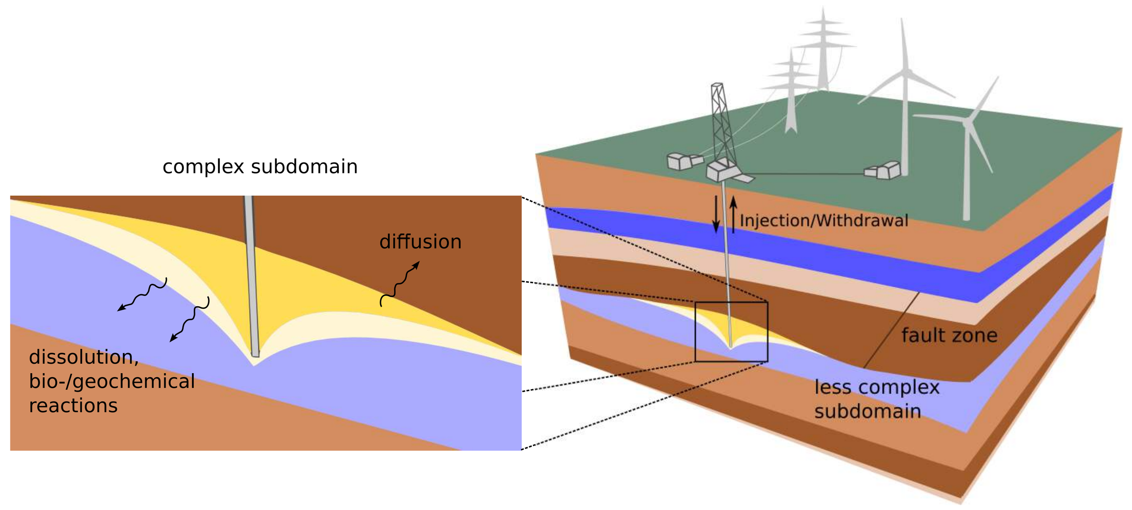
University of Stuttgart
Institute for Modelling Hydraulic and Environmental Systems
Department for Hydromechanics and Modelling of Hydrosystems

Adaptive Coupling of a Full-Dimensional Model with a Vertical Equilibrium Model

Beatrix Becker*, B. Guo°, K.W. Bandilla°, M.A. Celia°, B. Flemisch*, R. Helmig*
*University of Stuttgart, °Princeton University

1. Introduction



Underground storage system

Modeling challenges:

- large domains and limited data,
- locally complex processes,
- dynamic boundary conditions.

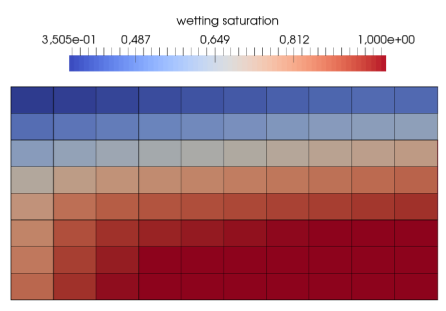
Here, we present a coupled model that adaptively applies:

- a full-dimensional model in regions of higher complexity and where the vertical equilibrium assumption does not hold,
- a vertical equilibrium model in the rest of the domain.

2. Models

2.1 Full-dimensional model:

- Mass balance equation:
$$\frac{\partial}{\partial t}(\varrho_\alpha \phi s_\alpha) + \nabla \cdot (\varrho_\alpha \mathbf{u}_\alpha) = \varrho_\alpha \psi^\alpha,$$
- Darcy's law:
$$\mathbf{u}_\alpha = -\frac{\mathbf{k} k_{r\alpha}}{\mu_\alpha}(\nabla p_\alpha - \varrho_\alpha \mathbf{g}),$$

