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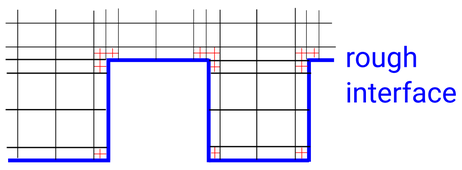
Melanie Lipp, Rainer Helmig

A locally refined quadtree finite-volume staggered-grid scheme

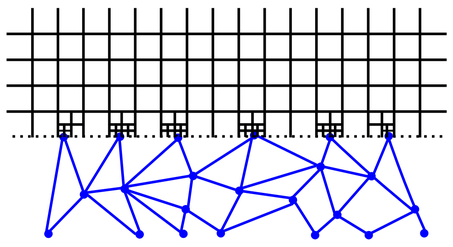
Motivation

free flow domain



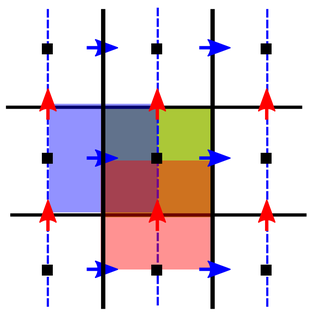
porous medium

free flow domain



pore-network model

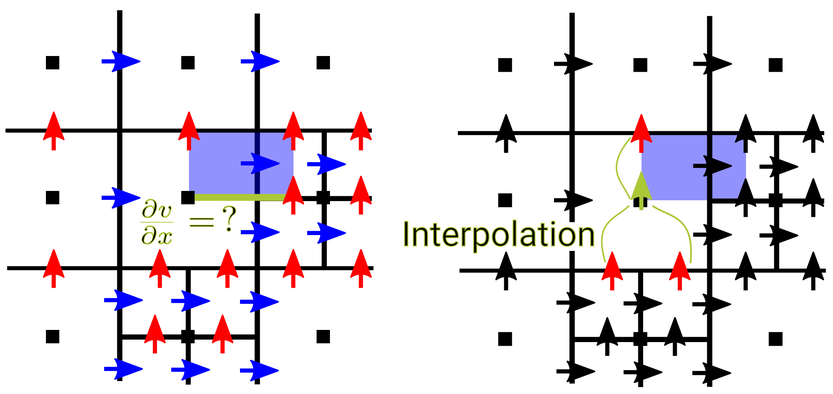
Staggered grid



Navier-Stokes equation

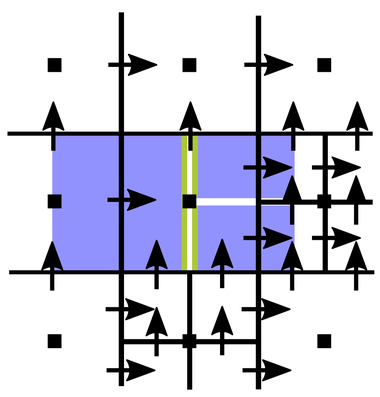
$$\frac{\partial(\varrho \mathbf{v})}{\partial t} + \nabla \cdot (\varrho \mathbf{v}\mathbf{v}^{\mathrm{T}}) - \nabla \cdot (\mu(\nabla \mathbf{v} + \nabla \mathbf{v}^{\mathrm{T}})) + \nabla p - \varrho \mathbf{g} - q_{\mathrm{v}} = 0$$

Interpolations



- many geometries
- larger stencils

Conservation of mass/momentum



Test case

[Jean Donea and Antonio Huerta. Finite element methods for flow problems. John Wiley & Sons, 2003.]

