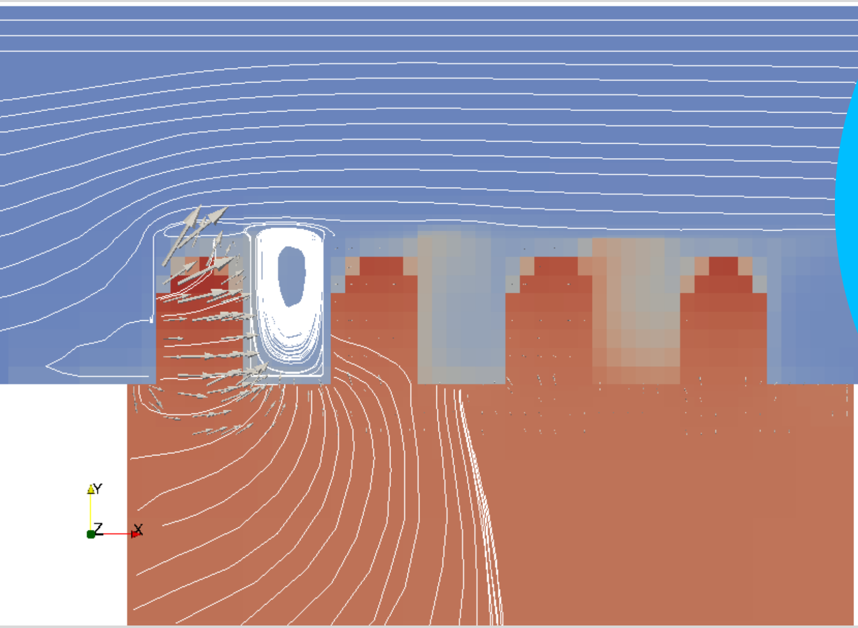
University of Stuttgart
Institute for Modelling Hydraulic and Environmental Systems
Department of Hydromechanics and Modelling of Hydrosystems

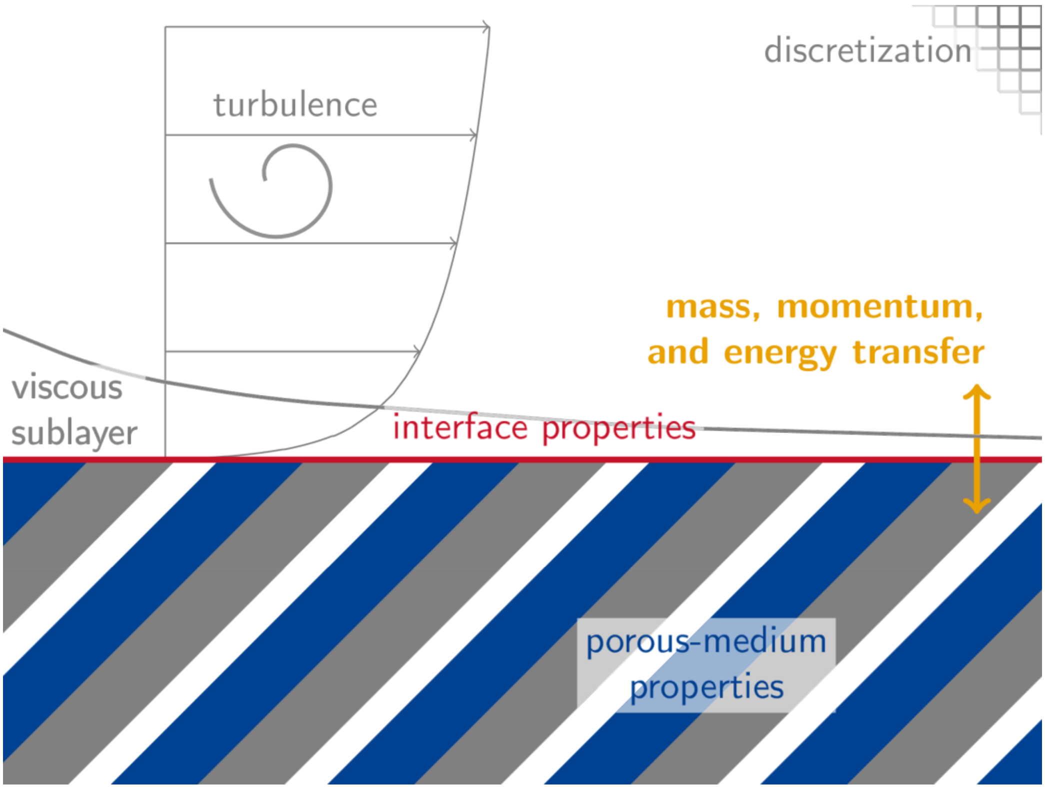
How Turbulence and Interface Properties Affect Porous-Medium / Free Flow Exchange Processes



T. Fetzer, K. M. Smits, C. Grüninger, B. Flemisch, R. Helmig

Introduction

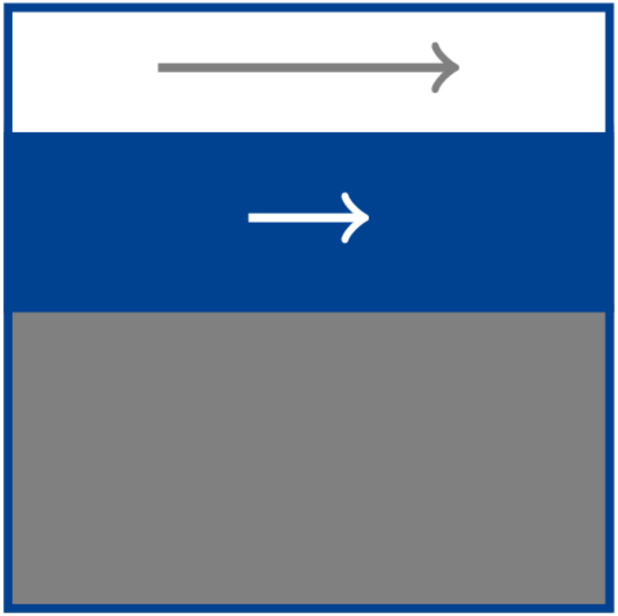
Challenges – Evaporation from a Porous Medium



