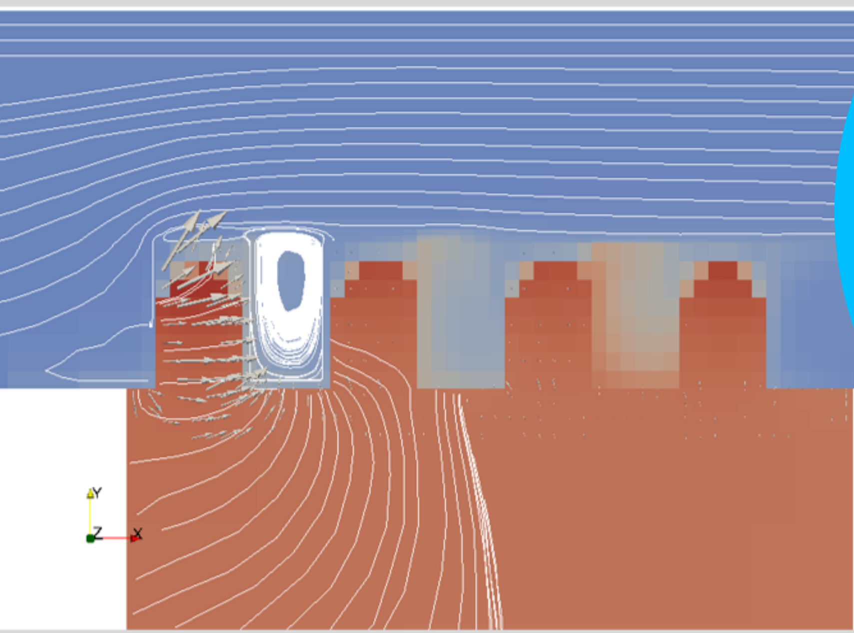
University of Stuttgart
Institute for Modelling Hydraulic and Environmental Systems
Department of Hydromechanics and Modelling of Hydrosystems

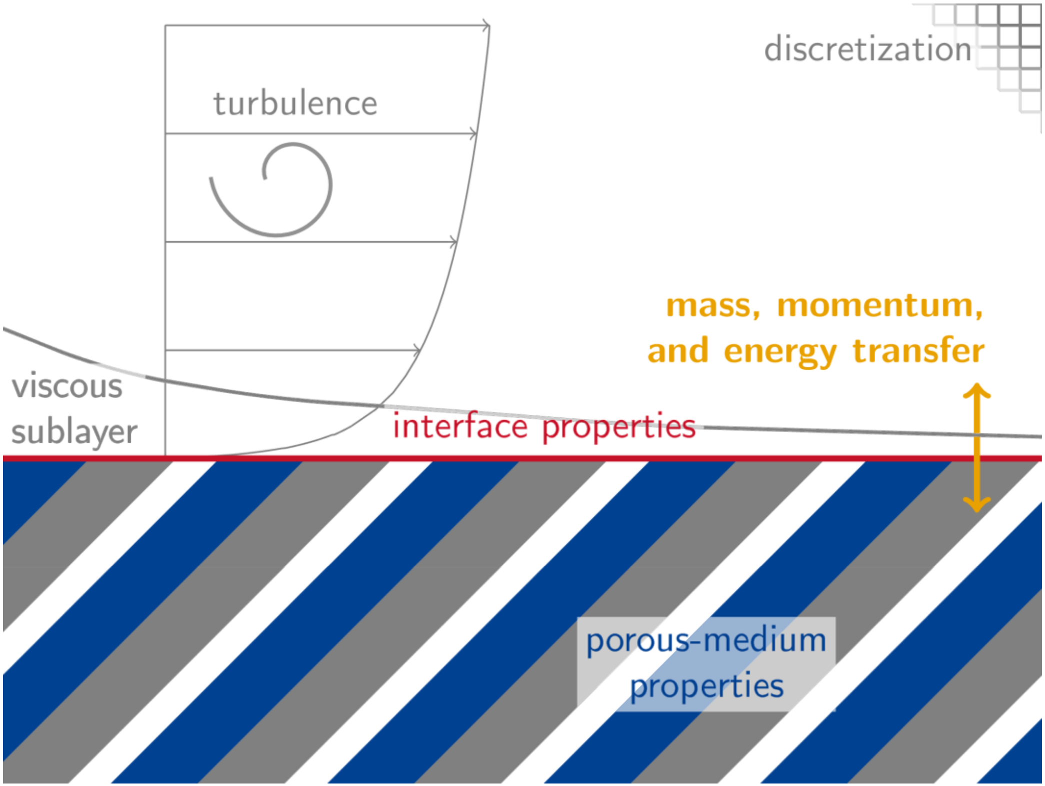


Influence of Turbulence and Interface Properties on Porous-Medium / Free Flow Exchange Processes

T. Fetzer, K. M. Smits, C. Grüninger, B. Flemisch, R. Helmig

Introduction

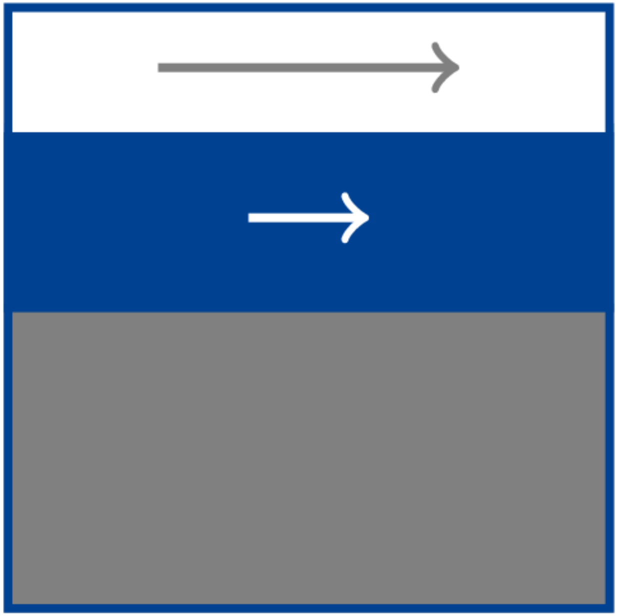
Challenges – Evaporation from a Porous Medium



