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Topic of Master's Thesis for

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SIMPLE-type preconditioners for the incompressible Navier-Stokes equations

Flow and transport processes in domains composed of a porous medium and an adjacent free-flow region appear in a wide range of industrial, environmental and medical applications. The LH^2 group is for instance working on predicting evaporation dynamics from soils or some processes in fuel cells by modeling free-flow coupled with porous-medium flow. Efficiently gaining information about such coupled flow phenomena requires good solution methods in both domains.

This Master's Thesis focuses on examining some solution methods for the free flow domain. More specifically, SIMPLE-type preconditioners for solving the incompressible Navier-Stokes equations should be studied. The SIMPLE (semi-implicit method for pressure-linked equations) algorithm is a widely-used method to solve the Navier-Stokes equations. Instead of solving the coupled system of the momentum and the mass balance simultaneously for the pressure and the velocity, every iteration step of the SIMPLE algorithm involves solving first one equation for a velocity correction and then another equation for a pressure correction. This way, velocity and pressure are corrected further in every iteration step, starting from an initial guess. The SIMPLE method, as well as some variants, have recently been implemented in DuMu^x using a matrix formulation as in [5]. Coefficients are mostly formulated as in [4]. So far, the SIMPLE algorithm itself does not behave advantageously compared to the Newton algorithm using a direct linear solver for the fully-coupled system. The task of this thesis is to use the SIMPLE-method as a preconditioner (as e.g. in [1, 2, 3]) and to, this way, likely let the method actually become valuable for DuMu^x.

The main subtasks will be:

- Understanding the relevant aspects about the currently implemented SIMPLE-type algorithms in DuMu^x.
- Studying literature on SIMPLE-type preconditioners.
- Extending the current implementation of the SIMPLE algorithm in DuMu^x to be usable as a preconditioner, using the existing DUNE solver functionalities as well as their backends in DuMu^x.
- Comparing different solution methods.

- Studying some test problem(s), e.g. lid-driven cavity flow (see e.g. [3] for this test problem in the SIMPLE-type preconditioner context).

The Master's Thesis shall be summarized in a report and presented in an oral presentation.

Examiner(s): apl. Prof. Dr. rer. nat. Bernd Flemisch, Prof. Dr. rer. nat. Dominik Göddeke
Supervisor(s): Wietse Boon, Melanie Lipp

Date of issue: any time
Date of submission: 6 months thereafter

References

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[2] M ur Rehman, C Vuik, and G Segal. Preconditioners for the steady incompressible navier-stokes problem. *International Journal of Applied Mathematics*, 38(4):1–10, 2008.

[3] M Ur Rehman, Cornelis Vuik, and G Segal. Simple-type preconditioners for the oseen problem. *International Journal for Numerical methods in fluids*, 61(4):432–452, 2009.

[4] H.K. Versteeg and W. Malalasekera. *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*. Pearson Education Limited, 2007.

[5] Pieter Wesseling. *Principles of computational fluid dynamics*, volume 29. Springer Science & Business Media, 2009.