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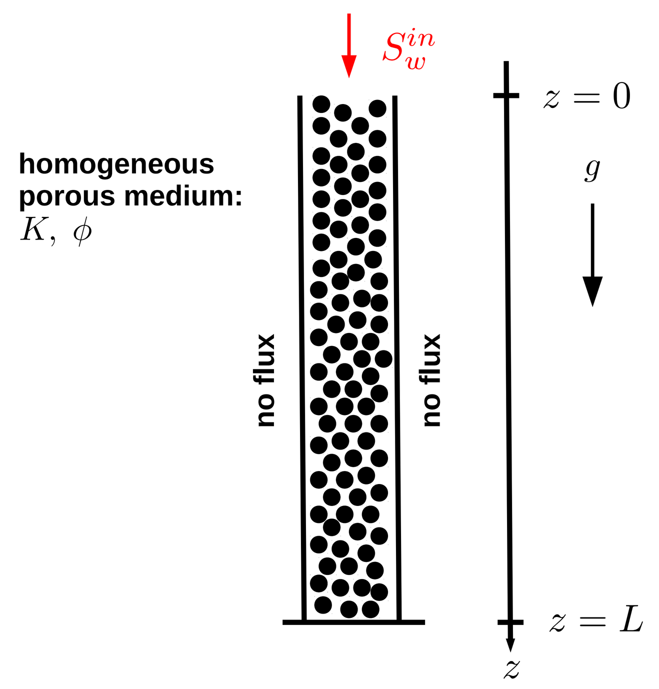
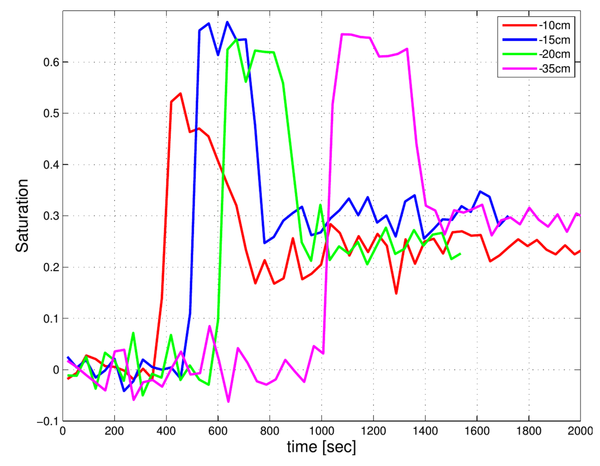
Stability of saturation overshoots for two-phase flow in porous media

Motivation

Setup of an infiltration experiment. At the top of a sand column having the height L water is injected, resulting in a water saturation S_w^{in} at $z = 0$:





(experiments by S. Fritz, 2012)

- **Measurements of water saturations at different depths of the sand column reveal that the water saturation curves exhibit overshoots containing a plateau [2, 3].**
- **The overshoot region consists of an imbibition front and a drainage front, inbetween these fronts a plateau is formed.**

Central issues:

1. **How can the standard two-phase flow equations for porous media be extended such that non-monotonous saturation profiles can be simulated?**

   Wide-spread approach: Extend the standard capillary pressure relationship by a dynamic τ-term [1]:

$$\tau \frac{\partial S_w}{\partial t} = p_n - p_w - p_c.$$

2. **How is the behavior of the saturation overshoot depending on S_w^{in}?**

Mathematical methods:

1. **Numerical methods: Discretize the standard two-phase flow equations in 1D using finite volumes and implicit euler:**

$$\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial z}\left(\frac{k_{rw} \cdot K}{\mu_w \cdot \phi}\left(\frac{\partial p_w}{\partial z} - \rho_w g\right)\right) = 0,\; z \in (0, L)\,,\; t > 0,$$

$$-\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial z}\left(\frac{k_{rn} \cdot K}{\mu_n \cdot \phi}\left(\frac{\partial p_n}{\partial z} - \rho_n g\right)\right) = 0,\; z \in (0, L)\,,\; t > 0,$$

$$p_c - p_n + p_w = 0,\; z \in (0, L)\,,\; t > 0,$$

   Create an overshoot not by a τ-term, but by a time dependent boundary condition at $z = 0$:

$$S_w(z = 0, t) = \begin{cases} S_w^P, & t \leq 500\,\mathrm{s}, \\ S_w^{in}, & t > 500\,\mathrm{s}. \end{cases}$$

