Numerical analysis of multi-dimensional models for network flow in biological systems

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Network structures can be detected in almost every biological system, since they are often responsible for the transport of fluids, nutrients or oxygen [1]. Such a network structure is for example a blood vessel network supplying organs with oxygenated blood or removing metabolic waste from the tissue [2]. A further example is the root network of a plant, ensuring the water supply of the plant [3] (see Figure 1).

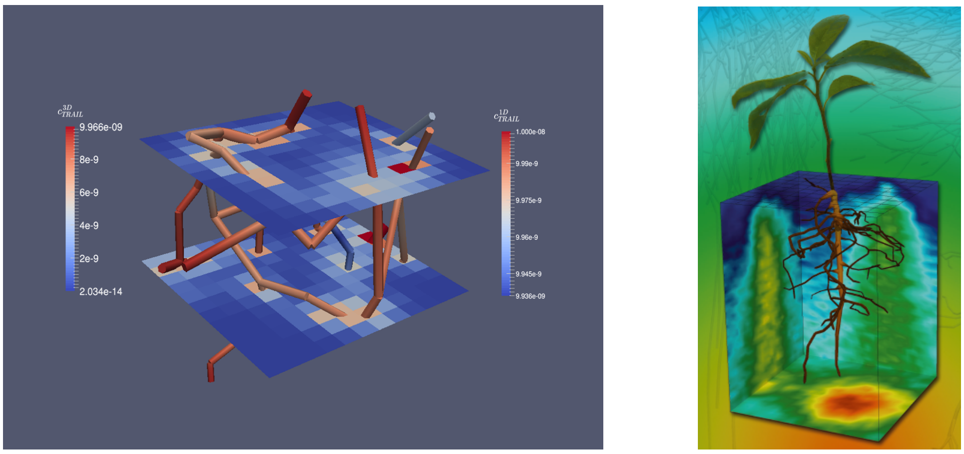

Figure 1: Mass transport in a capillary network embedded in tumor tissue (left), root network supplying a plant (right).

One way to obtain a realistic model for such processes is based on a decomposition approach. Thereby, the network structure is separated from the surrounding medium and different models are assigned to both domains. Quite often the surrounding medium (e.g. tissue or soil) can be considered as a three-dimensional (3D) porous medium. In order to decrease computational costs while maintaining a certain degree of accuracy, flow and transport processes within the networks are modeled by one-dimensional (1D) PDE-systems. A coupling of the network and the porous medium model is achieved by first averaging the 3D quantities and projecting them onto the 1D network structure. As a next step, the difference of the averaged 3D and 1D quantities is computed and incorporated into the source terms of both the network and the porous medium model, where the source term of the 3D problem exhibits a Dirac measure concentrated on the 1D network [4, 6].

In this talk we are concerned with the numerical analysis of PDE systems arising in the context of this model concept [5, 7]. In particular, it is investigated how the Dirac source terms and averaging operators affect the convergence behavior of standard finite element methods. Therefore, elliptic and parabolic model problems with Dirac source terms and averaging operators are investigated. Our theoretical results are confirmed by numerical tests.

References

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