

# University of Stuttgart

Department of Hydromechanics and Modelling of Hydrosystems

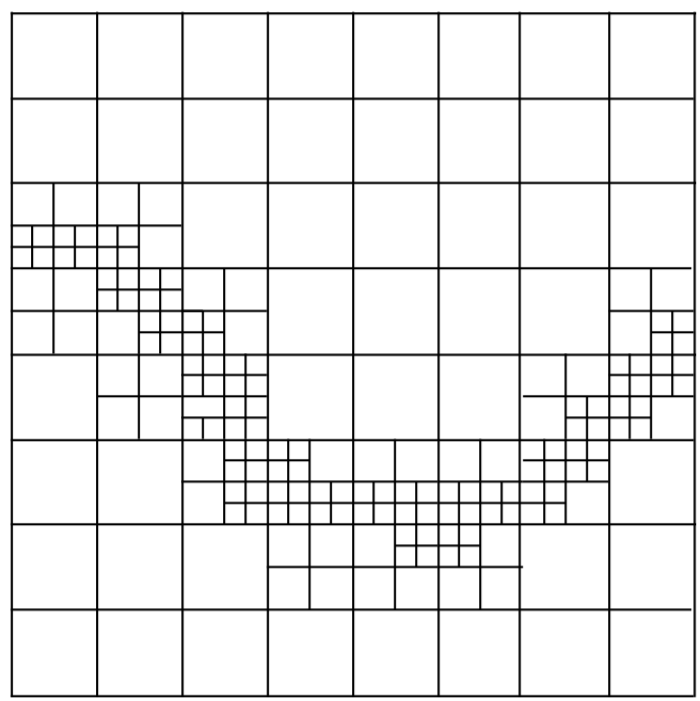
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## Adaptive Staggered 2D Grids for DuMux – Plans/Ideas

### Motivation

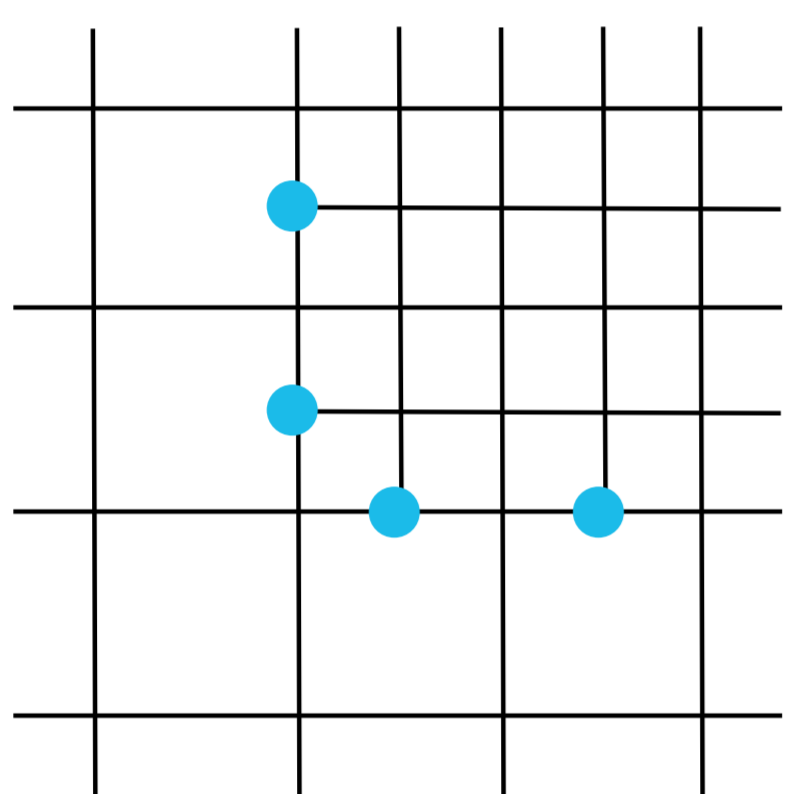
Implement Grid Adaptivity for 2D Staggered Grids in DuMux