Model

Porous Medium

- REV concept
- multiphase Darcy‘s law
- two fluid phases (gas, liquid)
- two components (air, water)
- compositional, non-isothermal
- local thermodynamic equilibrium
- $p_\mathrm{g}, S_\mathrm{l}, T$ or $p_\mathrm{g}, X_\mathrm{g}^\mathrm{w}, T$



Model

Porous Medium Equations

- mass balance

$$\sum_{\alpha \in \{\mathrm{l},\mathrm{g}\}} \left\{ \underbrace{\phi \frac{(\partial \varrho_\alpha S_\alpha)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot (\varrho_\alpha \mathbf{v}_\alpha)}_{\text{advection}} - q_\alpha \right\} = 0$$

- momentum balance (Darcy)

$$\frac{k_{\mathrm{r},\alpha}}{\nu_\alpha \varrho_\alpha} \mathbf{K} \left(\underbrace{\nabla p_\alpha}_{\text{pressure}} - \underbrace{\varrho_\alpha \mathbf{g}}_{\text{gravity}} \right) + \mathbf{v}_\alpha = 0$$

- component mass balance

$$\sum_{\alpha \in \{\mathrm{l},\mathrm{g}\}} \left\{ \underbrace{\phi \frac{\partial \left(\varrho_\alpha S_\alpha X_\alpha^\kappa \right)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot \left(\varrho_\alpha X_\alpha^\kappa \mathbf{v}_\alpha \right)}_{\text{advection}} - \underbrace{\nabla \cdot \left(D_{\alpha,\mathrm{pm}}^\kappa \varrho_{\alpha,\mathrm{mol}} M^\kappa \nabla x_\alpha^{\kappa,\mathrm{if}} \right)}_{\text{diffusion}} - \underbrace{q_\alpha^\kappa}_{\text{source}} \right\} = 0$$

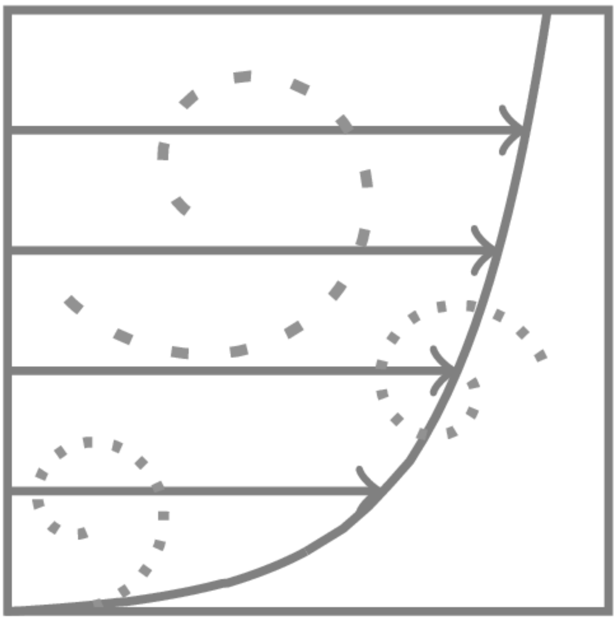
- energy balance

$$\sum_{\alpha \in \{\mathrm{l},\mathrm{g}\}} \left\{ \underbrace{\phi \frac{\partial \left(\varrho_\alpha S_\alpha u_\alpha \right)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot \left(\varrho_\alpha h_\alpha \mathbf{v}_\alpha \right)}_{\text{advection}} \right\} + \underbrace{(1 - \phi) \frac{(\partial \varrho_\mathrm{s} c_\mathrm{s} T)}{\partial t}}_{\text{storage (solid)}} - \underbrace{\nabla \cdot (\lambda_\mathrm{pm} \nabla T)}_{\text{conduction}} - \underbrace{q^T}_{\text{source}} = 0$$

Model

Free Flow

- laminar / turbulent (RANS)
- single phase (gas)
- two components (air / water)
- compositional, non-isothermal
- Ma < 0.3
- $p_{\mathrm{g}}, \mathbf{v}_{\mathrm{g}}, X_{\mathrm{g}}^{\mathrm{w}}, T$



Model

Free Flow Equations

- mass balance

$$\underbrace{\frac{\partial \varrho_\mathrm{g}}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot \left(\varrho_\mathrm{g} \bar{\mathbf{v}}_\mathrm{g} \right)}_{\text{advection}} = 0$$

- momentum balance (RANS)

$$\underbrace{\frac{\partial \left(\varrho_\mathrm{g} \bar{\mathbf{v}}_\mathrm{g} \right)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot \left(\varrho_\mathrm{g} \bar{\mathbf{v}}_\mathrm{g} \bar{\mathbf{v}}_\mathrm{g}^\mathsf{T} \right)}_{\text{inertia}} - \nabla \cdot \left(\left[\underbrace{\varrho_\mathrm{g} \nu_\mathrm{g}}_{\text{viscous}} + \underbrace{\varrho_\mathrm{g} \nu_\mathrm{g,t}}_{\text{eddy viscosity}} \right] \nabla \left(\bar{\mathbf{v}}_\mathrm{g} + \bar{\mathbf{v}}_\mathrm{g}^\mathsf{T} \right) \right) + \underbrace{\nabla \bar{p}_\mathrm{g}}_{\text{pressure}} - \underbrace{\varrho_\mathrm{g} \mathbf{g}}_{\text{gravity}} = 0$$