- 2D domain [0,1]x[0,1]
- stationary
- Stokes equation
- no slip, no flow boundary condition
- zero initial condition
- $\rho(\boldsymbol{r}) = \mu(\boldsymbol{r}) = 1, \forall \boldsymbol{r}$
-

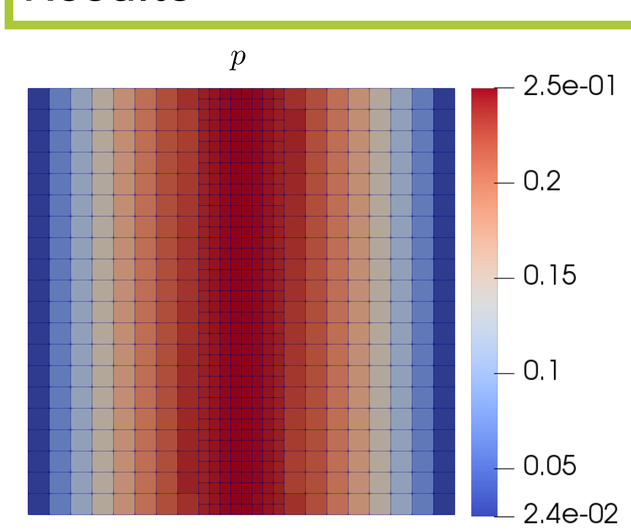
$$\boldsymbol{q} = \begin{pmatrix} (12-24y)x^4 + (-24+48y)x^3 + (-48y+72yr-48y^3+12)x^2 \\ +(-2+24y-72y^2+48y^3)x + 1 - 4y + 12y^2 - 8y^3 \\ (8-48y+48y^2)x^3 + (-12+72y-72y^2)x^2 \\ +(4-24y+48y^2-48y^3+24y^4)x - 12y^2 + 24y^3 - 12y^4 \end{pmatrix}$$