Model

Porous Medium

- REV concept
- multiphase Darcy‘s law
- two fluid phases (gas, liquid)
- two components (air, water)
- compositional, non-isothermal
- local thermodynamic equilibrium
- p_g, S_l, T or p_g, X_g^w, T



Model

Porous Medium Equations

- mass balance

$$\sum_{\alpha \in \{l,g\}} \left\{ \phi \frac{(\partial \varrho_\alpha S_\alpha)}{\partial t} + \nabla \cdot (\varrho_\alpha \mathbf{v}_\alpha) - q_\alpha \right\} = 0$$

storage advection

- momentum balance (Darcy)

$$\frac{k_{r,\alpha}}{\nu_\alpha \varrho_\alpha} \mathbf{K} \left(\nabla p_\alpha - \varrho_\alpha \mathbf{g} \right) + \mathbf{v}_\alpha = 0$$

pressure gravity

- component mass balance

$$\sum_{\alpha \in \{l,g\}} \left\{ \phi \frac{\partial \left(\varrho_\alpha S_\alpha X_\alpha^\kappa \right)}{\partial t} + \nabla \cdot (\varrho_\alpha X_\alpha^\kappa \mathbf{v}_\alpha) - \nabla \cdot \left(D_{\alpha,pm}^\kappa \varrho_{\alpha,mol} M^\kappa \nabla x_\alpha^{\kappa,if} \right) - q_\alpha^\kappa \right\} = 0$$

storage advection diffusion source

- energy balance

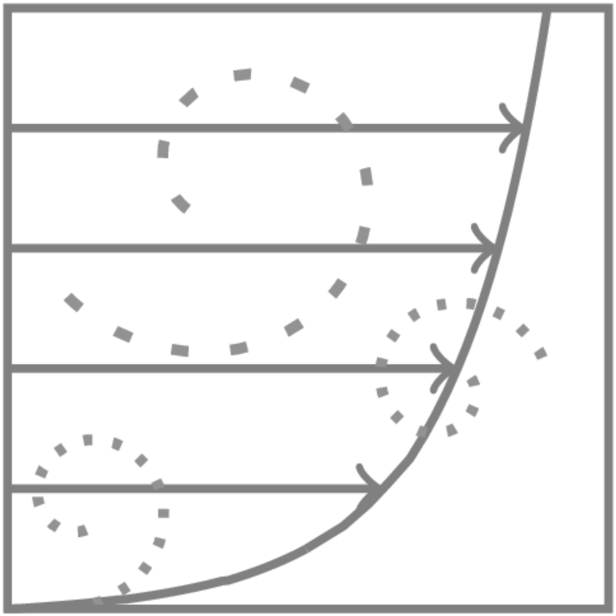
$$\sum_{\alpha \in \{l,g\}} \left\{ \phi \frac{\partial \left(\varrho_\alpha S_\alpha u_\alpha \right)}{\partial t} + \nabla \cdot (\varrho_\alpha h_\alpha \mathbf{v}_\alpha) \right\} + (1 - \phi) \frac{(\partial \varrho_s c_s T)}{\partial t} - \nabla \cdot (\lambda_{pm} \nabla T) - q^T = 0$$

storage advection storage (solid) conduction source

Model

Free Flow

- laminar / turbulent (RANS)
- single phase (gas)
- two components (air / water)
- compositional, non-isothermal
- Ma < 0.3
- $p_\mathrm{g}, \mathbf{v}_\mathrm{g}, X^\mathrm{w}_\mathrm{g}, T$



Model

Free Flow Equations

- mass balance

$$\underbrace{\frac{\partial \varrho_g}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot (\varrho_g \bar{\mathbf{v}}_g)}_{\text{advection}} = 0$$

- momentum balance (RANS)

$$\underbrace{\frac{\partial (\varrho_g \bar{\mathbf{v}}_g)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot (\varrho_g \bar{\mathbf{v}}_g \bar{\mathbf{v}}_g^{\mathsf{T}})}_{\text{inertia}} - \nabla \cdot \left(\left[\underbrace{\varrho_g \nu_g}_{\text{viscous}} + \underbrace{\varrho_g \nu_{g,t}}_{\text{eddy viscosity}} \right] \nabla \left(\bar{\mathbf{v}}_g + \bar{\mathbf{v}}_g^{\mathsf{T}} \right) \right) + \underbrace{\nabla \bar{p}_g}_{\text{pressure}} - \underbrace{\varrho_g \mathbf{g}}_{\text{gravity}} = 0$$

