# Numerical methods for fractured porous media systems

International Research Training Group nupus

**Nicolas Schwenck***, Bernd Flemisch*, Barbara Wohlmuth
Jan Nordbotten, Inga Berre
*Department of Hydromechanics and Modelling of Hydrosystems

LH2
University of Stuttgart
Germany

# Motivation

- Flow in fractured porous-media systems is often dominated by their heterogeneities and discontinuities. Such systems characterise many applications, e.g.
  - $CO_2$ capture and storage,
  - reservoir engineering,
  - groundwater resource management.

*Figure 1: Fractured Rock, Pliezhausen, GER*

- At the scale of interest the material properties differ in orders of magnitude for the fracture network and the surrounding rock matrix. Furthermore, the characteristic flow behaviour of the whole system crucially depends on both the fractures and the rock matrix.
- The exact fracture structure on the field scale cannot be determined. Thus the fracture-network model has to be stochastically generated. To get meaningful results several (> 100) realisations have to be simulated.

⇒ *The discrete fractured porous-medium model has to be meshed fast and produce accurate results.*

# Current State

- Simulating fractured porous-media systems with fully resolved fractures is state of the art.
- At the Department of Hydromechanics and Modelling of Hydrosystems a fracture (*FRAC3D*) and a mesh generator (*ART3D*) were developed and regularly used.

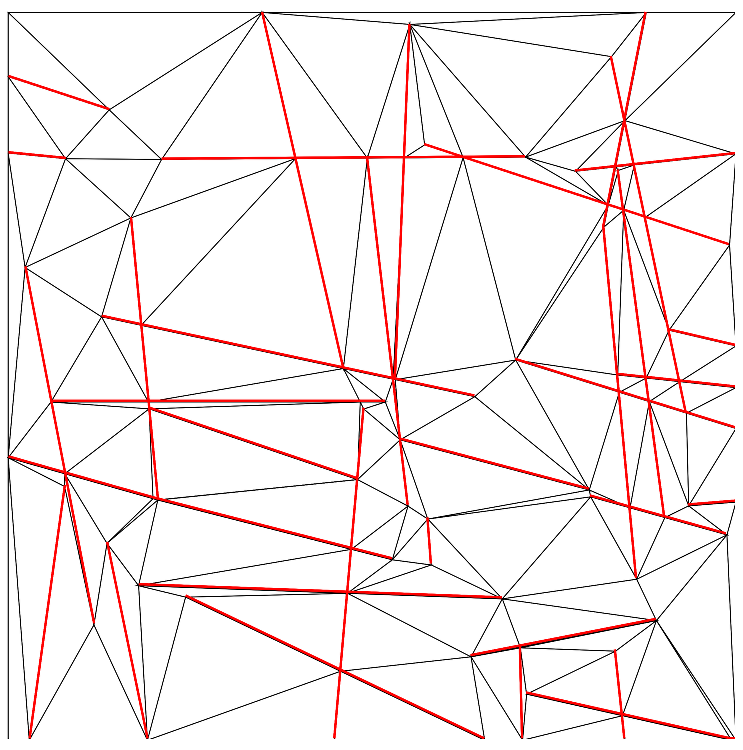

*Figure 2: Stochastically generated fractures meshed with a coarse grid leads to badly shaped elements.*

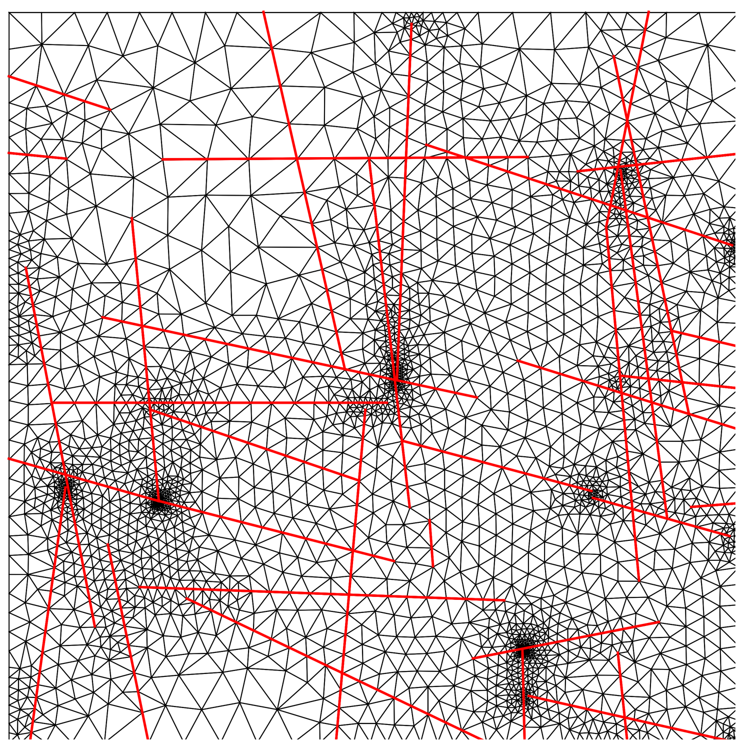

*Figure 3: To decrease the influence of badly shaped elements and to resolve the physically complex flow behaviour, a fine grid is needed.*

- Several different approaches exist to handle/resolve the fracture-matrix interaction,
- but they all require to resolve the fractures with a very fine grid to produce reasonable results.

