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A locally refined quadtree finite-volume staggered-grid scheme

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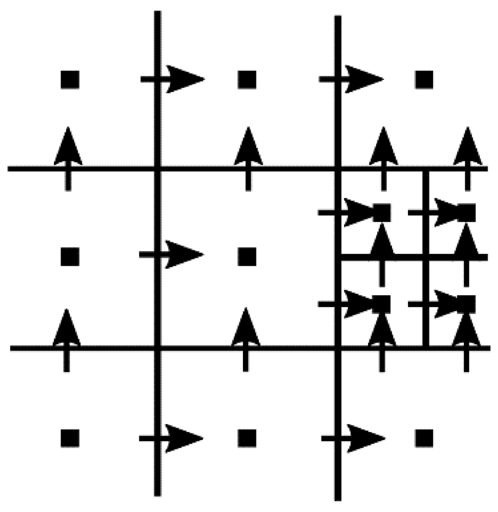
AX2

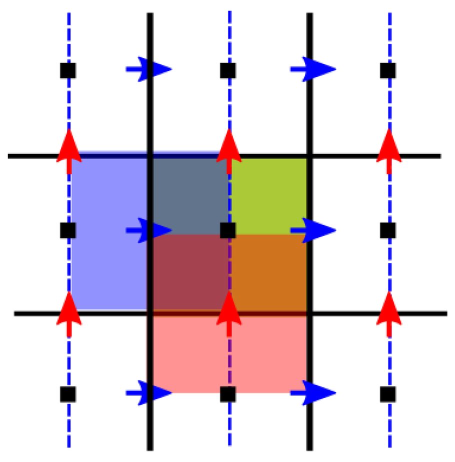
Motivation



Local refinement for Navier-Stokes equation
Goal: Find out if helpful for A02 (monolithic coupling)