- component mass balance

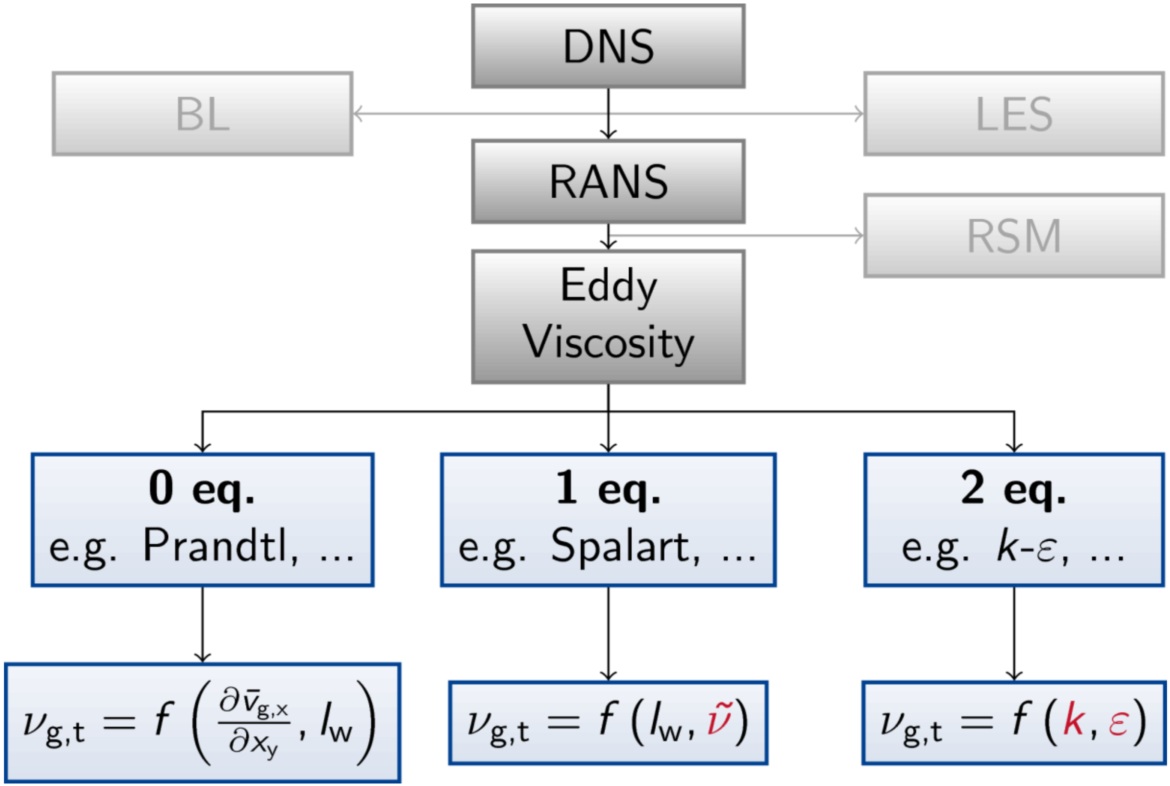
$$\underbrace{\frac{\partial (\varrho_g \bar{X}_g^\kappa)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot (\varrho_g \bar{X}_g^\kappa \bar{\mathbf{v}}_g)}_{\text{advection}} - \nabla \cdot \left(\left[\underbrace{D_g^\kappa}_{\text{diffusion}} + \underbrace{D_{g,t}^\kappa}_{\text{eddy diffusion}} \right] \varrho_{g,\mathsf{mol}} M^\kappa \nabla \bar{x}_g^\kappa \right) - \underbrace{q_g^\kappa}_{\text{source}} = 0$$

- energy balance

$$\underbrace{\frac{\partial (\varrho_g \bar{u}_g)}{\partial t}}_{\text{storage}} + \underbrace{\nabla \cdot (\varrho_g \bar{h}_g \bar{\mathbf{v}}_g)}_{\text{advection}} - \sum_{\kappa \in \{\mathsf{a},\mathsf{w}\}} \underbrace{\left\{ \nabla \cdot \left(\bar{h}_g^\kappa \bar{j}_{g,\mathsf{ff,t,diff}}^\kappa \right) \right\}}_{\text{(eddy-)diffusion}} - \nabla \cdot \left(\left[\underbrace{\lambda_g}_{\text{conduction}} + \underbrace{\lambda_{g,t}}_{\text{eddy conduction}} \right] \nabla \bar{T} \right) - \underbrace{q_g^T}_{\text{source}} = 0$$

Model

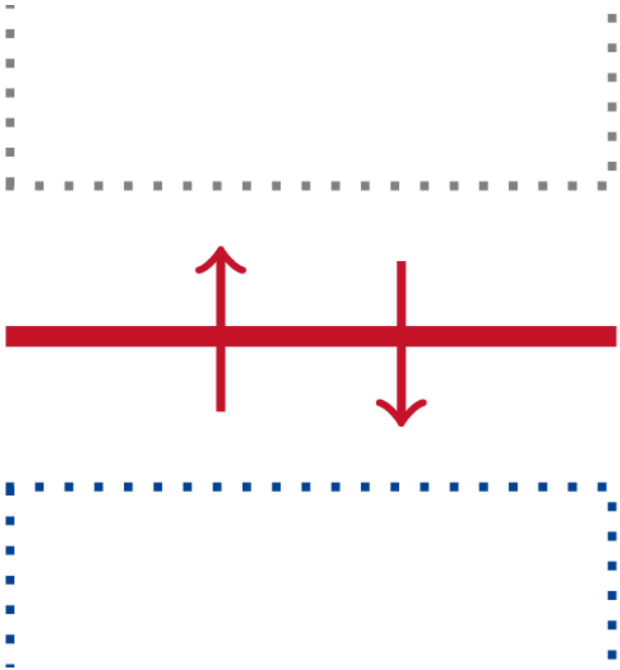
Turbulence



Model

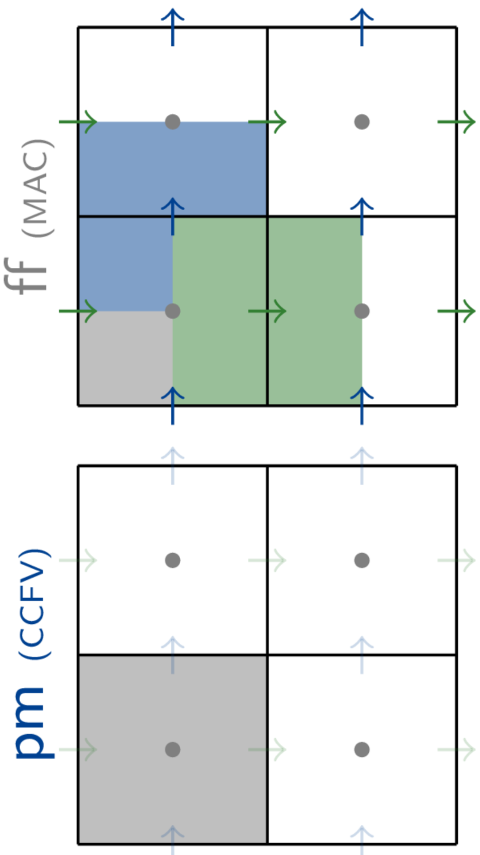
Coupling

- after [Mosthaf et al. 2011, Fetzer et al. 2016]
- local thermodynamic equilibrium
- continuity of fluxes
- continuity of primary variables

