{f},\mathbf{t}} = 10^4$, $\mathrm{d} = 10^{-4}$

outlook: for the discontinuous pressure, lower dimensional model also impermeable fractures are possible

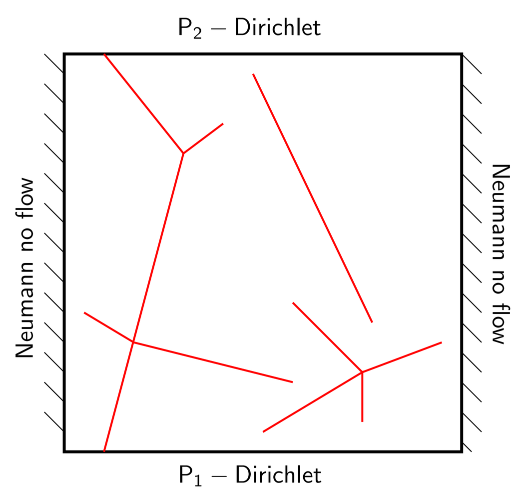


Figure 3: irregular network: domain and boundary conditions

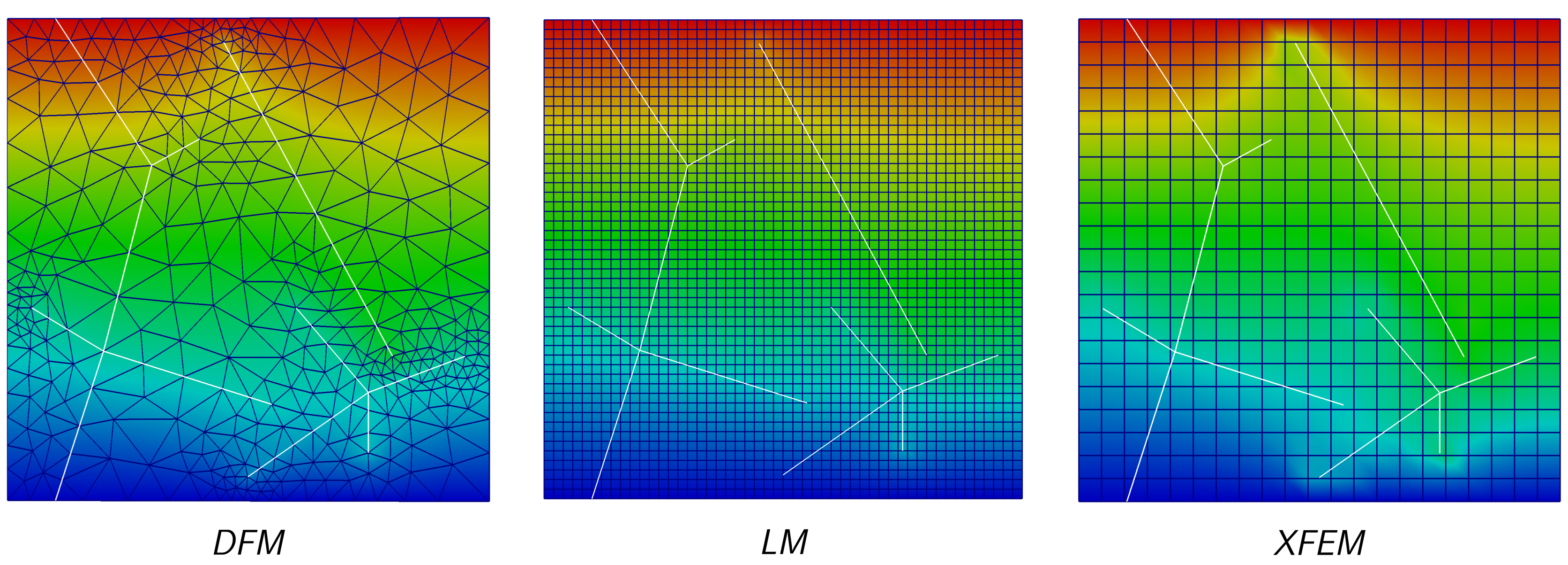


Figure 4: pressure distribution for the three models for the irregular network test case

