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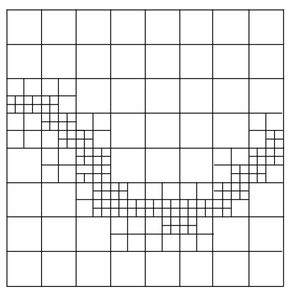
Adaptive Staggered 2D Grids for DuMux – Plans/Ideas

Motivation

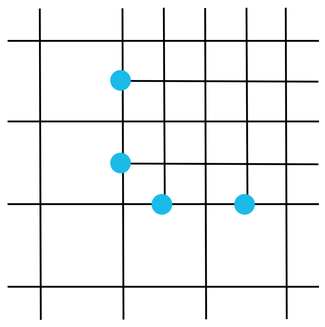
Implement Grid Adaptivity for 2D Staggered Grids in DuMuX



Refined Grid in the Regions of Interest => Significant Increase in Accuracy
With Moderate Increase in Computational Cost

Plans and Ideas for the Realization

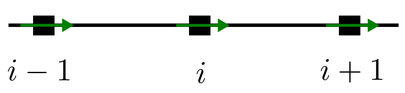
- Hanging Nodes Occur



- Staggered Grid (in Free Flow)
  [F.H. Harlow, J.E. Welch. Phys Fluids, **8**, 282 (1965)]

  Navier-Stokes Equation Requires ∇p at Position where $\mathbf{v}$ is Calculated

  If p and $\mathbf{v}$ Live on the Same Grid (Same Nodes)

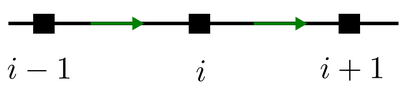


$$\partial_x p|_i = \frac{\frac{p_{i+1}+p_i}{2} - \frac{p_i+p_{i-1}}{2}}{\Delta x} = \frac{p_{i+1}-p_{i-1}}{2\Delta x}$$

  Neighboring Pressure Values Decouple => Unphysical

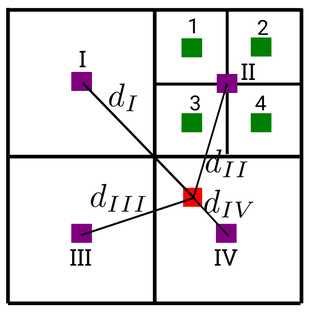
  Workaround: Staggered Grid: p and $\mathbf{v}$ Live on Separate, Staggered Grids

$$\partial_x p|_{i-\frac{1}{2}} = \frac{p_i - p_{i-1}}{2\Delta x}$$



- Interpolations are Required - Possible Interpolations
  [L. Vittoz et al. International Conference on Finite Volumes for Complex Applications. Springer, Cham, 2017,
  A. Theodorakakos, G. Bergeles. Int J Numer Meth Fluids, **45**:421, 2004]

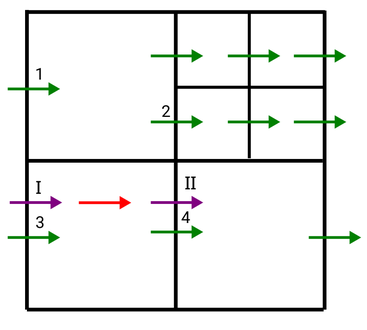
Cell Centered Quantities



$$p_{II} = \frac{p_1+p_2+p_3+p_4}{4}$$

$$p = \frac{p_I/d_I + p_{II}/d_{II} + p_{III}/d_{III} + p_{IV}/d_{IV}}{1/d_I + 1/d_{II} + 1/d_{III} + 1/d_{IV}}$$

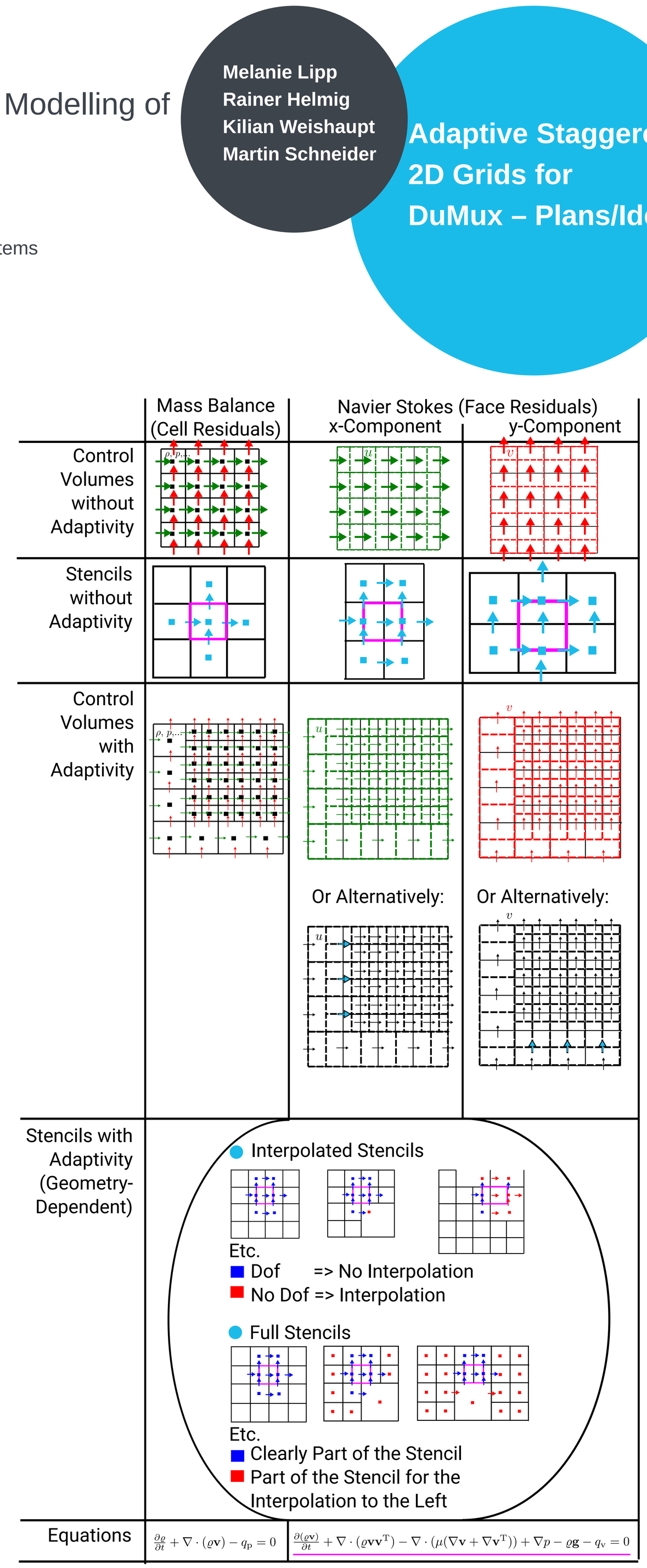
Face Centered Quantities



$$u_I = \frac{u_1+3u_3}{4} \quad u_{II} = \frac{u_2+2u_4}{3}$$

$$u = u_I + u_{II}$$

Open Question: What Is A Mass Conservative Interpolation?



Outlook

Possible Next Step: Dynamic Adaptivity, e.g. Similar To
T.V. Gerya, D.A. May, T. Duretz. Geochem Geophys, 14(4):1200, 2013

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