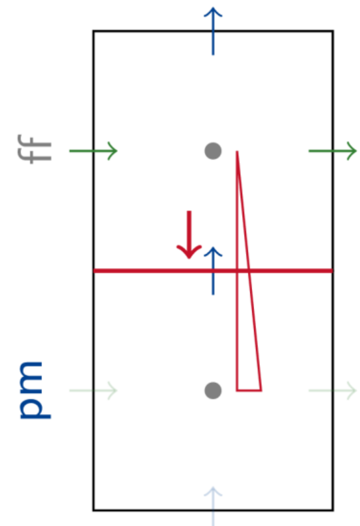


Model

Coupling Discretization



- • $p, S_\alpha, X_\alpha^\kappa, T, (k, \varepsilon)$
- → v_x
- ↑ v_y



$$q_n^{pm} = q_n^{ff}$$
$$q_n^{ff} = \checkmark$$
$$u^{if} = \checkmark$$
$$\nabla u^{if} = \checkmark$$

[Grüninger, Fetzer, Flemisch, Helmig (in preparation)]

Model

Coupling Equations 1

- mass

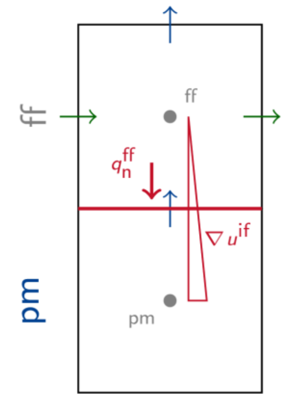
$$\left[\left(\varrho_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}}\right) \cdot \mathbf{n}\right]^{\mathrm{ff}} = -\left[\left(\varrho_{\mathrm{g}} \mathbf{v}_{\mathrm{g}} + \varrho_{\mathrm{l}} \mathbf{v}_{\mathrm{l}}\right) \cdot \mathbf{n}\right]^{\mathrm{pm}}$$

- component mass

$$\left[\left(\varrho_{\mathrm{g}} \bar{X}_{\mathrm{g}}^{\kappa} \bar{\mathbf{v}}_{\mathrm{g}} - \left(D_{\mathrm{g}}^{\kappa} + D_{\mathrm{g,t}}^{\kappa}\right) \varrho_{\mathrm{g,mol}} M^{\kappa} \nabla x_{\mathrm{g}}^{\kappa,\mathrm{if}}\right) \cdot \mathbf{n}\right]^{\mathrm{ff}}$$
$$= -\left[\left(\varrho_{\mathrm{g}} X_{\mathrm{g}}^{\kappa} \mathbf{v}_{\mathrm{g}} - \mathbf{j}_{\mathrm{g,pm,diff}}^{\kappa} + \varrho_{\mathrm{l}} \mathbf{v}_{\mathrm{l}} X_{\mathrm{l}}^{\kappa} - \mathbf{j}_{\mathrm{l,pm,diff}}^{\kappa}\right) \cdot \mathbf{n}\right]^{\mathrm{pm}}$$

- energy

$$\left[\left(\varrho_{\mathrm{g}} \bar{h}_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}} - \bar{h}_{\mathrm{g}}^{\mathrm{a}} \bar{\mathbf{j}}_{\mathrm{g,ff,t,diff}}^{\mathrm{a}} - \bar{h}_{\mathrm{g}}^{\mathrm{w}} \bar{\mathbf{j}}_{\mathrm{g,ff,t,diff}}^{\mathrm{w}} - \lambda_{\mathrm{g,t}} \nabla \bar{T}^{\mathrm{if}}\right) \cdot \mathbf{n}\right]^{\mathrm{ff}}$$
$$= -\left[\left(\varrho_{\mathrm{g}} h_{\mathrm{g}} \mathbf{v}_{\mathrm{g}} + \varrho_{\mathrm{l}} h_{\mathrm{l}} \mathbf{v}_{\mathrm{l}} - \lambda_{\mathrm{pm}} \nabla T\right) \cdot \mathbf{n}\right]^{\mathrm{pm}}$$

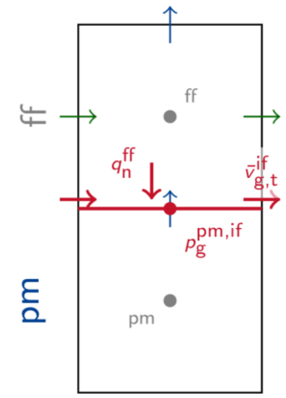


Model

Coupling Equations 2

- tangential momentum (only free flow)

$$\left[\bar{v}^{\mathrm{if}}_{\mathrm{g,t}} + \frac{\sqrt{K}}{\alpha_{\mathrm{BJ}}} \frac{\bar{v}^{\mathrm{ff}}_{\mathrm{g,t}} - \bar{v}^{\mathrm{if}}_{\mathrm{g,t}}}{\mathbf{x}^{\mathrm{ff}} - \mathbf{x}^{\mathrm{if}}} \cdot \mathbf{n}\right]^{\mathrm{ff}} = 0$$



- normal momentum (only free flow)