Refined Grid in the Regions of Interest => Significant Increase in Accuracy With Moderate Increase in Computational Cost

### Plans and Ideas for the Realization

- Hanging Nodes Occur



- Staggered Grid (in Free Flow)

[F.H. Harlow, J.E. Welch. Phys Fluids, 8, 282 (1965)]

Navier-Stokes Equation Requires  $\nabla p$  at Position where  $\mathbf{v}$  is Calculated

If  $p$  and  $\mathbf{v}$  Live on the Same Grid (Same Nodes)

$$\partial_x p|_i = \frac{p_{i+1} + p_i - p_i - p_{i-1}}{2 \Delta x} = \frac{p_{i+1} - p_{i-1}}{2 \Delta x}$$

Neighboring Pressure Values Decouple => Unphysical

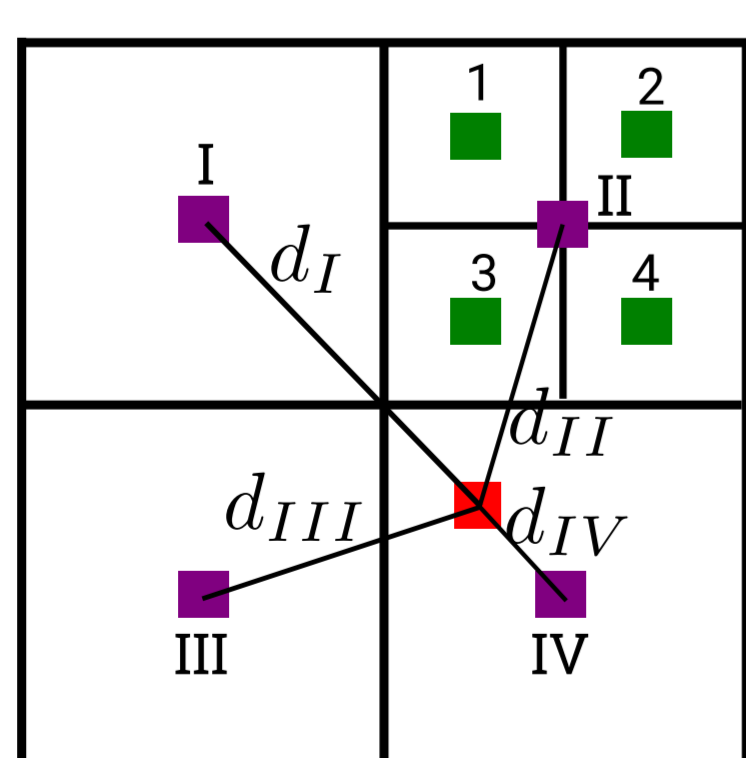
Workaround: Staggered Grid:  $p$  and  $\mathbf{v}$  Live on Separate, Staggered Grids

$$\partial_x p|_{i-\frac{1}{2}} = \frac{p_i - p_{i-1}}{2 \Delta x}$$

- Interpolations are Required - Possible Interpolations

[L. Vittoz et al. International Conference on Finite Volumes for Complex Applications. Springer, Cham, 2017, A. Theodorakakos, G. Bergeles. Int J Numer Meth Fluids, 45:421, 2004]