- component mass balance

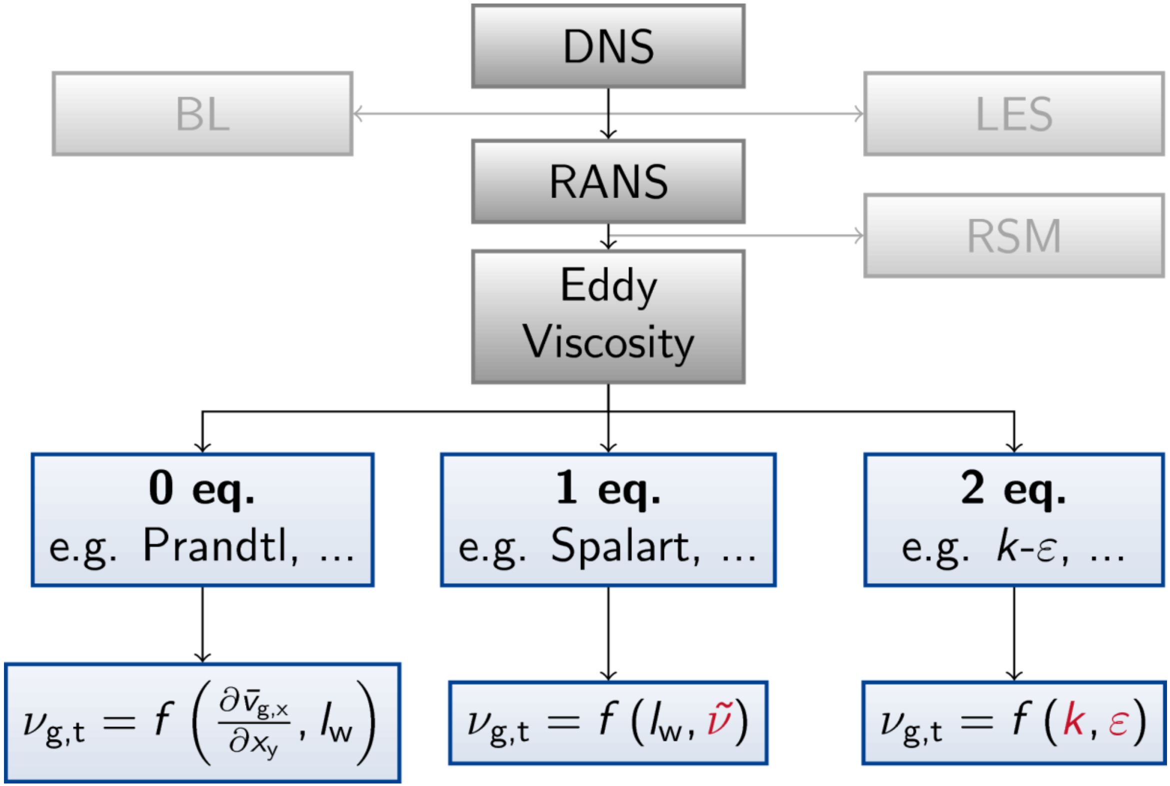
$$\underbrace{\frac{\partial \left(\varrho_\mathrm{g} \bar{X}_\mathrm{g}^\kappa \right)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot \left(\varrho_\mathrm{g} \bar{X}_\mathrm{g}^\kappa \bar{\mathbf{v}}_\mathrm{g} \right)}_{\text{advection}} - \nabla \cdot \left(\left[\underbrace{D_\mathrm{g}^\kappa}_{\text{diffusion}} + \underbrace{D_\mathrm{g,t}^\kappa}_{\text{eddy diffusion}} \right] \varrho_\mathrm{g,mol} M^\kappa \nabla \bar{x}_\mathrm{g}^\kappa \right) - \underbrace{q_\mathrm{g}^\kappa}_{\text{source}} = 0$$

- energy balance

$$\underbrace{\frac{\partial \left(\varrho_\mathrm{g} \bar{u}_\mathrm{g} \right)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot \left(\varrho_\mathrm{g} \bar{h}_\mathrm{g} \bar{\mathbf{v}}_\mathrm{g} \right)}_{\text{advection}} - \sum_{\kappa \in \{\mathrm{a,w}\}} \underbrace{\left\{ \nabla \cdot \left(\bar{h}_\mathrm{g}^\kappa \bar{j}_\mathrm{g,ff,t,diff}^\kappa \right) \right\}}_{\text{(eddy-)diffusion}} - \nabla \cdot \left(\left[\underbrace{\lambda_\mathrm{g}}_{\text{conduction}} + \underbrace{\lambda_\mathrm{g,t}}_{\text{eddy conduction}} \right] \nabla \bar{T} \right) - \underbrace{q_\mathrm{g}^T}_{\text{source}} = 0$$