⇒ *Time consuming meshing processes and simulations*

# Future Work

## Goals

Development of robust, flexible and consistent weakly coupled schemes for:

- porous media and fracture networks of co-dimension one and
- overlapping non-conforming grids for the rock matrix and the fracture network.

The implementation of these schemes will be

- integrated into the porous-media simulation toolbox DuMu$^X$ and
- based on DUNE (the Distributed and Unified Numerics Environment).

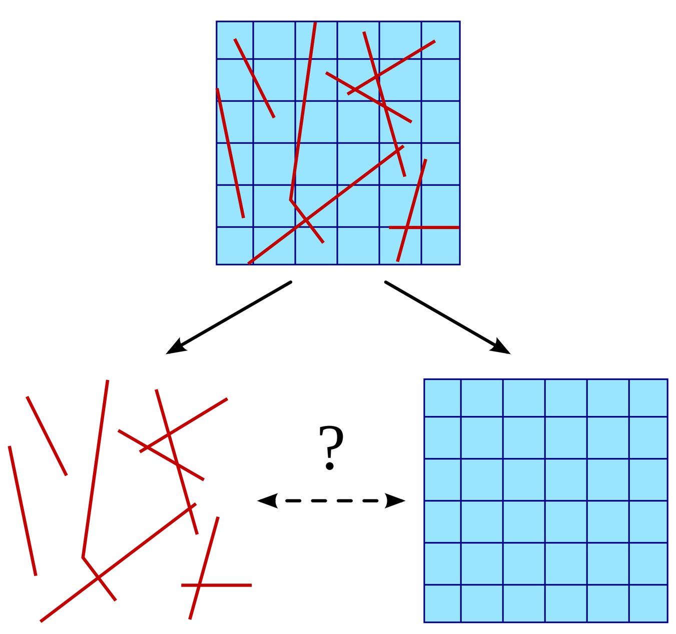


*Figure 4: Coupling between the fracture network and the rock matrix*

## Techniques

- The interface problem (discontinuities) is handled by a XFEM-based (eXtended Finite Element Method) approach.
  - ▸ Nodes of fracture-containing elements are enriched, i.e.,
  - ▸ the degrees of freedom are doubled.
  - ▸ With specific shape functions, discontinuities within elements can be handled.

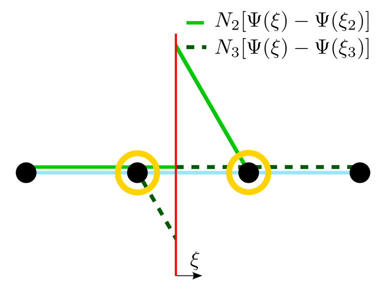


*Figure 5: 1D example of XFEM shape functions*

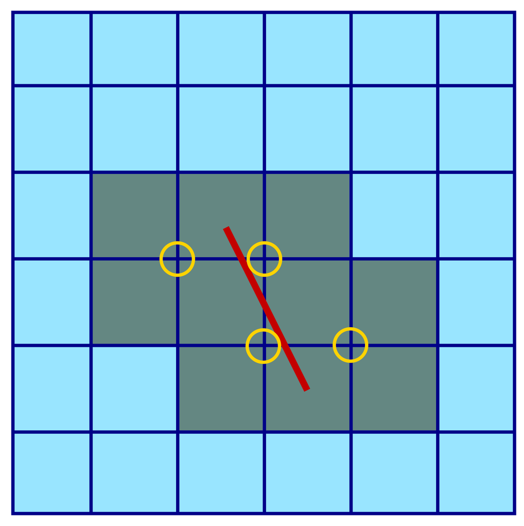

*Figure 6: 2D XFEM grid with enriched nodes (yellow)*

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{k=1}^{m} N_k(\mathbf{x})(\Psi(\xi) - \Psi(\xi_k))\mathbf{a}_k$$

- Discrete conservative conditions for the coupling between the fracture system and the rock matrix based on Lagrange-multiplier techniques have to be developed.