FF
PM

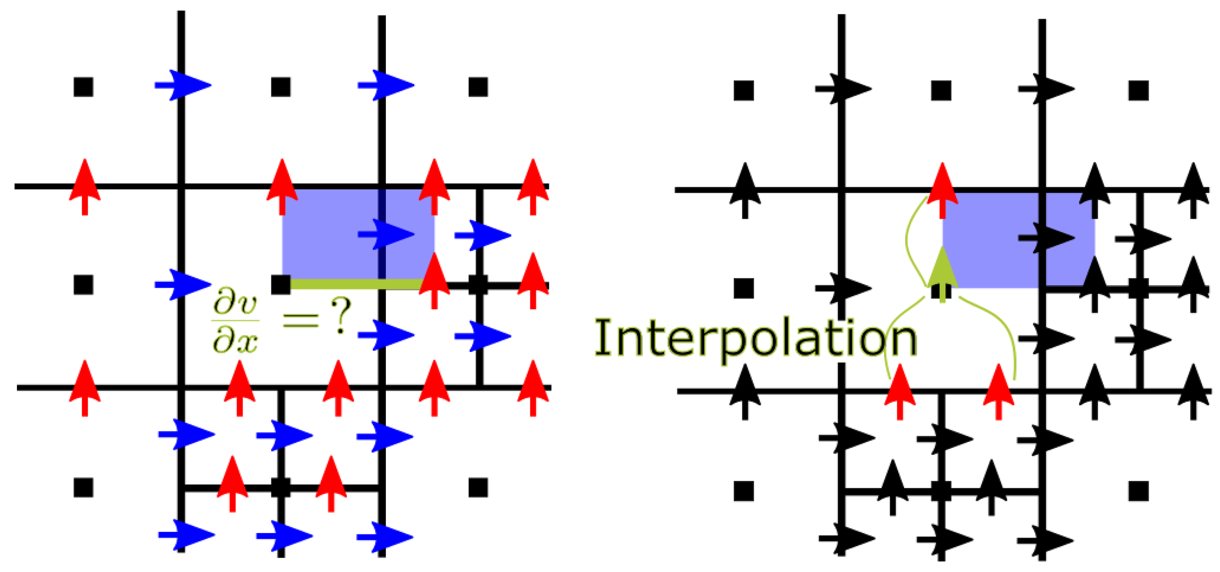
Staggered grid



Navier-Stokes equation

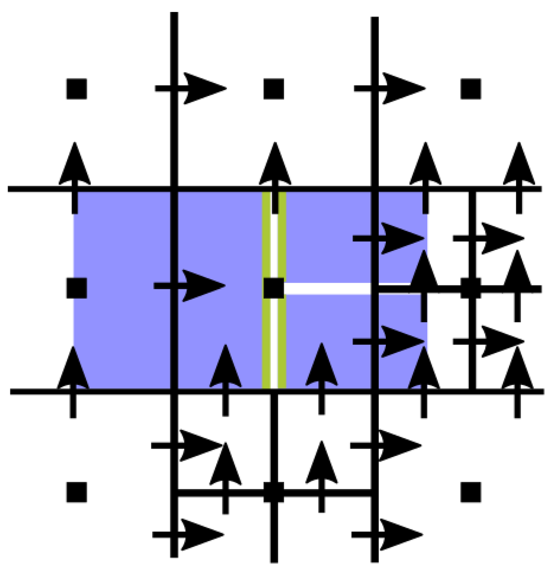
$$\frac{\partial(\varrho \mathbf{v})}{\partial t} + \nabla \cdot (\varrho \mathbf{v}\mathbf{v}^T) - \nabla \cdot (\mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)) + \nabla p - \varrho \mathbf{g} - q_v = 0$$

Interpolations



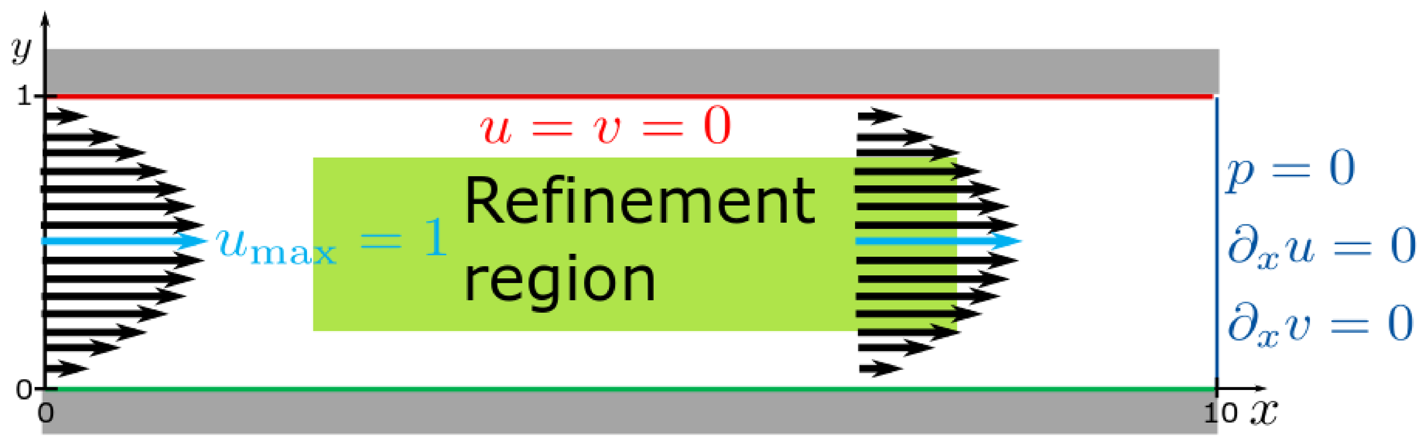
- Many geometries
- Larger stencils

Conservation of mass/momentum



-Complicates assembly

Pure free flow: Channel flow



Stationary Stokes equation, no gravity, no sources/sinks, $\varrho = 1\ \frac{kg}{m^3}$, $\nu = 1\ \frac{m^2}{s}$

L2 errors

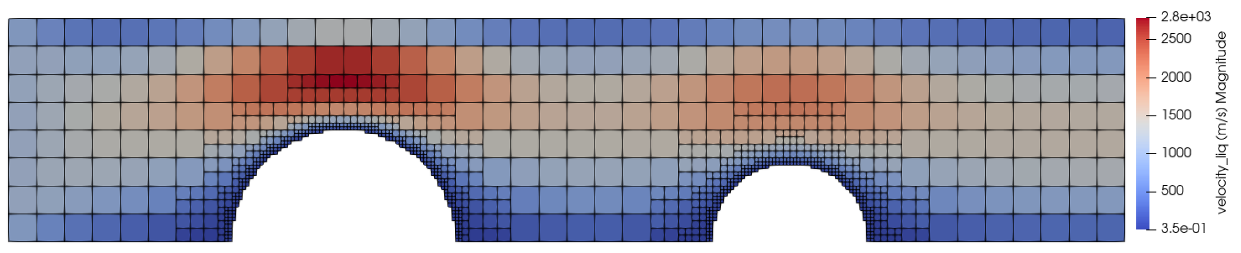
Rough grid		$\|\|\epsilon_p\|\|_2$	Order	$\|\|\epsilon_p\|\|_\infty$	Order
20x20	unrefined	1.72e-1	-	2.98e-1	-
20x20	refined	1.01e-1	-	1.82e-1	-
40x40	refined	2.60e-2	2.0	5.07e-2	1.8
80x80	refined	6.57e-3	2.0	1.49e-2	1.8
160x160	refined	1.66e-3	2.0	4.88e-3	1.6