Model

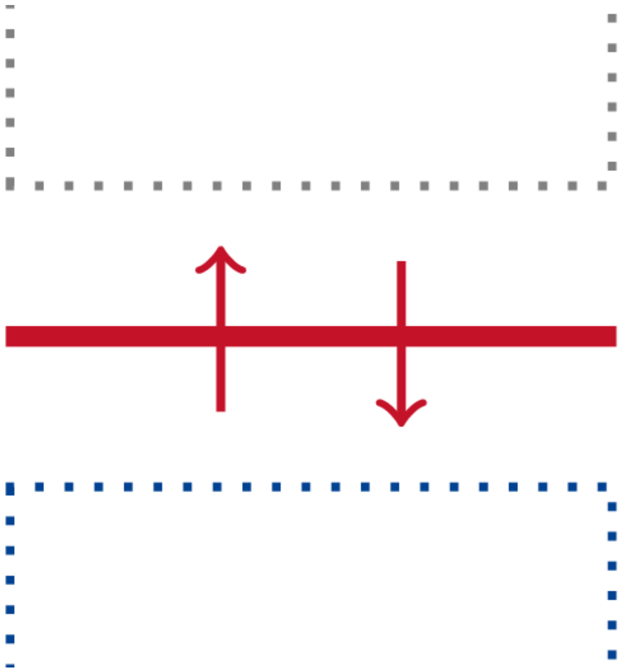
Turbulence



Model

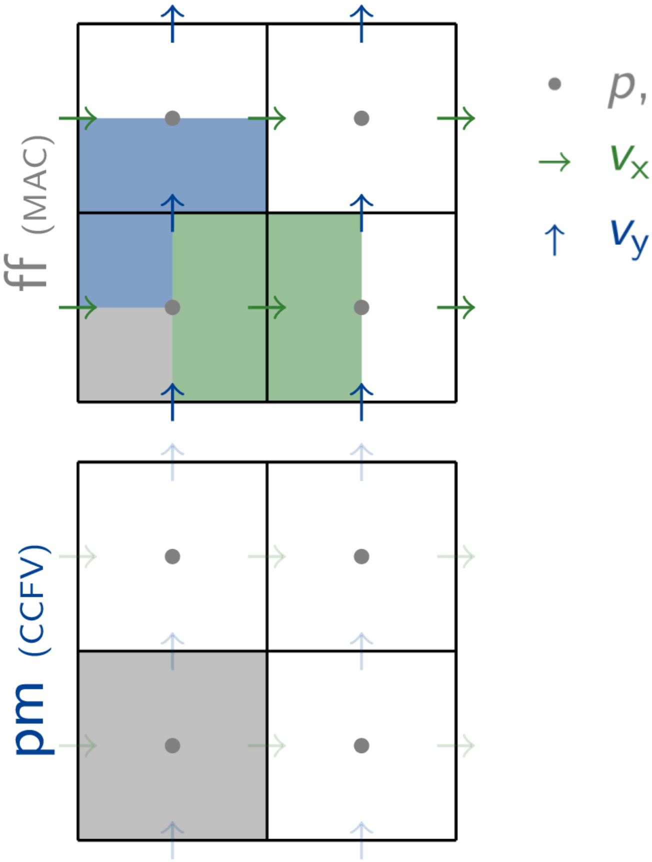
Coupling

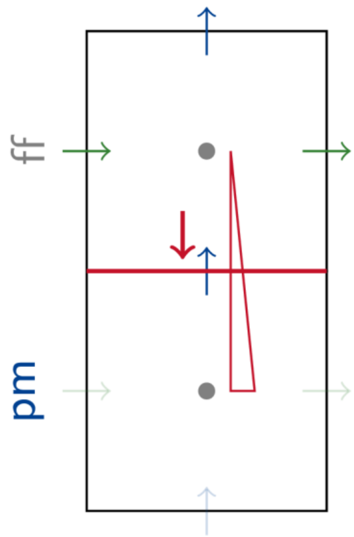
- after [Mosthaf et al. 2011, Fetzer et al. 2016]
- local thermodynamic equilibrium
- continuity of fluxes
- continuity of primary variables



Model

Coupling Discretization





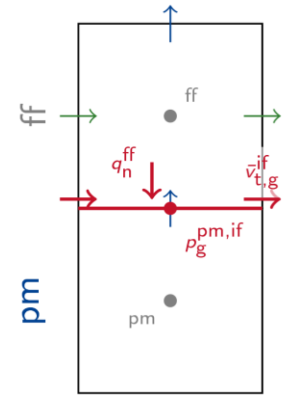
$q_n^{pm} = q_n^{ff}$
$q_n^{ff} = \checkmark$
$u^{if} = \checkmark$
$\nabla u^{if} = \checkmark$

[Grüninger, Fetzer, Flemisch, Helmig, in preparation]

Model

Coupling Equations

- mass $$\left[\left(\varrho_\mathrm{g}\bar{\mathbf{v}}_\mathrm{g}\right)\cdot\mathbf{n}\right]^\mathrm{ff} = -\left[\left(\varrho_\mathrm{g}\mathbf{v}_\mathrm{g} + \varrho_\mathrm{l}\mathbf{v}_\mathrm{l}\right)\cdot\mathbf{n}\right]^\mathrm{pm}$$



- tangential momentum (only free flow) $$\left[\bar{v}_\mathrm{t,g}^\mathrm{if} - \frac{\sqrt{K}}{\alpha_\mathrm{BJ}}\frac{\bar{v}_\mathrm{t,g}^\mathrm{ff} - \bar{v}_\mathrm{t,g}^\mathrm{if}}{1/2\,\Delta y}\right]^\mathrm{ff} = 0$$

- normal momentum (only free flow) $$\left[\left(\left\{\varrho_\mathrm{g}\bar{\mathbf{v}}_\mathrm{g}\bar{\mathbf{v}}_\mathrm{g}^\mathsf{T} - \bar{\boldsymbol{\tau}}_\mathrm{g,t} + \bar{p}_\mathrm{g}\mathbf{I}\right\}\mathbf{n}\right)\cdot\mathbf{n}\right]^\mathrm{ff} = \left[p_\mathrm{g}^\mathrm{if}\right]^\mathrm{pm}$$

$$\left[\varrho_\mathrm{g} v_\mathrm{n,g}^\mathrm{if}\right]^\mathrm{ff} = -\left[-\varrho_\mathrm{g}\frac{k_\mathrm{r,g}K}{\mu_\mathrm{g}}\frac{p_\mathrm{g}^\mathrm{if} - p_\mathrm{g}^\mathrm{pm}}{1/2\,\Delta y}n_\mathrm{y} - \varrho_\mathrm{l}\frac{k_\mathrm{r,l}K}{\mu_\mathrm{l}}\frac{p_\mathrm{l}^\mathrm{if} - p_\mathrm{l}^\mathrm{pm}}{1/2\,\Delta y}n_\mathrm{y}\right]^\mathrm{pm}$$

Model

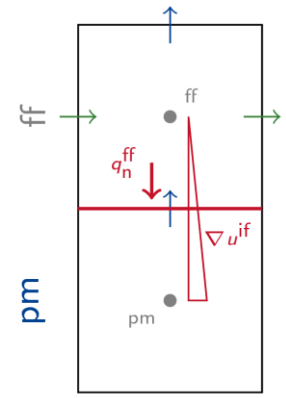
Coupling Equations 2

- component mass

$$\left[\left(\varrho_\mathrm{g} \bar{X}_\mathrm{g}^\kappa \bar{\mathbf{v}}_\mathrm{g} - \left(D_\mathrm{g}^\kappa + D_\mathrm{g,t}^\kappa\right) \varrho_\mathrm{g,mol} M^\kappa \nabla x_\mathrm{g}^{\kappa,\mathrm{if}}\right) \cdot \mathbf{n}\right]^\mathrm{ff}$$
$$= -\left[\left(\varrho_\mathrm{g} X_\mathrm{g}^\kappa \mathbf{v}_\mathrm{g} - \mathbf{j}_\mathrm{g,pm,diff}^\kappa + \varrho_\mathrm{l} \mathbf{v}_\mathrm{l} X_\mathrm{l}^\kappa - \mathbf{j}_\mathrm{l,pm,diff}^\kappa\right) \cdot \mathbf{n}\right]^\mathrm{pm}$$