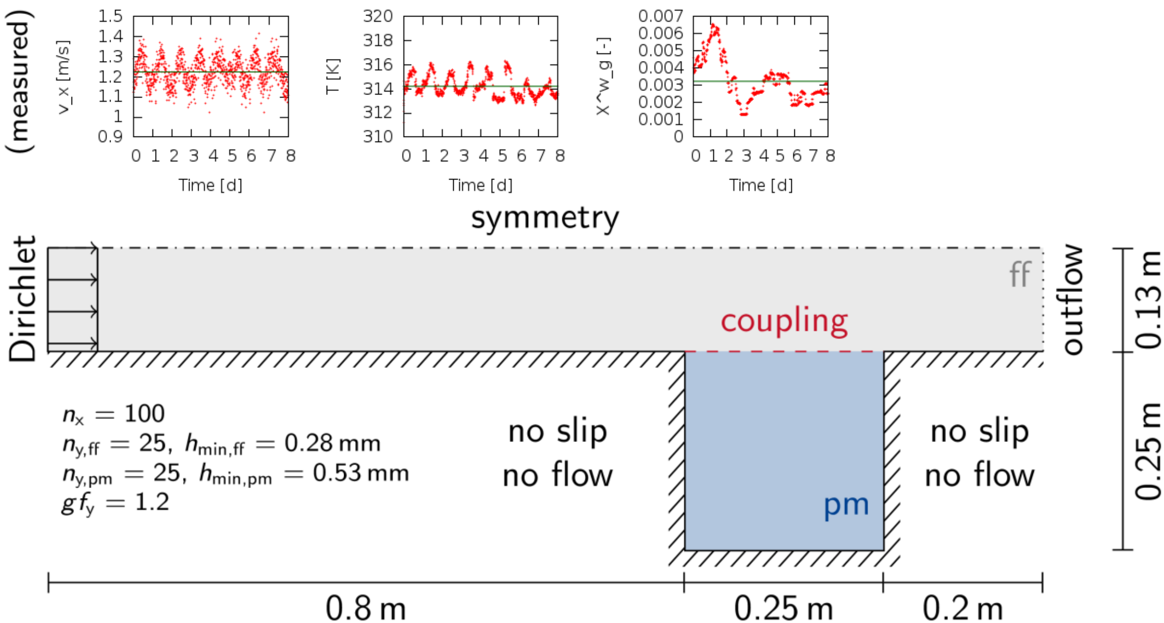
$$\left[\left(\left\{\varrho_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}} \bar{\mathbf{v}}_{\mathrm{g}}^{\mathsf{T}} - \bar{\boldsymbol{\tau}}_{\mathrm{g,t}} + \bar{p}_{\mathrm{g}} \mathbf{I}\right\}\mathbf{n}\right)\cdot\mathbf{n}\right]^{\mathrm{ff}} = \left[p^{\mathrm{if}}_{\mathrm{g}}\right]^{\mathrm{pm}}$$

$$\left[p_{\mathrm{g}}^{\mathrm{if}}\right]^{\mathrm{pm}} = \frac{\left[\varrho_{\mathrm{g}} v^{\mathrm{if}}_{\mathrm{g,n}}\right]^{\mathrm{ff}}}{K\left(k_{\mathrm{r,g}}/\nu_{\mathrm{g}} + k_{\mathrm{r,l}}/\nu_{\mathrm{l}}\right)} \frac{\left(\mathbf{x}^{\mathrm{if}}-\mathbf{x}^{\mathrm{pm}}\right)}{\mathbf{n}^{\mathrm{pm}}} + p_{\mathrm{g}}^{\mathrm{pm}}$$

Results

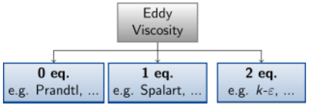
Implementation and Experimental Setup

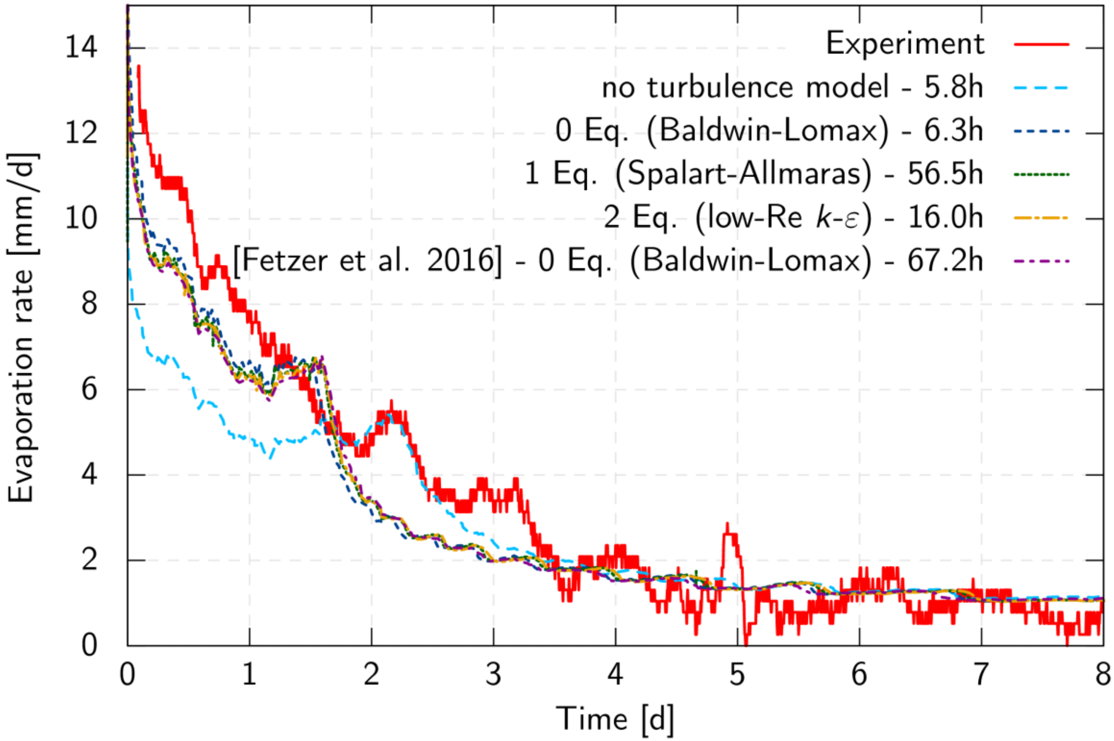
- numerical simulator: DuMu^x
  - MAC / cell-centered FV
  - fully implicit, monolithic
- experiments: [Davarzani et al. 2014]