   In order to account for imbibition and drainage processes, we introduce for each process a constitutive relation for the relative permeabilities as well as the capillary pressure: $k_{rw}^i,\ k_{rn}^i,\ p_c^i,\ i \in \{im, dr\}$. A transition between the two processes is described by means of smooth scanning curves (hysteresis model).

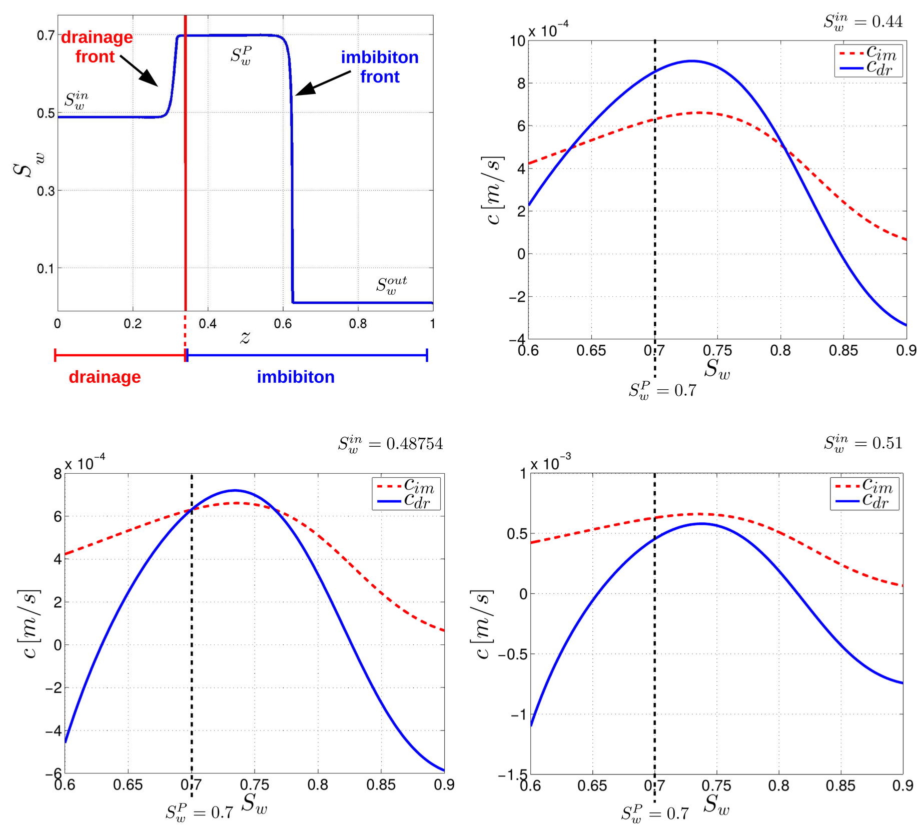
2. **Analytical methods: Assume that the total velocity $v_t = v_w + v_n$ is constant in space and time and derive a fractional flow formulation from the two-phase flow equations:**

$$\frac{\partial S_w}{\partial t} + \frac{v_t}{\phi}\frac{\partial}{\partial z}\left[f(S_w) - D(S_w)\frac{\partial S_w}{\partial z}\right] = 0,\; z \in (0, L)\,,\; t > 0.$$