- energy

$$\left[\left(\varrho_\mathrm{g} \bar{h}_\mathrm{g} \bar{\mathbf{v}}_\mathrm{g} - \bar{h}_\mathrm{g}^\mathrm{a} \bar{\mathbf{j}}_\mathrm{g,ff,t,diff}^\mathrm{a} - \bar{h}_\mathrm{g}^\mathrm{w} \bar{\mathbf{j}}_\mathrm{g,ff,t,diff}^\mathrm{w} - \lambda_\mathrm{g,t} \nabla \bar{T}^\mathrm{if}\right) \cdot \mathbf{n}\right]^\mathrm{ff}$$
$$= -\left[\left(\varrho_\mathrm{g} h_\mathrm{g} \mathbf{v}_\mathrm{g} + \varrho_\mathrm{l} h_\mathrm{l} \mathbf{v}_\mathrm{l} - \lambda_\mathrm{pm} \nabla T\right) \cdot \mathbf{n}\right]^\mathrm{pm}$$



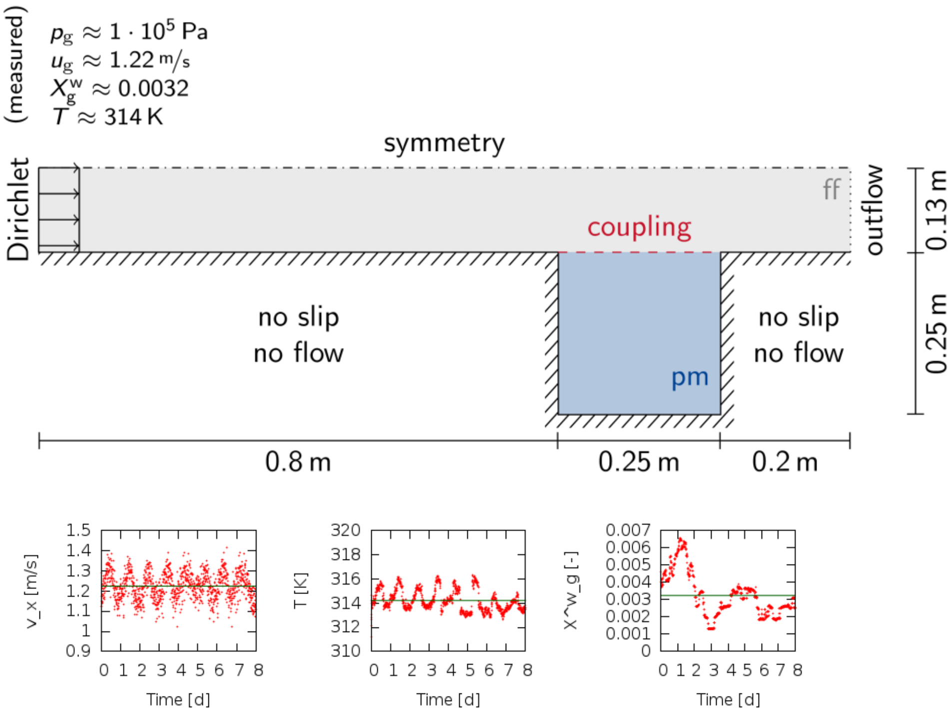
Results

Implementation and Setup

- MAC / cell-centered FV
- fully implicit
- monolithic
- numerical simulator

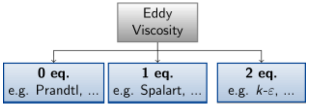
DuMu^x

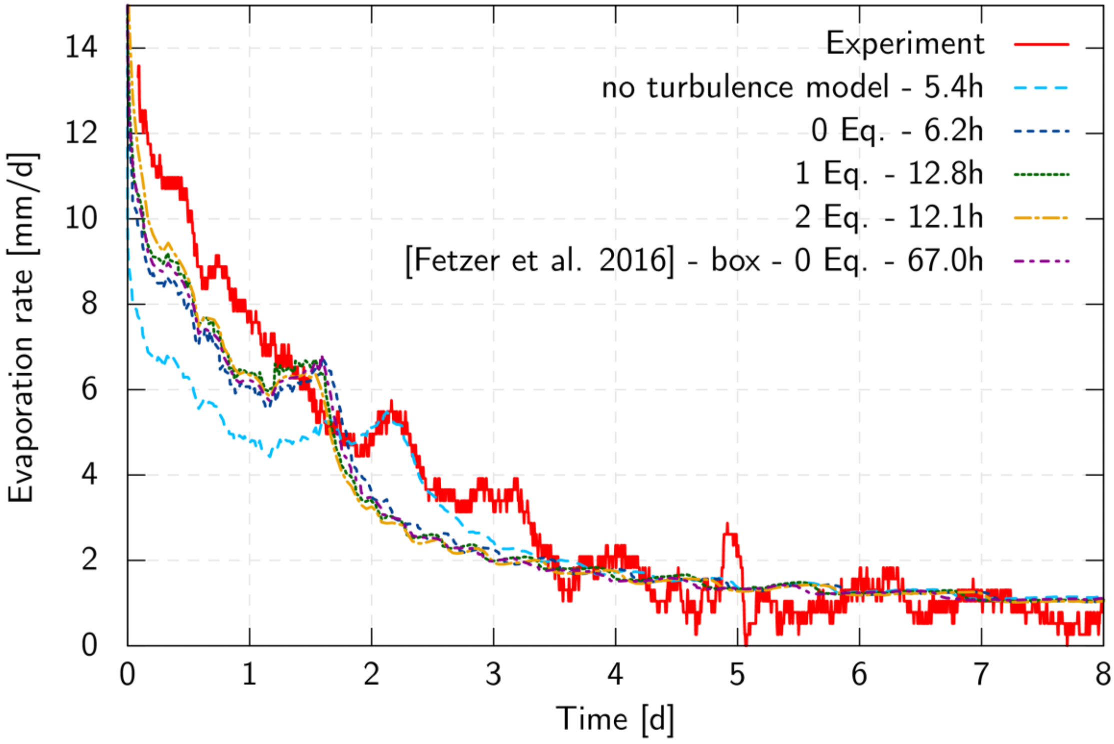
- experiments:
  [Davarzani et al. 2014]