Models

Equi-Dimensional

Darcy's Law, continuity equation for isothermal, incompressible fluids (strong formulation for single phase flow)

$$\nabla \cdot (\, -\mathbf{K}\, \nabla\, p\,) = 0 \qquad \text{in } \Omega$$

with fractures (heterogeneities):

$$\nabla \cdot (\, -\mathbf{K}_\mathrm{m}\, \nabla\, p_\mathrm{m}\,) = 0 \quad \text{in } \Omega_i$$
$$\nabla \cdot (\, -\mathbf{K}_\mathrm{f}\, \nabla\, p_\mathrm{f}\,) = 0 \quad \text{in } \Gamma$$

with coupling conditions so that at any point on $\bar{\Omega}_i \cap \bar{\Gamma}$

$$p_\mathrm{m} = p_\mathrm{f}$$
$$\nabla \cdot (\, -\mathbf{K}_\mathrm{m}\, \nabla\, p_\mathrm{m}\,) \cdot \mathbf{n}_\mathrm{m} = -\nabla \cdot (\, -\mathbf{K}_\mathrm{f}\, \nabla\, p_\mathrm{f}\,) \cdot \mathbf{n}_\mathrm{f}$$

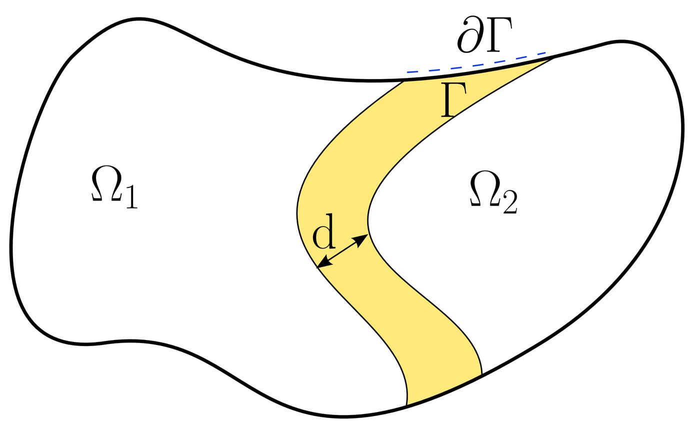


Figure 5: Domain decomposition into fracture and rock matrix

Implemented in DuMu^x as DFM [3] discretized with the BOX method with two-point flux stencil.

Lower-Dimensional, Continuous Pressure

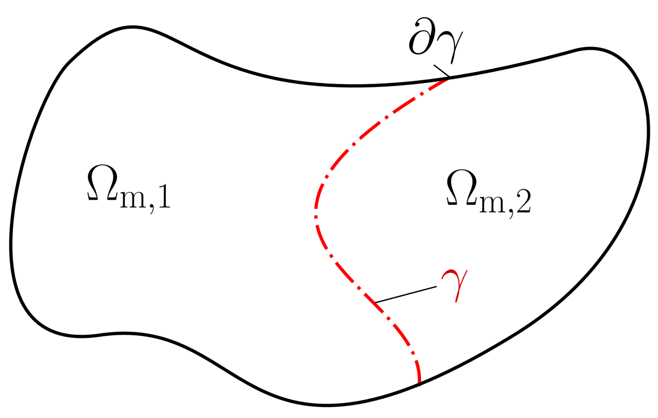


Figure 6: Domain decomposition into lower dimensional fracture and rock matrix

Lagrangian multiplier coupling – strong formulation:

$$\nabla \cdot (\, -\mathbf{K}_\mathrm{m}\, \nabla\, p_\mathrm{m}\,) = -\lambda \qquad \text{in } \Omega_\mathrm{m}$$
$$\nabla \cdot (\, -\mathrm{K}_\mathrm{f}\mathrm{d}\, \nabla\, p_\mathrm{f}\,) = +\lambda \qquad \text{in } \gamma$$

Weak formulation:

$$\int_\Omega \mathbf{K}_\mathrm{m}\, \nabla\, p \cdot \nabla\, \phi_\mathrm{m} - \int_\gamma \lambda\, \phi_\mathrm{m} = 0$$
$$\int_\gamma \mathrm{K}_\mathrm{f}\mathrm{d}\, \nabla\, p \cdot \nabla\, \phi_\mathrm{f} + \int_\gamma \lambda\, \phi_\mathrm{f} = 0$$
$$\int_\gamma (p_\mathrm{m} - p_\mathrm{f})\phi_\lambda = 0$$

Implemented in SciLab with mixed FEM in [2].