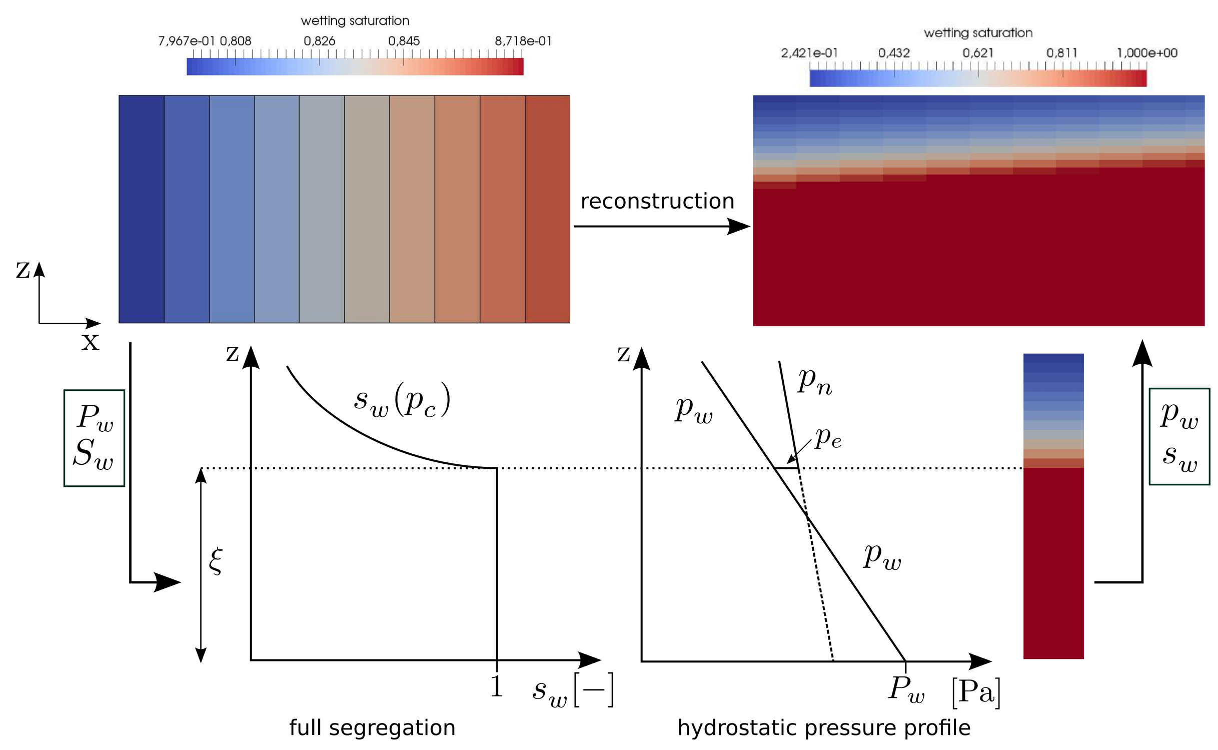


with wetting/non-wetting phase α, saturation s, pressure p, density ϱ_α, porosity ϕ, permeability tensor k, relative permeability $k_{r\alpha}$ viscosity μ_α, sink/source ψ^α.

2.2 Vertical equilibrium model:

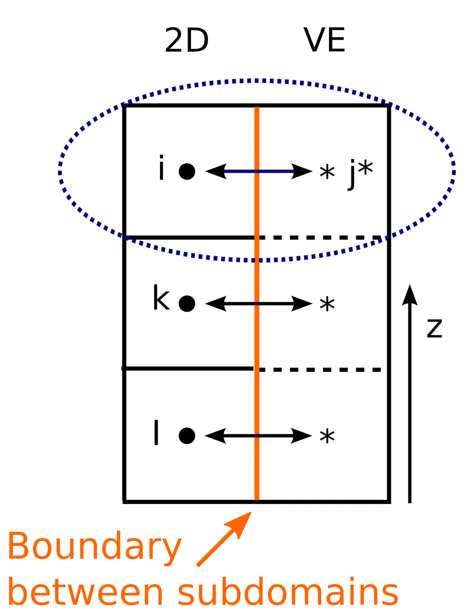
- Mass balance equation: $\frac{\partial}{\partial t}(\varrho_\alpha \Phi S_\alpha) + \nabla_{||} \cdot (\varrho_\alpha \mathbf{U}_\alpha) = \varrho_\alpha \Psi^\alpha,$
- Darcy's law: $\mathbf{U}_\alpha = -\mathbf{K}\Lambda_\alpha(\nabla_{||} P_\alpha - \varrho_\alpha \mathbf{G}),$

with vertically integrated variables and reference pressure.