Results

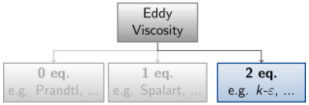
Turbulence Modeling

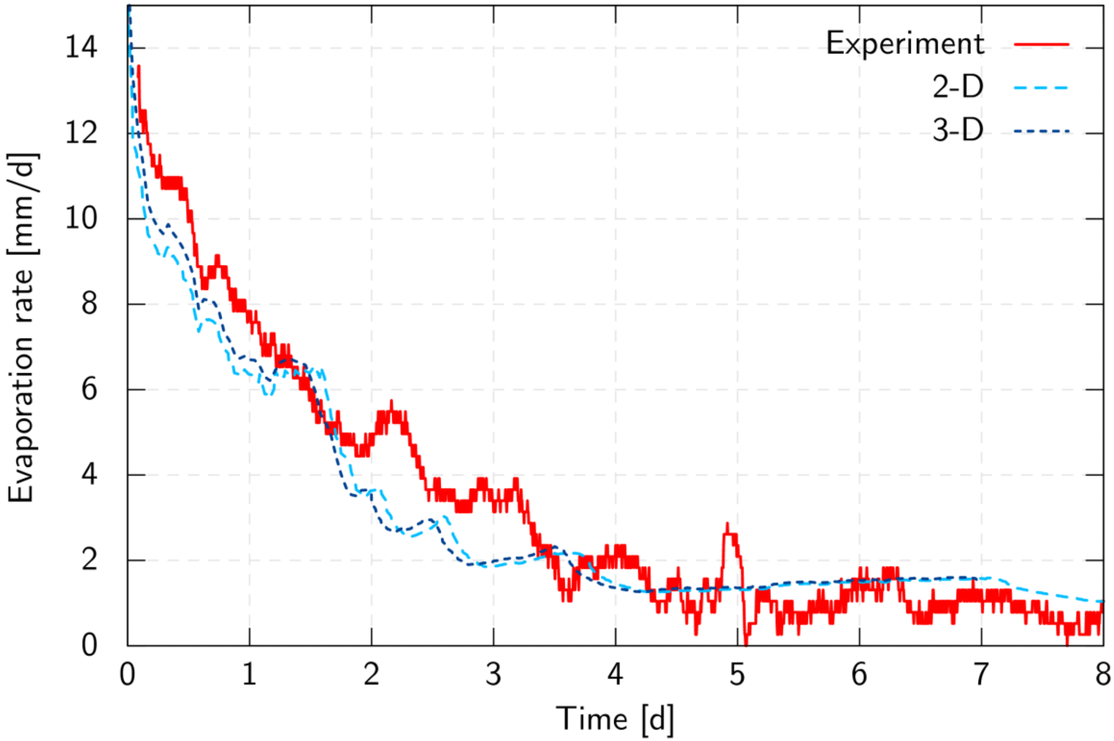


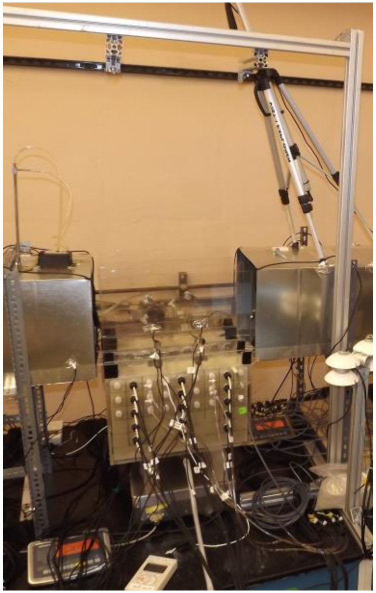


Results

2-D vs. 3-D

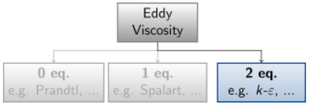


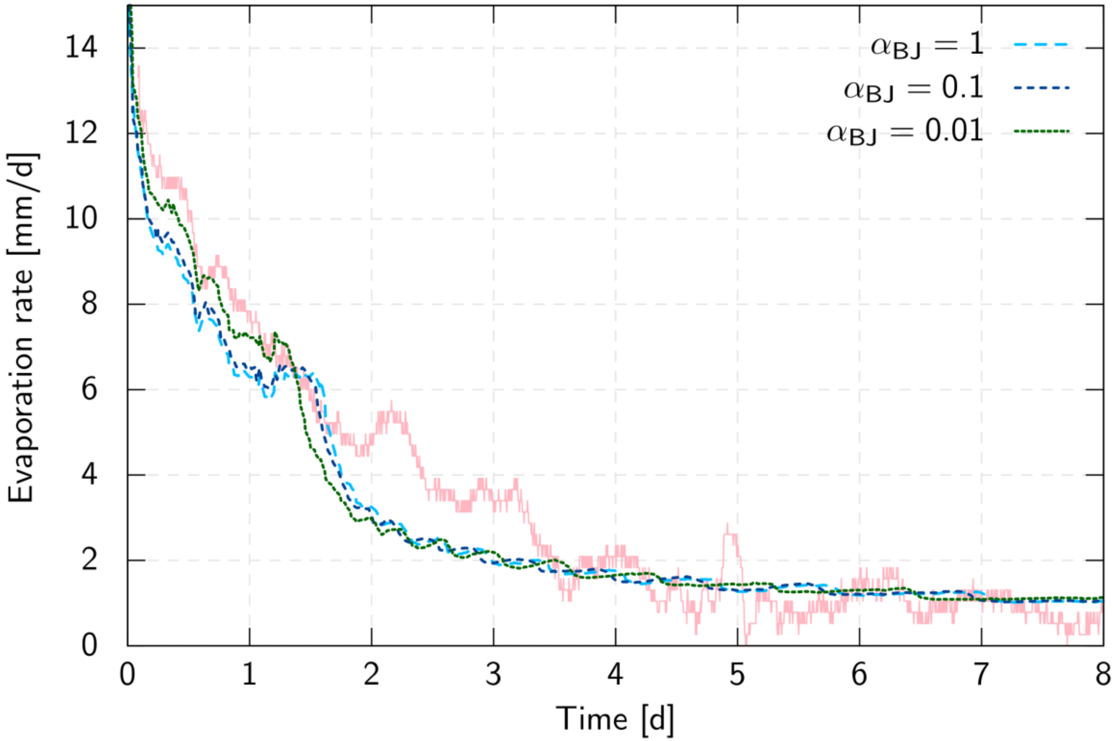




Results