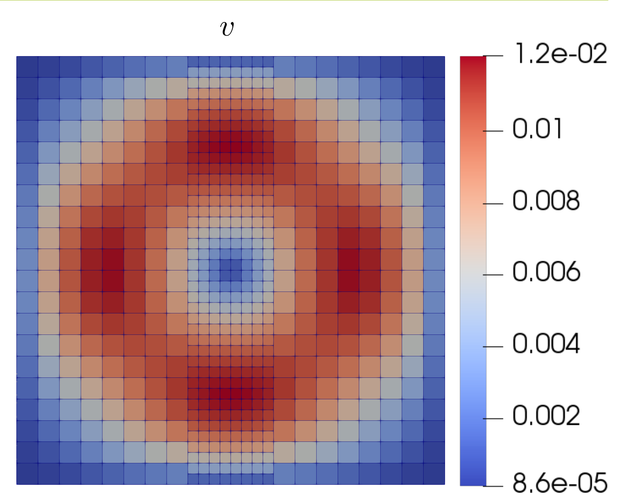
Analytical solution

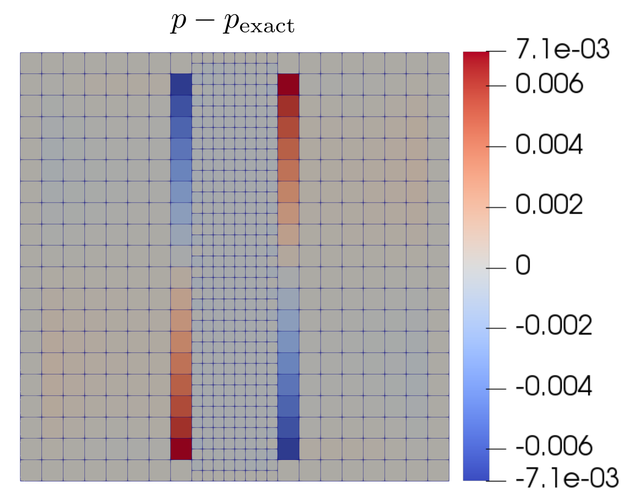
$$p = x(1-x)$$

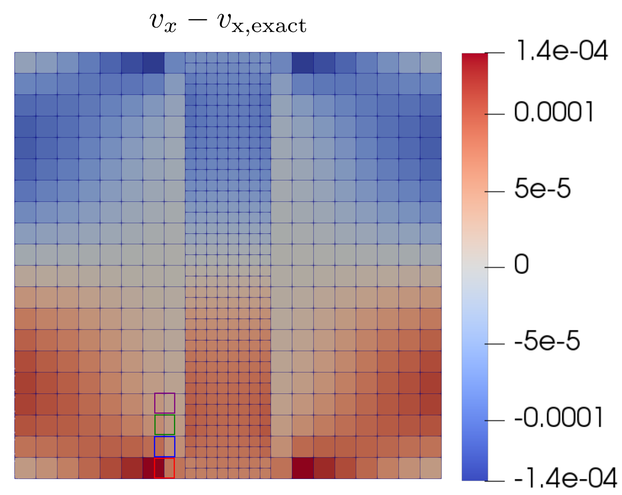
$$\boldsymbol{v} = \begin{pmatrix} x^2(1-x)^2(2y-6y^2+4y^3 \\ -y^2(1-y)^2(2x-6x^2+4x^3) \end{pmatrix}$$

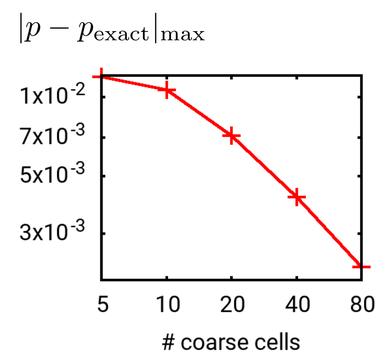
Results

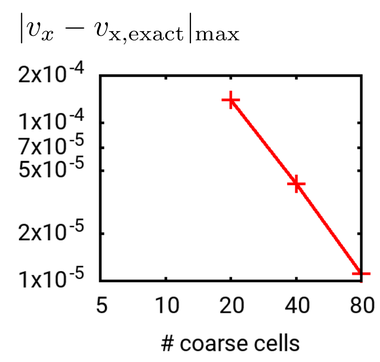




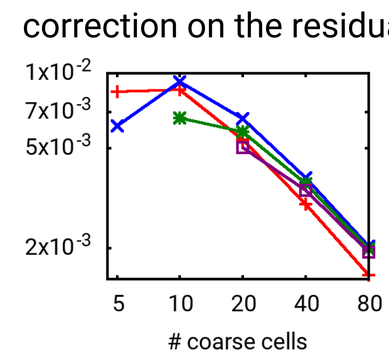








Influence of the flux correction on the residual

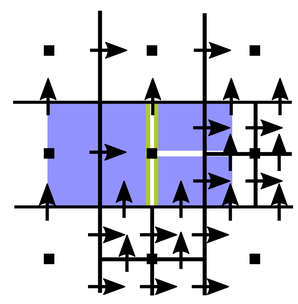


With refinement of 0.4 < x < 0.6

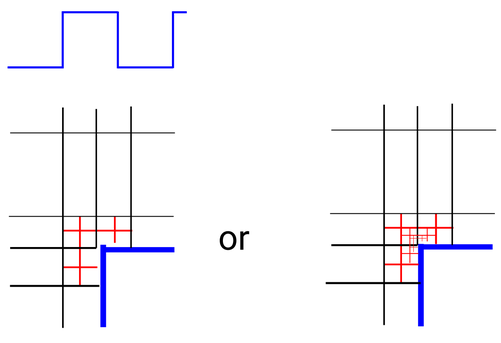
Conclusion and outlook

Conclusion

Significant influence of



Outlook



or

Dynamic adaptivity

a) $\mathrm{Re} = \sqrt{\left(\frac{|u|\Delta x}{\nu}\right)^2 + \left(\frac{|v|\Delta y}{\nu}\right)^2}$ in a certain range of values

b) Criterion based on normalized gradients

Literature

- Eric Chénier, Robert Eymard, Thierry Gallouët, and Raphaele Herbin. Calcolo, 52(1):69–107, 2015.
- TV Gerya, DA May, and Thibault Duretz. Geochemistry, Geophysics, Geosystems, 14(4):1200–1225, 2013.
- Louis Vittoz, Guillaume Oger, Zhe Li, Matthieu De Leffe, and David Le Touzé. In International Conference on Finite Volumes for Complex Applications, pages 73–89. Springer, 2017.

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