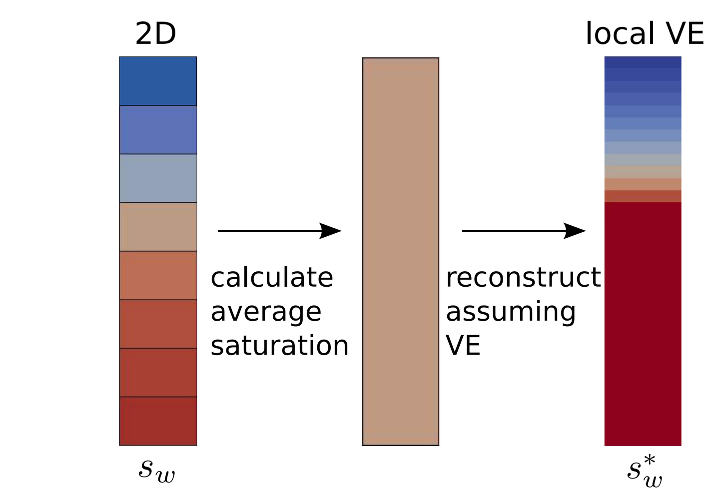
Reconstruction of solution in vertical direction

3. Model coupling

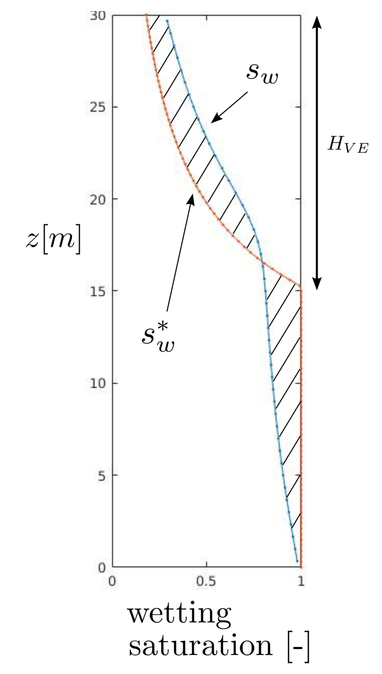


- Discretized mass balance equation (Finite Volume Method):
$\sum_j q_{tot,ij} = \sum_j v_{tot,ij} q_{ij} = q_{tot,i}$,
with source/sink $q_{tot,i}$.
- Total velocity from VE-cell i to 2D cell j:
$v_{tot,ij} = -k\lambda_{tot}\left(\frac{p^*_{wj} - p_{wi}}{\Delta x} + f_n \frac{p^*_{cj} - p_{ci}}{\Delta x}\right).$
- Reconstructed pressures in VE ghost cells:
$p^*_{wj} = P_{wj} - \varrho_w g z,$
$p^*_{cj} = p_c(s^*_w).$
- Calculation of secondary variables in VE ghost cells:
Total mobility $\lambda_{tot} = \lambda_w + \lambda_n$ and fractional flow function $f_n = \lambda_n / \lambda_{tot}$ based on averaged saturation in ghost cell saturation $\bar{s}^*_w$.