In a first approach the following assumptions are made:

- isothermal, single-phase, incompressible Darcy flow[1],
- the pressure in the fracture is known,
- and pressure discontinuities across the fractures can occur,

so that the following classical FE equation has to be solved:

$$a(p, \phi) = l(\phi)$$

with

$$a(p, \phi) = \sum (\mathbf{K}_i \nabla p_i, \nabla \phi_i)_{\Omega_i} + \left(\frac{\alpha_f}{\xi - 1/2}\{p\}, \{\phi\}\right)_\gamma + \left(\frac{\alpha_f}{2}[p], [\phi]\right)_\gamma$$

and

$$l(\phi) = \left(\frac{\alpha_f}{\xi - 1/2} p_f, \{\phi\}\right)_\gamma .$$

⇒ *Later an extension to multi-phase, compositional, non-isothermal, compressible flow within the fractures and the rock matrix is desired.*

# Cooperation

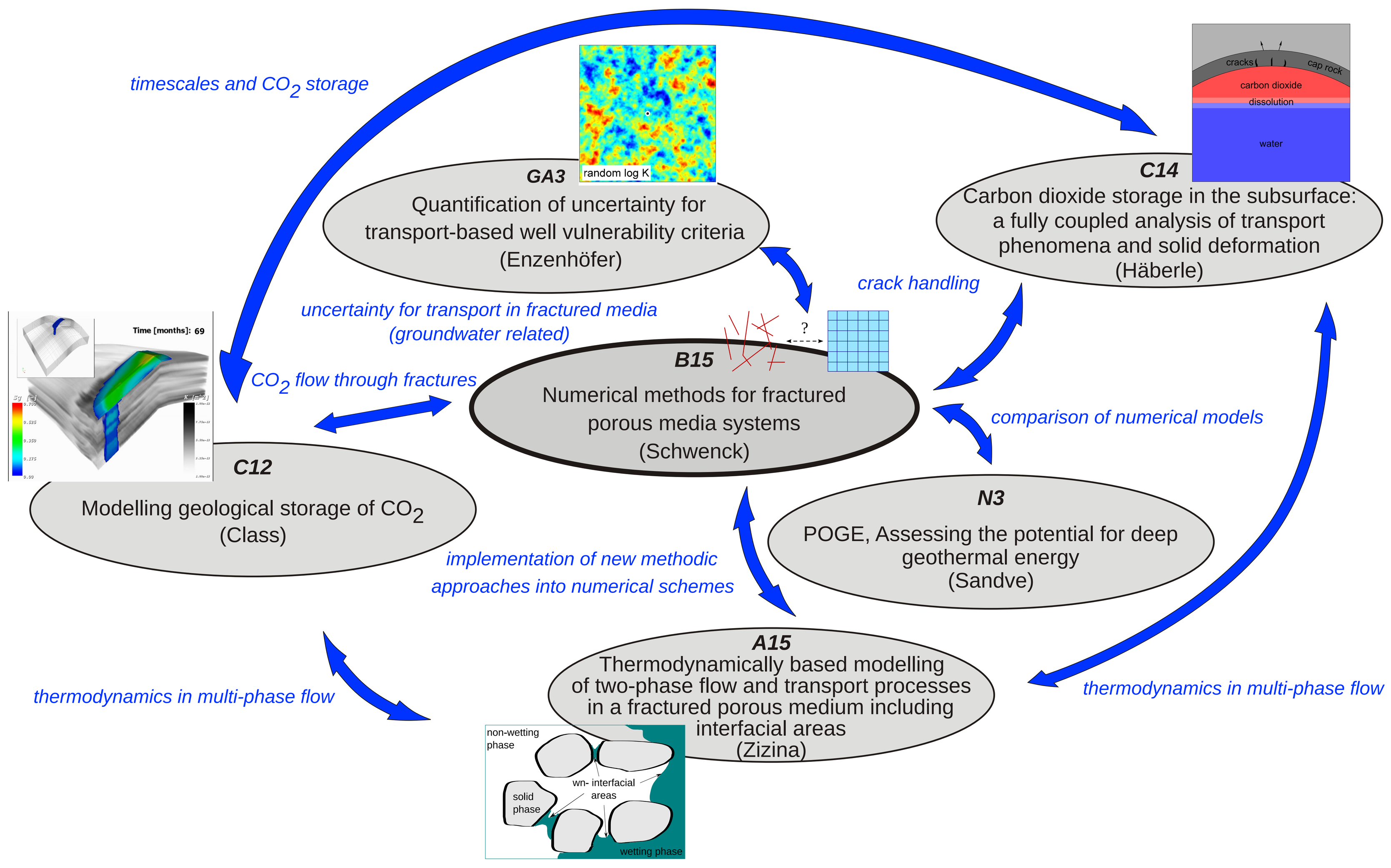


*Figure 7: Research framework within NUPUS around project B15*

# Literature

[1] *Martin, V., Jaffré, J. and Roberts, J.E.*: Modeling fractures and barriers as interfaces for flow in porousmedia. *SIAM Journal on Scientific Computing*, 26 (2005), p. 1667–1691.

[2] *Dietrich, P., Helmig, R., Sauter, M., Hötzl, H., Köngeter, J., and Teutsch, G. eds.*: Flow and transport in fractured porous media. *Springer-Verlag Berlin Heidelberg, 2005.*

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