Results

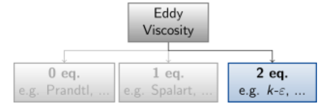
Turbulence Modeling

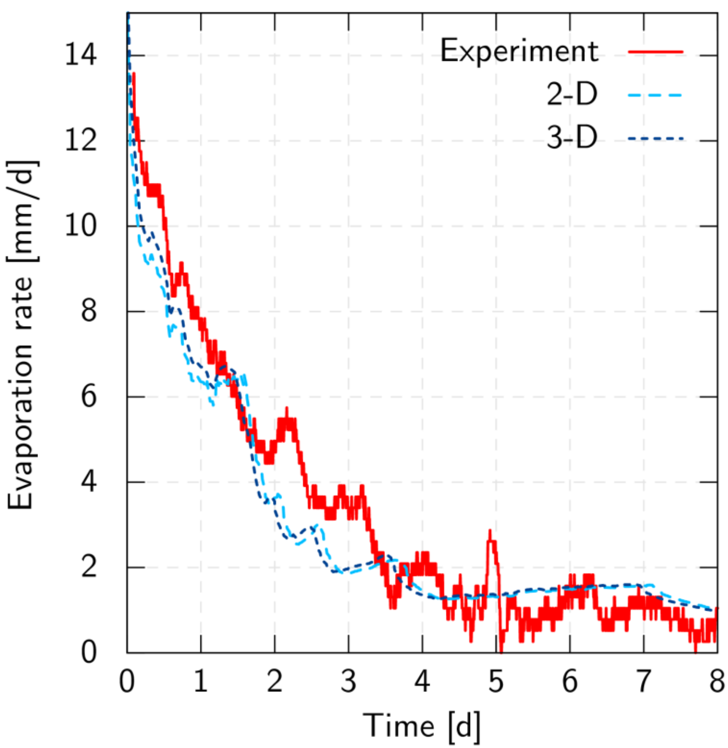


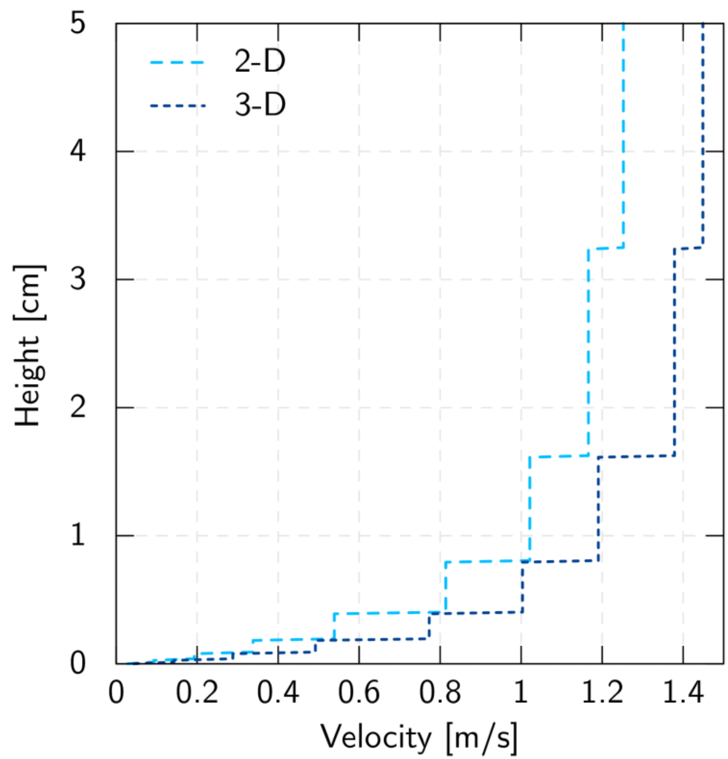


Results

2-D vs. 3-D

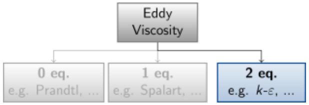


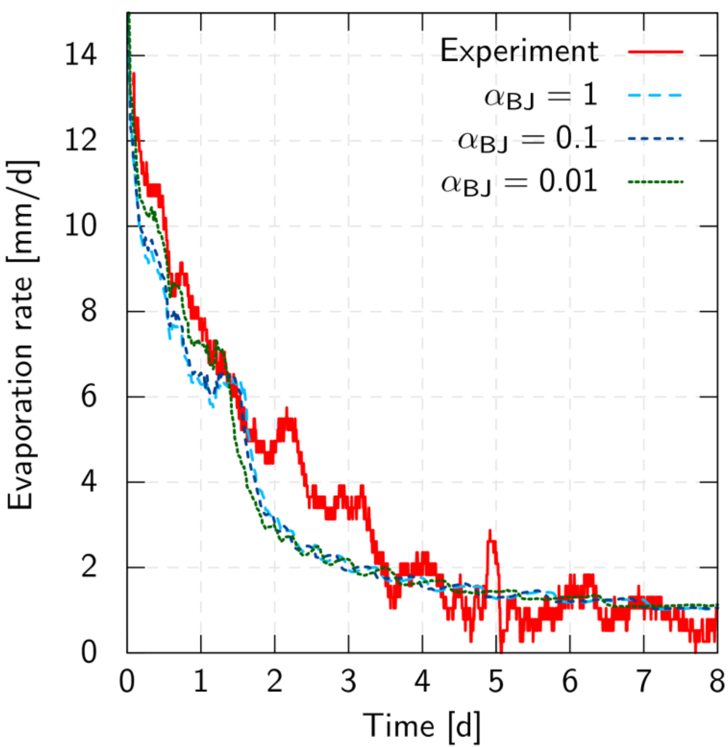


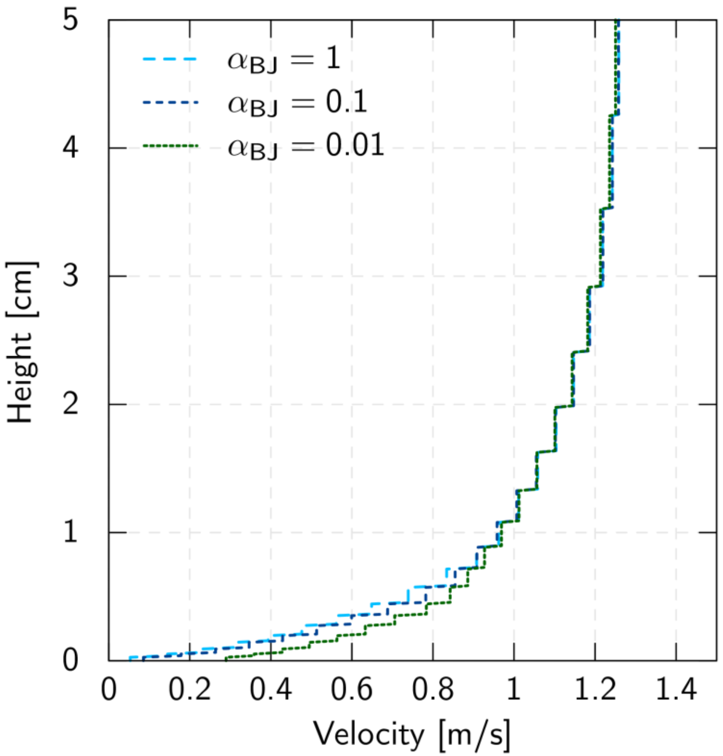


Results

Beavers-Joseph Coefficient



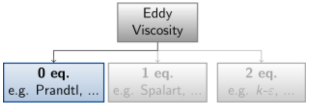


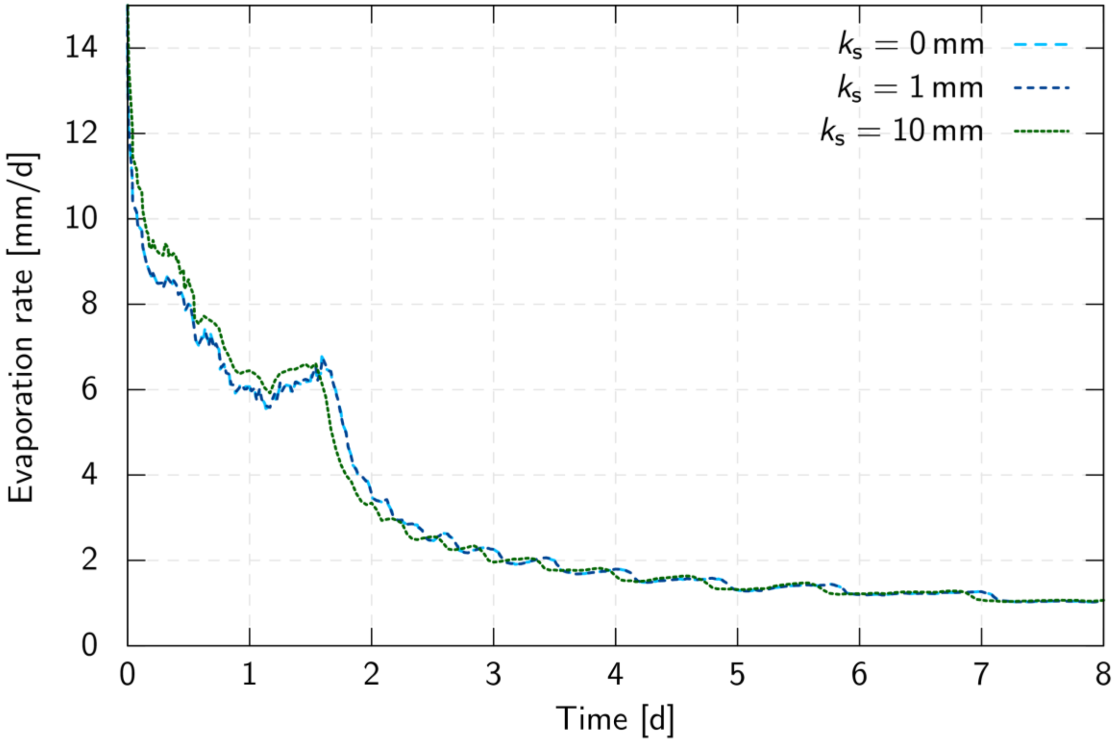


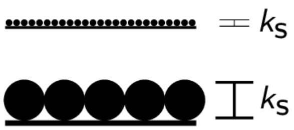
LH²

Results

Sand-Grain Roughness

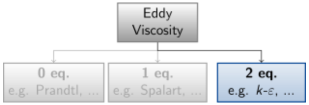


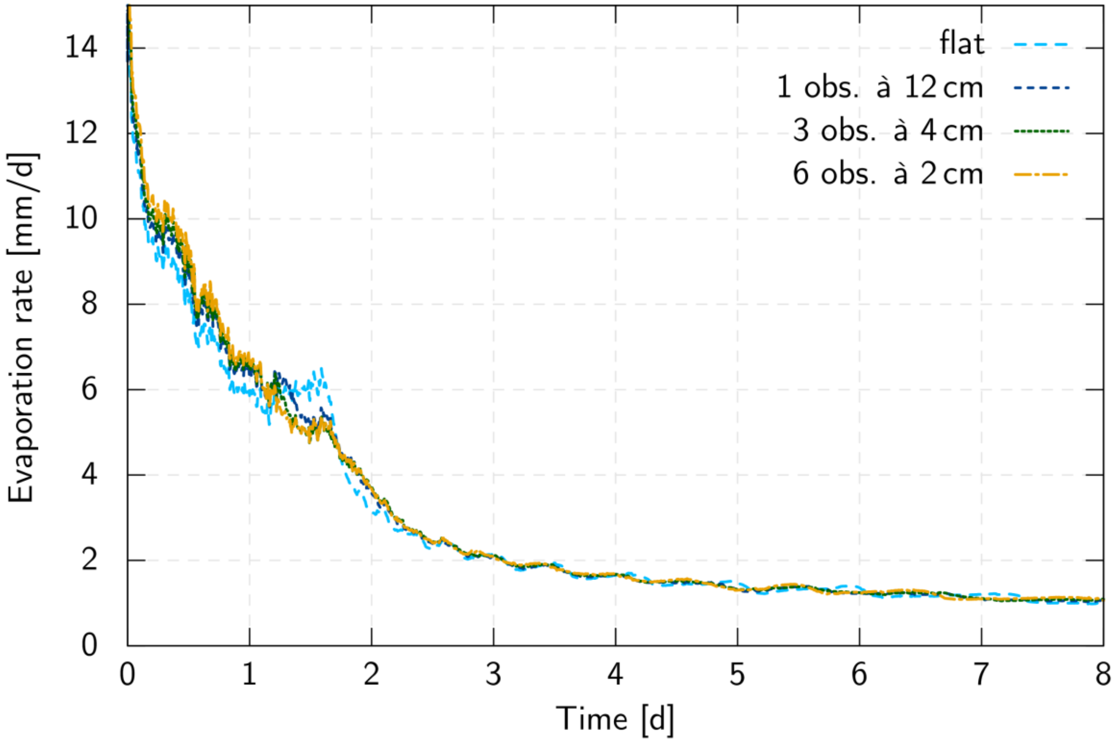


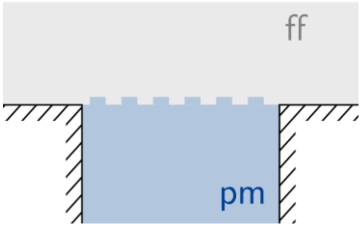


Results

Obstacles with H=10mm