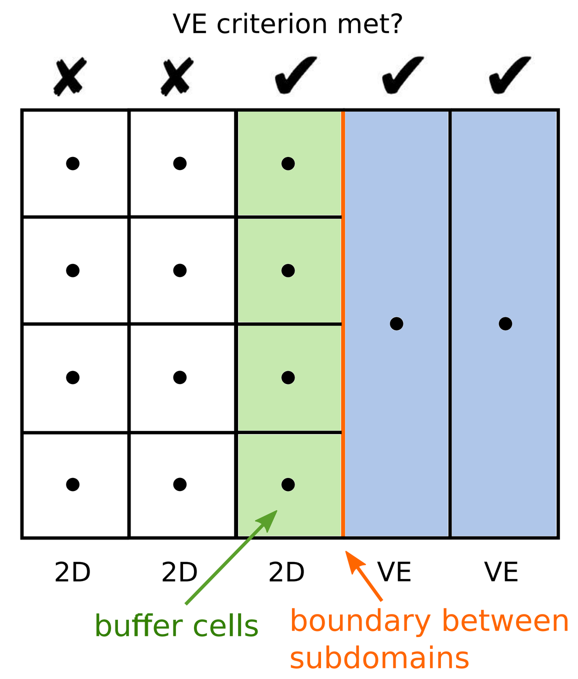
4. Model adaptation



Construction of VE saturation profile



Area between profiles



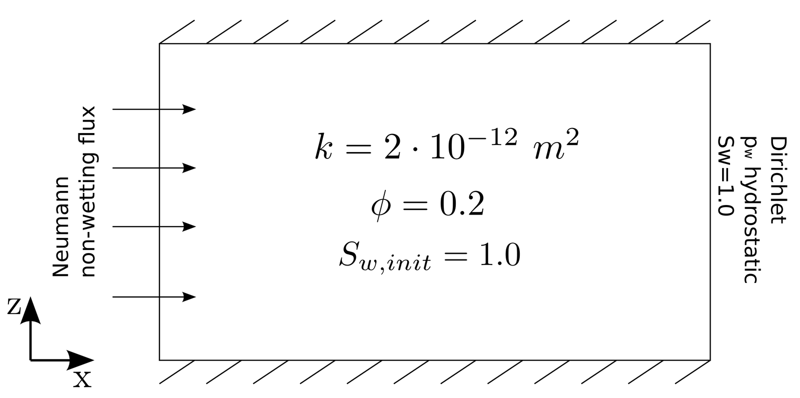
The criterion is normalized by the height H_{VE} of the VE gas plume.

$$c_{sat} = \frac{\int_0^H |s_w - s^*_w| \mathrm{d}z}{H_{VE}} < \epsilon_S,$$

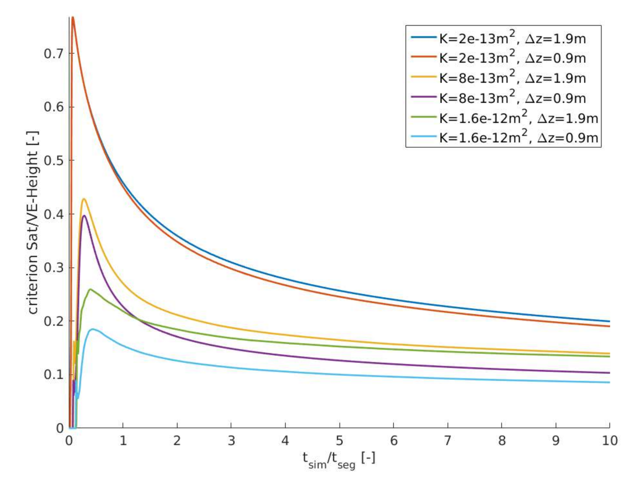
$$c_{relPerm} = \frac{\int_0^H |k_{rw} - k^*_{rw}| \mathrm{d}z}{H_{VE}} < \epsilon_r.$$

A buffer zone is introduced between the subdomains.