Beavers-Joseph Coefficient

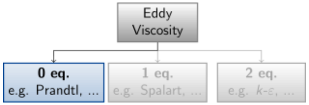


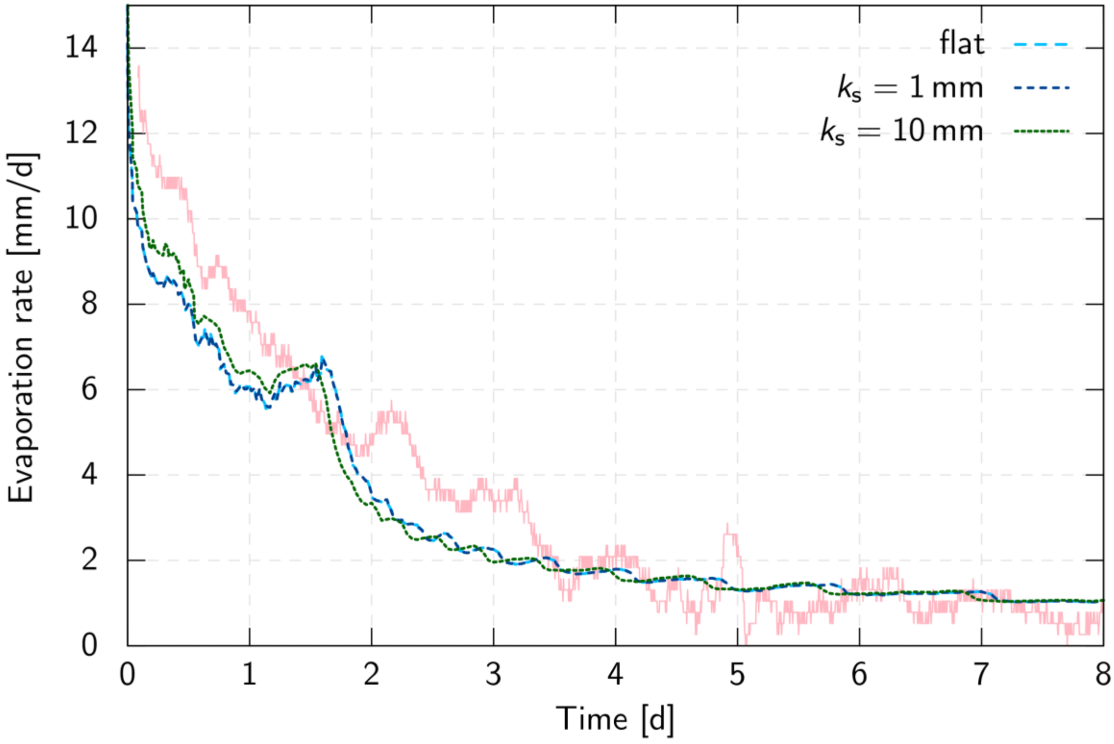


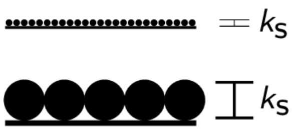
$$\left[\bar{v}_{t,g}^{if} - \frac{\sqrt{K}}{\alpha_{BJ}} \frac{\bar{v}_{t,g}^{ff} - \bar{v}_{t,g}^{if}}{{}^{1}/{}_{2}\Delta y} \right]^{ff} = 0$$