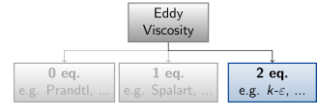


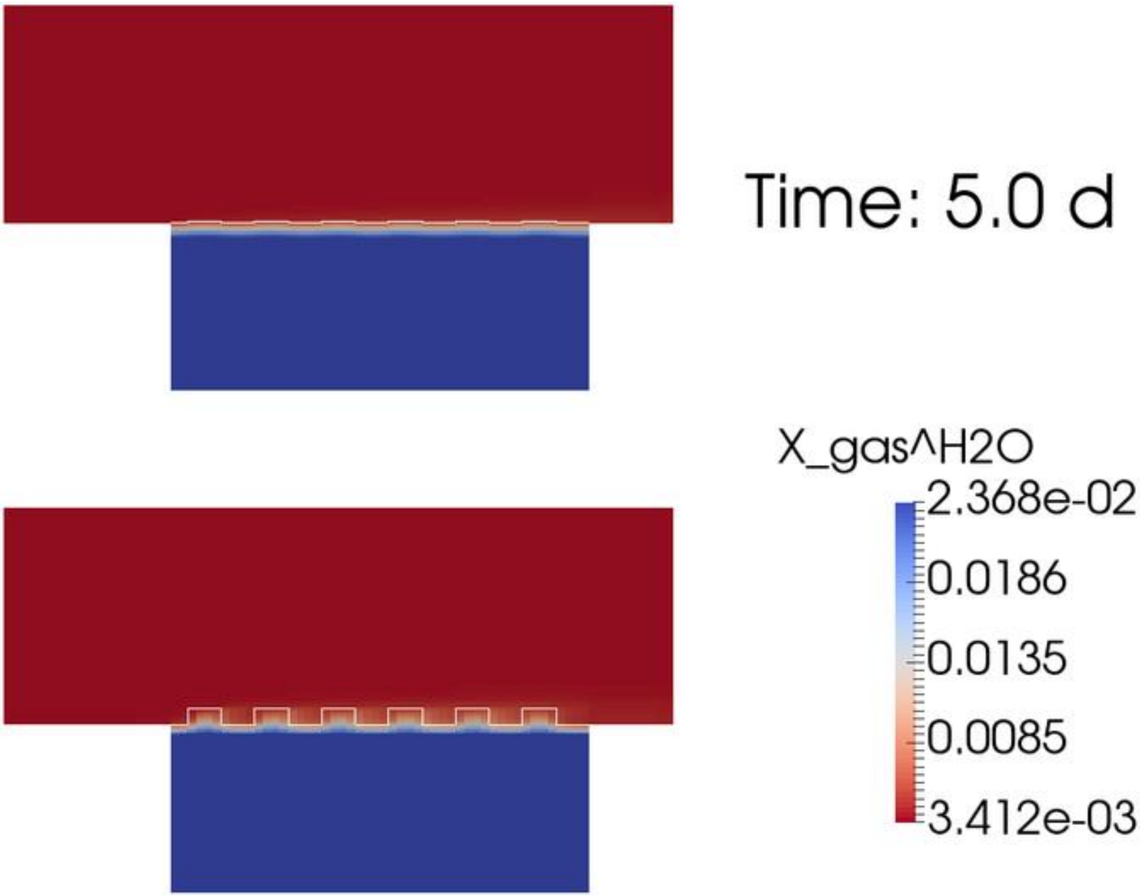




Results

Obstacles





Summary and Outlook

Summary

- coupling of free flow (MAC) to porous-medium flow (CCFV)
- comparison to laboratory experiments
  - turbulent models of different complexity
  - influence of interface properties

Outlook

- turbulent / barometric pumping
- Forchheimer extension to Darcy‘s law
- local thermodynamic non-equilibrium
- pore network model at the interface

Thank you for your attention!

- [Beavers, G. S. & Joseph, D. D. 1967] - Boundary conditions at a naturally permeable wall
  Journal of Fluid Mechanics, **1967**, *30*, 197-207
- [Davarzani et al. 2014] - Study of the effect of wind speed on evaporation from soil through integrated modeling of the atmospheric boundary layer and shallow subsurface
  Water Resources Research, **2014**, *50*, 1-20
- [Fetzer et al. 2016] - Effect of Turbulence and Roughness on Coupled Porous-Medium/Free Flow Exchange Processes
  Transport in Porous Media, **2016**
- [Grüninger, Fetzer, Flemisch, Helmig (in preparation)] - Coupling DuMuX and DUNE-PDELab to investigate evaporation at the interface between Darcy and Navier-Stokes flow
- [Mosthaf et al. 2011] - A coupling concept for two-phase compositional porous-medium and single-phase compositional free flow
  Water Resources Research, **2011***, 47,* W10522