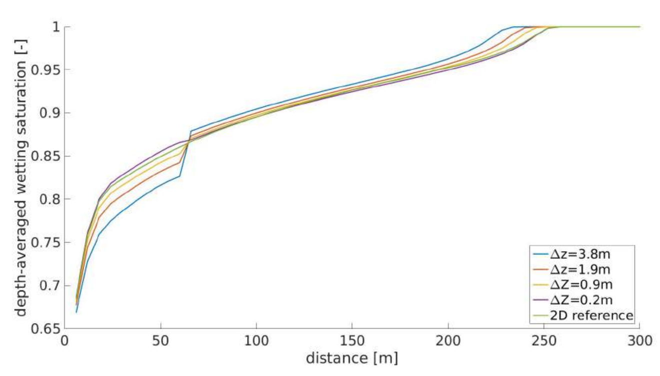
5. Results



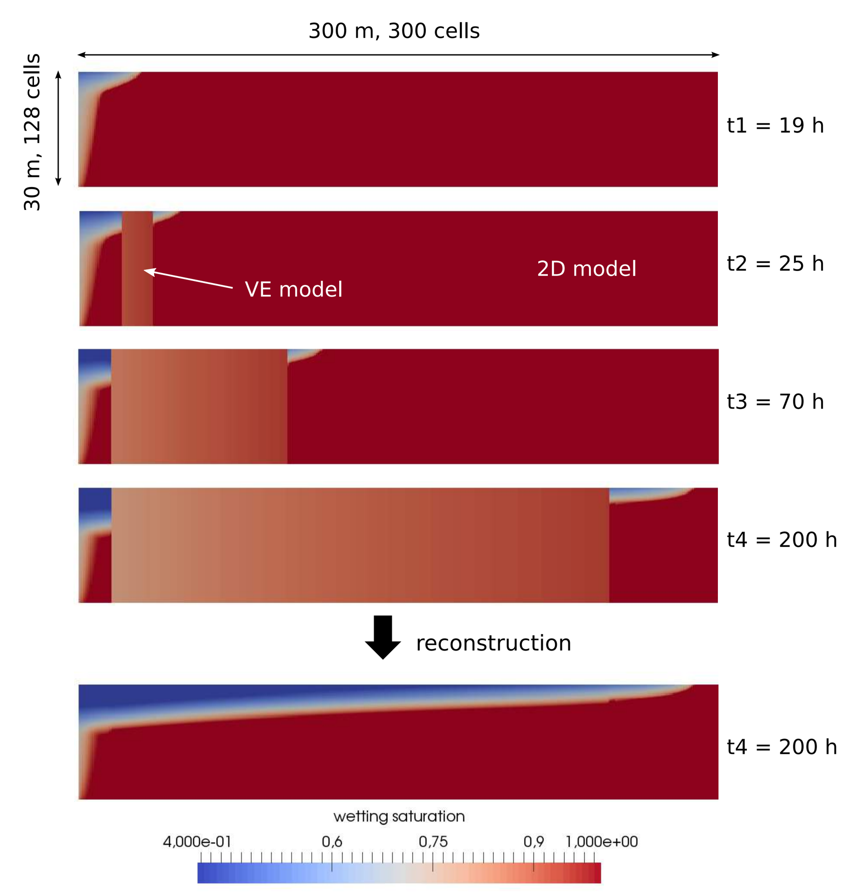
Brooks-Corey cap. pressure:
$\lambda = 2.0$, $p_e = 1$ bar
Phase properties (CH_4, water):
$\varrho_n = 59.2\ kg/m^3$
$\varrho_w = 991\ kg/m^3$
$\mu_n = 1.2 \cdot 10^{-5}\ Pa\,s$
$\mu_w = 5.2 \cdot 10^{-4}\ Pa\,s$
Injection rate: $Q_{nw} = 552\ t/m/a$



Saturation criterion in one column



Influence of grid size, non-adaptive



Adaptive modelcoupling

6. Outlook

- Analysis of advantages and disadvantages of adaptive concept.
- Include hysteresis in the model.
- Test concept for field scale case of underground energy storage.

Literature

[1] Nordbotten, J. and Celia, M. (2011). *Geological Storage of CO2.* John Wiley and Sons, New York.

[2] Weishaupt, K., Beck, M., Becker, B., Class, H., Fetzer, T., Flemisch, B., Futter, G., Gläser, D., Grüninger, C., Hommel, J., Kissinger, A., Koch, T., Schneider, M., Schröder, N., Schwenck, N., and Seitz, G. (2016). DuMuX 2.9.0.