Results

Sand-Grain Roughness

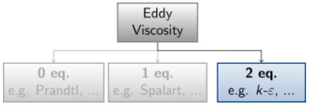


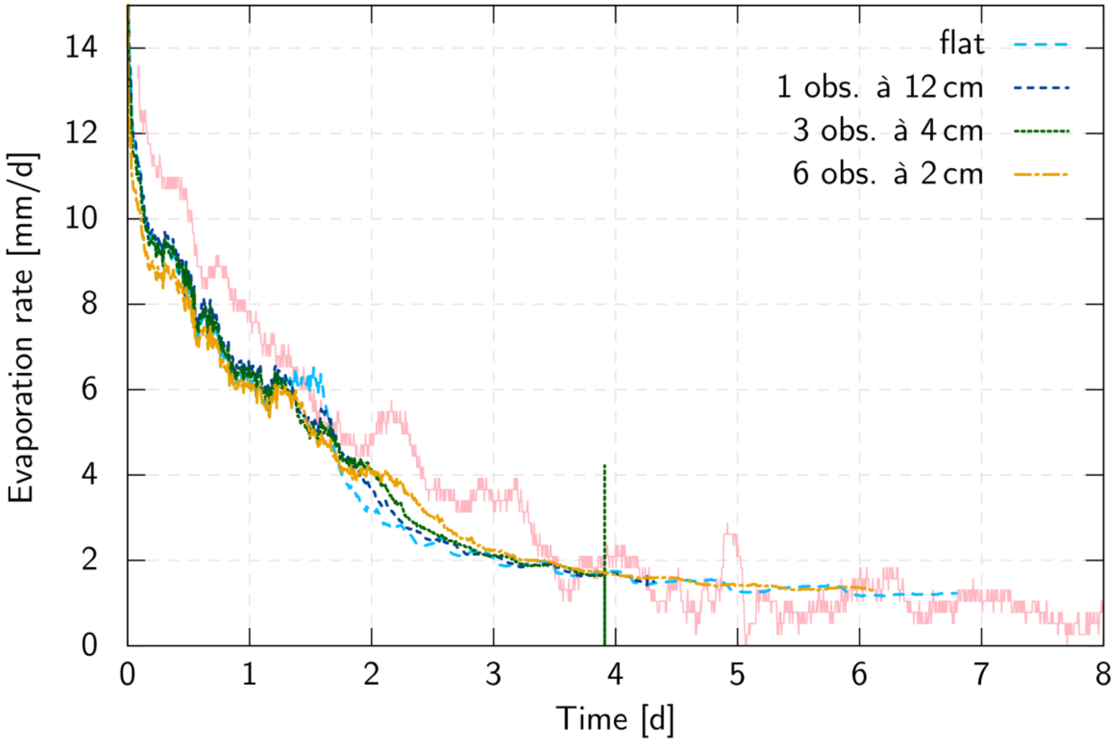


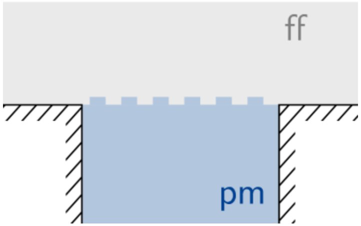


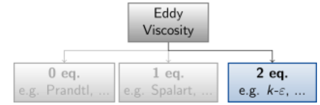
Results

Obstacles with H=10mm



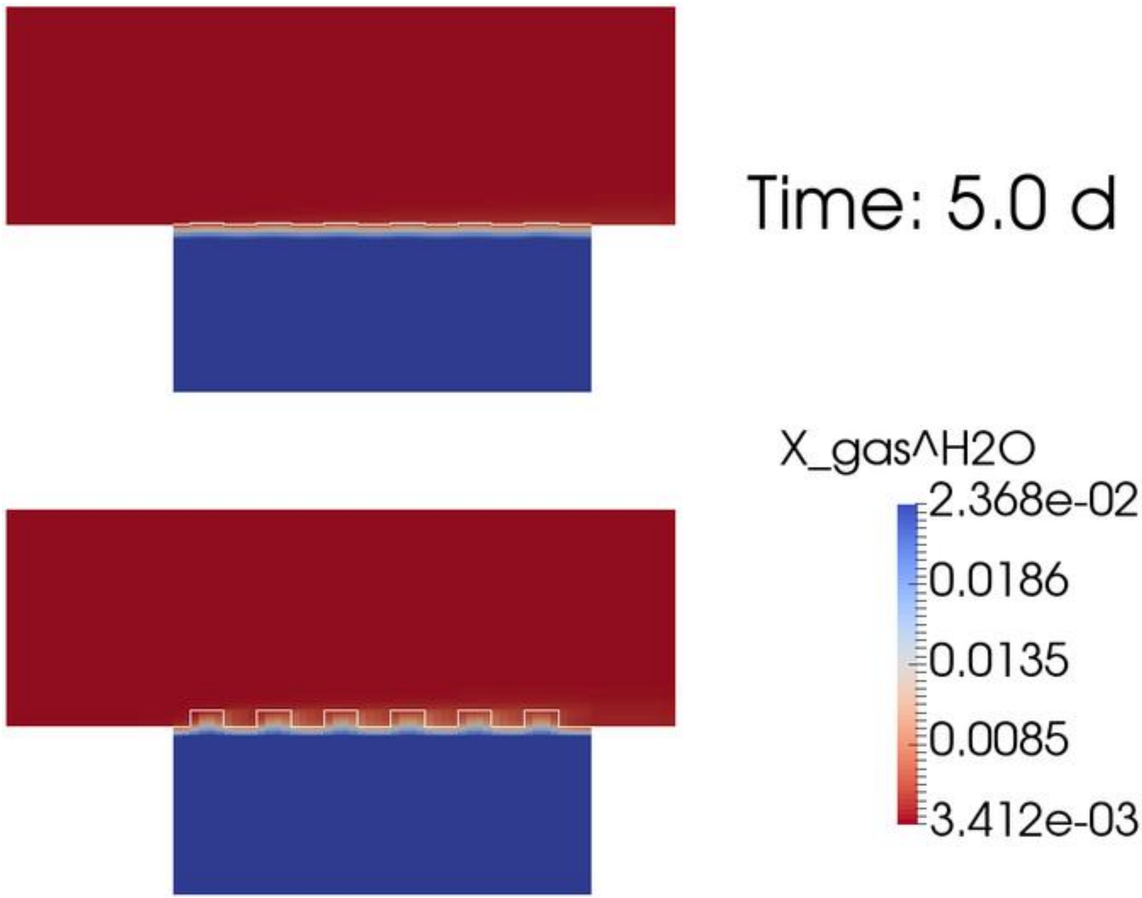






Results

Obstacles



Summary and Outlook

Summary

- coupling of free flow (MAC) to porous-medium flow (CCFV)
- turbulent models of different complexity
- influence of interface properties

Outlook

- turbulent / barometric pumping
- Forchheimer extension to Darcy‘s law
- local thermodynamic non-equilibrium
- pore network model at the interface

Thank you for your attention!

- [Beavers, G. S. & Joseph, D. D. 1967] - Boundary conditions at a naturally permeable wall
  Journal of Fluid Mechanics, **1967**, *30*, 197-207
- [Davarzani et al. 2014] - Study of the effect of wind speed on evaporation from soil through integrated modeling of the atmospheric boundary layer and shallow subsurface
  Water Resources Research, **2014**, *50*, 1-20
- [Fetzer et al. 2016] - Effect of Turbulence and Roughness on Coupled Porous-Medium/Free Flow Exchange Processes
  Transport in Porous Media, **2016**
- [Grüninger, Fetzer, Flemisch, Helmig (in preparation)] - Coupling DuMuX and DUNE-PDELab to investigate evaporation at the interface between Darcy and Navier-Stokes flow
- [Mosthaf et al. 2011] - A coupling concept for two-phase compositional porous-medium and single-phase compositional free flow
  Water Resources Research, **2011***, 47,* W10522