A priori error estimates for parabolic problems with Dirac right-hand side

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In this talk, we investigate the numerical approximation of a parabolic problem in three space dimensions with a Dirac measure concentrated on a line as a source term. Such a problem can be derived from a model that describes the flow in fractured porous media, like blood flow in vascularized human tissue. To approximate the problem, we use the cG(1)dG(0)-method. This method is given by a standard linear finite element approximation in space and a constant discontinuous Galerkin approximation in time. Due to the singular source term, the solution of this problem is not smooth enough to apply a standard error analysis. Motivated by some numerical experiments, the convergence behavior of the cG(1)dG(0)-method is investigated. In particular, we are interested in local error estimates, i.e., we consider in space a L^2-norm on a fixed subdomain which excludes a neighborhood of the Dirac measure. The choice of such a norm is motivated by the observation that in many applications the error at the singularity is dominated by the model error (e.g., in dimension reduced settings) or is not the quantity of interest (e.g., in optimal control problems).

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