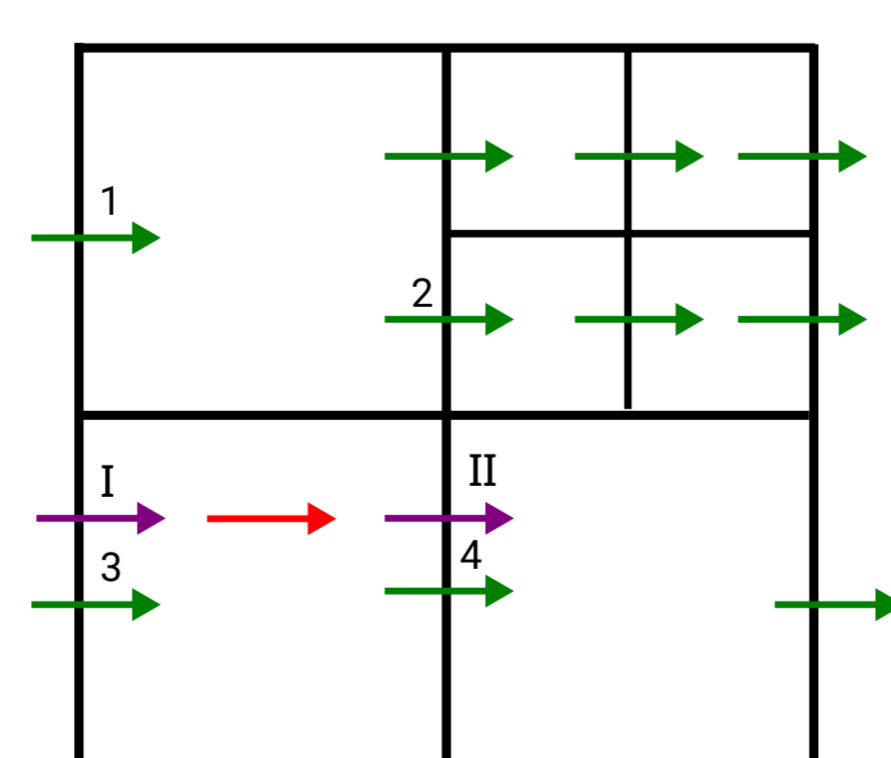
Cell Centered Quantities



$$p_{II} = \frac{p_1 + p_2 + p_3 + p_4}{4}$$

$$p = \frac{p_I/d_I + p_{II}/d_{II} + p_{III}/d_{III} + p_{IV}/d_{IV}}{1/d_I + 1/d_{II} + 1/d_{III} + 1/d_{IV}}$$

Face Centered Quantities



$$u_I = \frac{u_1 + 3u_3}{4} \quad u_{II} = \frac{u_2 + 2u_4}{3}$$

$$u = u_I + u_{II}$$

Open Question: What Is A Mass Conservative Interpolation?

	Mass Balance (Cell Residuals)	Navier Stokes (Face Residuals) x-Component	Navier Stokes (Face Residuals) y-Component
Control Volumes without Adaptivity			
Stencils without Adaptivity			
Control Volumes with Adaptivity			
		Or Alternatively:	Or Alternatively:
Stencils with Adaptivity (Geometry-Dependent)	<ul style="list-style-type: none"> <li>● Interpolated Stencils</li> <li></li> <li></li> <li></li> <li>Etc.</li> <li>■ Dof =&gt; No Interpolation</li> <li>■ No Dof =&gt; Interpolation</li> <li>● Full Stencils</li> <li></li> <li></li> <li></li> <li>Etc.</li> <li>■ Clearly Part of the Stencil</li> <li>■ Part of the Stencil for the Interpolation to the Left</li> </ul>		
Equations	$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) - q_p = 0$	$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}^T) - \nabla \cdot (\mu (\nabla \mathbf{v} + \nabla \mathbf{v}^T)) + \nabla p - \rho \mathbf{g} - q_v = 0$	

$\rho$  | Density  
 $t$  | Time  
 $\mathbf{v}$  | Velocity  
 $q_p/q_v$  | Source in Mass/Momentum Balance Cell  
 $\mu$  | Viscosity  
 $p$  | Pressure  
 $\mathbf{g}$  | Gravity

### Outlook

Possible Next Step: Dynamic Adaptivity, e.g. Similar To T.V. Gerya, D.A. May, T. Duretz. Geochem Geophys, 14(4):1200, 2013

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