   This formulation is the basis for estimating the speed c of the imbibition and drainage front by means of the Rankine-Hugoniot condition [4, 5]:

$$c_{im}\left(S_w^P\right) \approx \frac{v_t}{\phi}\frac{f_{im}\left(S_w^P\right) - f_{im}\left(S_w^{out}\right)}{S_w^P - S_w^{out}},\; c_{dr}\left(S_w^P\right) \approx \frac{v_t}{\phi}\frac{f_{im}\left(S_w^P\right) - f_{dr}\left(S_w^{in}\right)}{S_w^P - S_w^{in}}.$$

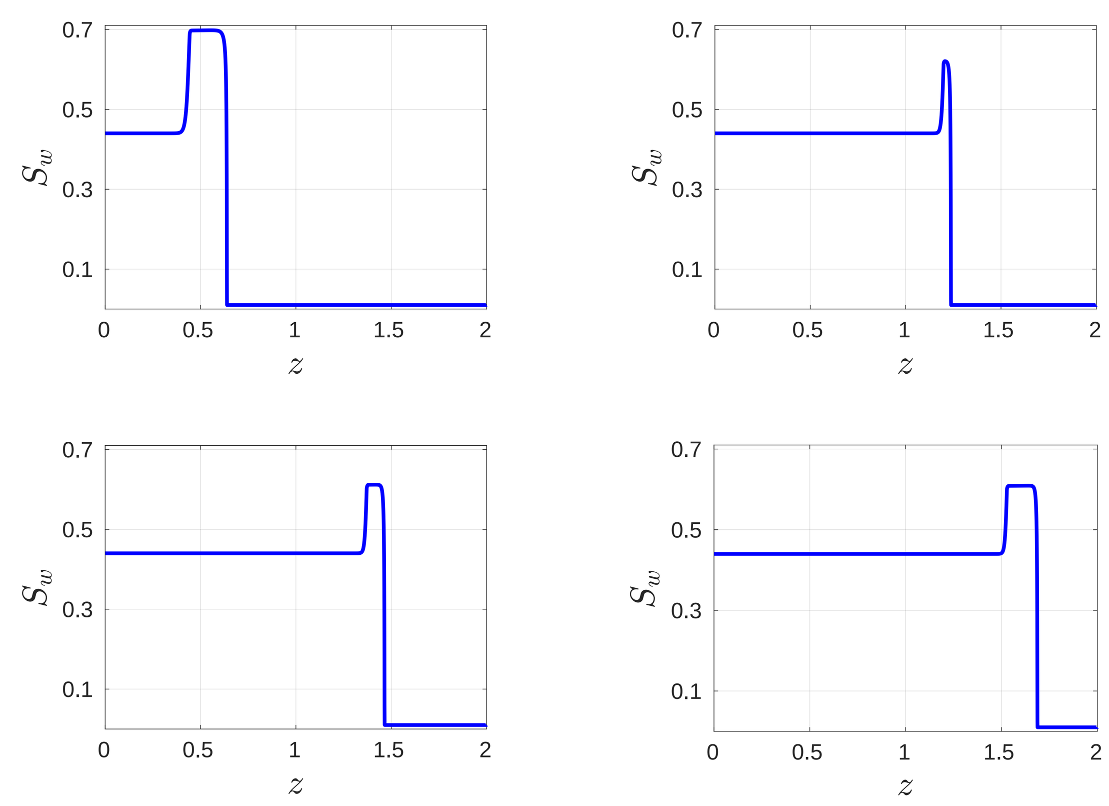
- Velocity curves for $S_w^{in} \in \{0.44,\ 0.48754,\ 0.51\}$ and $S_w^P = 0.7$:



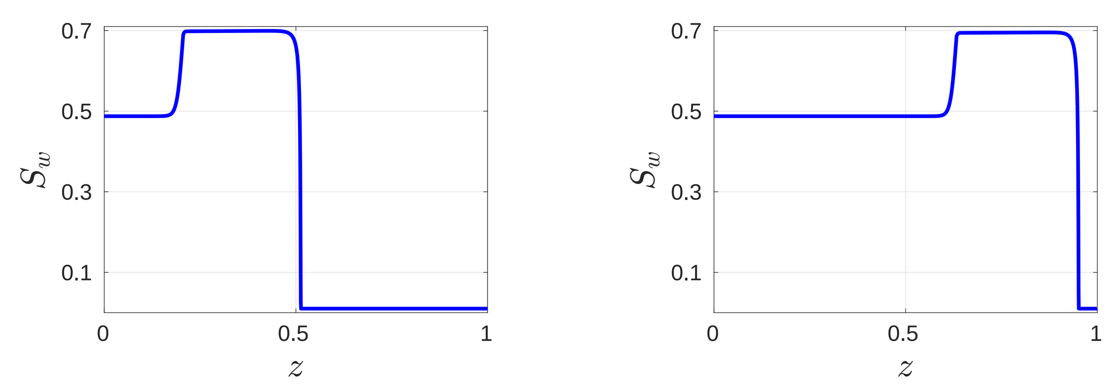
Results:

Variation of S_w^{in} leads to different front propagation behavior, which is analyzed for $S_w^{in} \in \{0.44,\ 0.48754,\ 0.51\}$.

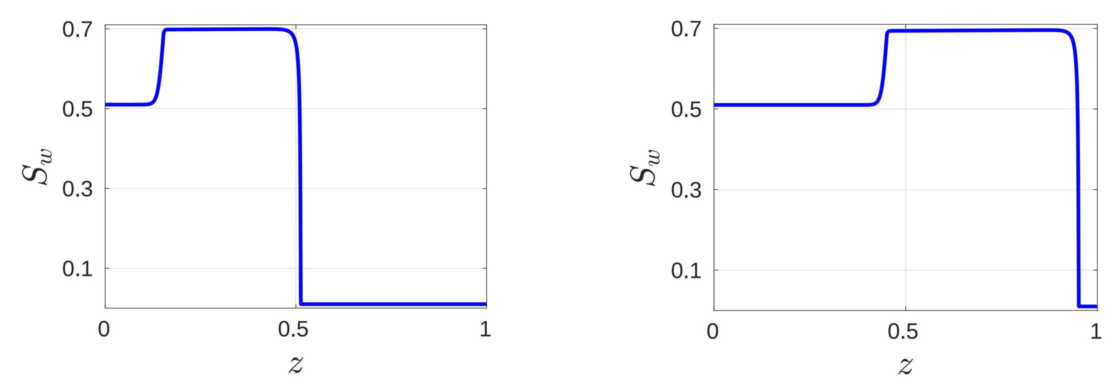
- Case 1: $S_w^{in} = 0.44$, $c_{im} < c_{dr}$, $t \in \{1000\,\mathrm{s},\ 2000\,\mathrm{s},\ 2500\,\mathrm{s},\ 3000\,\mathrm{s}\}$
  initial plateau vanishes and a second one is created at a lower level



- Case 2: $S_w^{in} = 0.48754$, $c_{im} = c_{dr}$, $t \in \{800\,\mathrm{s}, 1500\,\mathrm{s}\}$, **initial plateau is stable**



- Case 3: $S_w^{in} = 0.51$, $c_{im} > c_{dr}$, $t \in \{800\,\mathrm{s}, 1500\,\mathrm{s}\}$, **initial plateau is enlarged**



Conclusion:

The qualitative behavior of the simulated saturation overshoots is in accordance with the analytical considerations.

References

[1] A. Beliaev and S. Hassanizadeh. A theoretical model of hysteresis and dynamic effects in the capillary relation for two-phase flow in porous media. *Transport in Porous media*, 43(3):487–510, 2001.

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[4] R. Hilfer and R. Steinle. Saturation overshoot and hysteresis for twophase flow in porous media. *The European Physical Journal Special Topics*, 223(11):2323–2338, 2014.

[5] R. Steinle and R. Hilfer. Hysteresis in relative permeabilities suffices for propagation of saturation overshoot: A quantitative comparison with experiment. *Physical Review E*, 95(4):043112, 2017.