Rough grid		$\|\|\epsilon_u\|\|_2$	Order	$\|\|\epsilon_u\|\|_\infty$	Order
20x20	unrefined	1.09e-3	-	2.12e-3	-
20x20	refined	1.27e-3	-	2.37e-3	-
40x40	refined	3.19e-4	2.0	6.12e-4	2.0
80x80	refined	7.96e-5	2.0	1.55e-4	2.0
160x160	refined	1.99e-5	2.0	3.89e-5	2.0

Rough grid		$\|\|\epsilon_v\|\|_2$	Order	$\|\|\epsilon_v\|\|_\infty$	Order
20x20	unrefined	7.15e-5	-	4.45-4	-
20x20	refined	8.90e-5	-	4.44e-4	-
40x40	refined	2.86e-5	1.6	1.82e-4	1.3
80x80	refined	8.31e-6	1.8	8.19e-5	1.2
160x160	refined	2.06e-6	2.0	2.86e-5	1.5

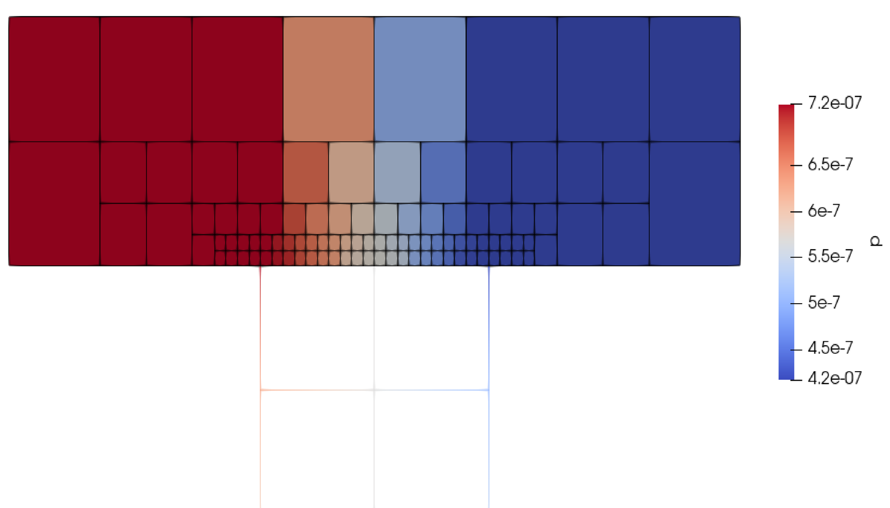
☺ Grid convergence
☹ Worse than underlying unrefined grid

Pure free flow: Channel flow with drops

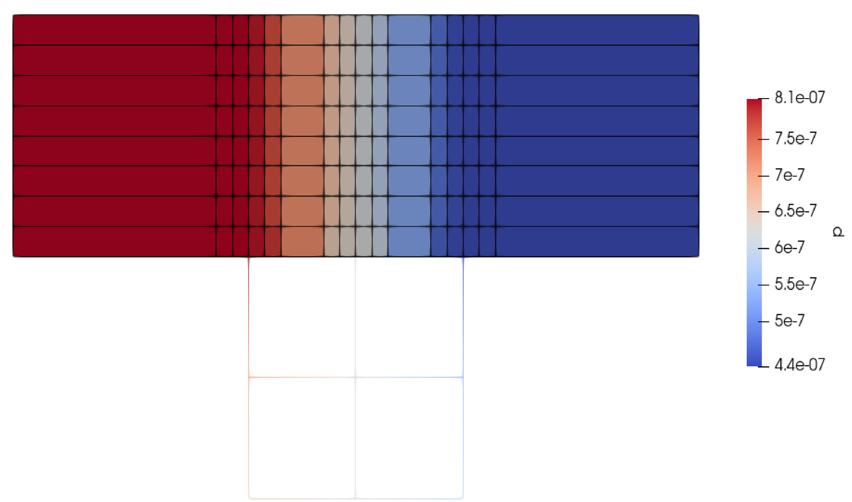


Resolve e.g. droplet-shaped interface forms (with help from Ivan Buntic)

Coupled porous-medium (Pore-network model from K. Weishaupt, A02) and free flow



Local refinement:
Pressure values in pore-network model further from all-fine free-flow grid



Global refinement:
Pressure values in pore-network model closer to all-fine free-flow grid

Conclusion and Outlook

With the examples calculated so far, the local refinement technique seems to converge but be inferior to global refinement in practical use. It will be further examined in which cases uniform, globally or locally refined grids are most efficient to use. It is planned to extend the local refinement in the free-flow regime to nonisothermal, compressible and compositional flow, and to couple with an REV-scale porous medium flow.

Literature

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