Lower-Dimensional, Discontinuous Pressure

Strong formulation:

$$\nabla \cdot (\, -\mathbf{K}_i\, \nabla\, p_i\,) = q_i \qquad \text{in } \Omega_i \qquad i = 1,2$$
$$\mathbf{K}_i\, \nabla\, p_i \cdot \mathbf{n}_i = \frac{2\mathrm{k}_{\mathrm{f},\mathbf{n}}}{\mathrm{a}}(\, p_\mathrm{f} - p_i) \qquad \text{on } \gamma \qquad i = 1,2$$
$$\nabla_\mathrm{t} \cdot (\, -\mathrm{k}_{\mathrm{f},\mathbf{t}}\, \mathrm{d}\, \nabla_\mathrm{t}\, p_\mathrm{f}\,) = \mathrm{q}_\mathrm{f} - (\mathbf{K}_1\, \nabla\, p_1 \cdot \mathbf{n}_1|_\gamma + \mathbf{K}_2\, \nabla\, p_2 \cdot \mathbf{n}_2|_\gamma) \qquad \text{on } \gamma$$

Weak Formulation – for the porous rock matrix:

$$\left(\frac{4\mathrm{k}_{\mathrm{f},\mathbf{n}}}{\mathrm{d}} p_\mathrm{f},\ \{\!\{\, q_\mathrm{m}\,\}\!\}\right)_\gamma = \sum_i (\mathbf{K}_i\, \nabla\, p_{\mathrm{m}i},\ \nabla\, q_{\mathrm{m}i})_{\Omega_{\mathrm{m},i}} + \left(\frac{4\mathrm{k}_{\mathrm{f},\mathbf{n}}}{\mathrm{d}} \{\!\{\, p_\mathrm{m}\,\}\!\},\ \{\!\{\, q_\mathrm{m}\,\}\!\}\right)_\gamma + \left(\frac{\mathrm{k}_{\mathrm{f},\mathbf{n}}}{\mathrm{d}} [\![\, p_\mathrm{m}\,]\!],\ [\![\, q_\mathrm{m}\,]\!]\right)_\gamma$$

and for the fracture network:

$$\left(\frac{4\mathrm{k}_{\mathrm{f},\mathbf{n}}}{\mathrm{d}} \{\!\{\, p_\mathrm{m}\,\}\!\},\ q_\mathrm{f}\right)_\gamma = (\mathrm{k}_{\mathrm{f},\mathbf{t}}\, \mathrm{d}\, \nabla_\mathrm{f}\, p_\mathrm{f},\ \nabla_\mathrm{f}\, q_\mathrm{f})_\gamma + \left(\frac{4\mathrm{k}_{\mathrm{f},\mathbf{n}}}{\mathrm{d}} p_\mathrm{f},\ q_\mathrm{f}\right)_\gamma$$

Conceptual model similar to [1], implemented in DUNE with XFEM.

Outlook

Benchmark based on "Swedish Nuclear Power Inspectorate (SKI), The international hydrocoin project – background and results. Paris, 1987"

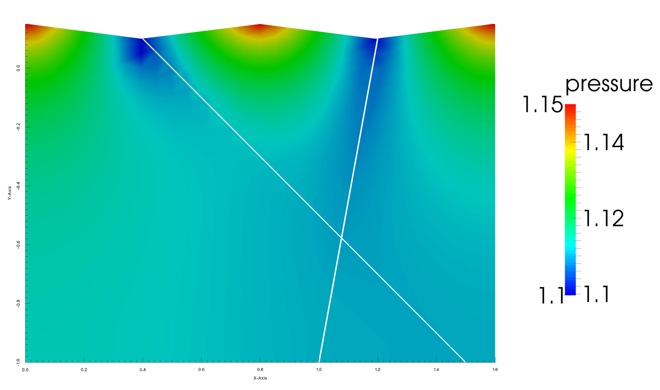


Figure 7: pressure distribution for the XFEM, discontinuous pressure method

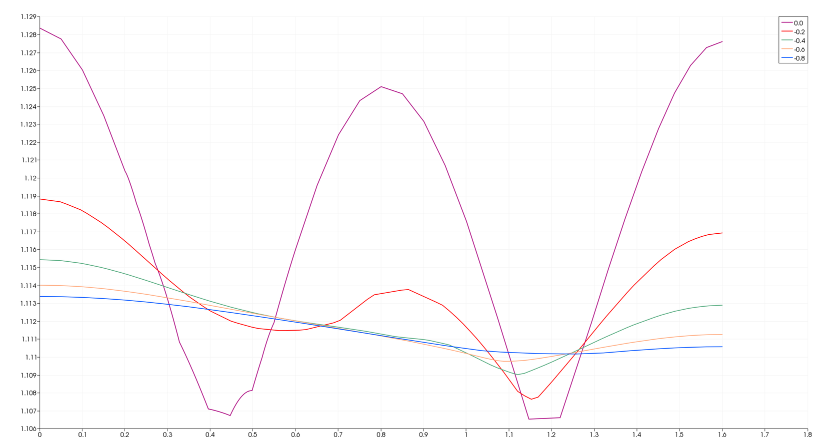


Figure 8: pressure plots at different depths

Literature

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