International Research Training Group nupus

Modeling of Multiphase Flow with a Multiphysics Framework on Adaptive Grids.

Interpore Conference

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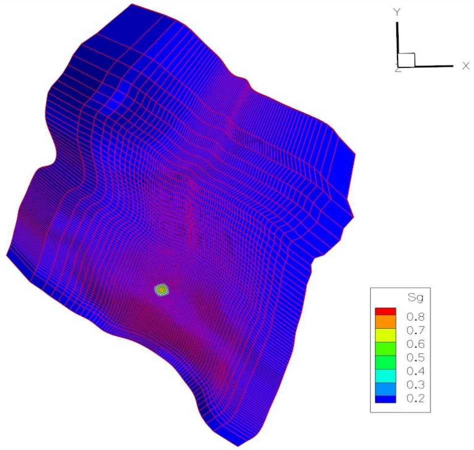
Large Scales

Computational demands are high:

- Large domain size.
- Large timespan of interest.



BGR (2010)

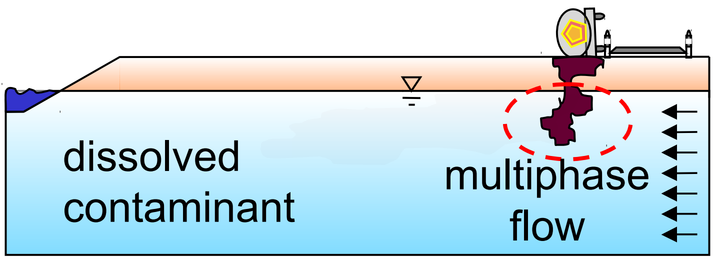
=> We need efficient models!
=> We need efficient models!

Complicated Processes

Most environmental applications of flow and transport in porous media are subject to...

… complex physics...

- Compressible, partly miscible substances
- Multi-phase behaviour



Fritz (2010)

… that differ over space & time:

- Complex multi-phase locally vs. single-phase in far-field.

=> We need good models!
=> We need good models!

Numerical Scheme

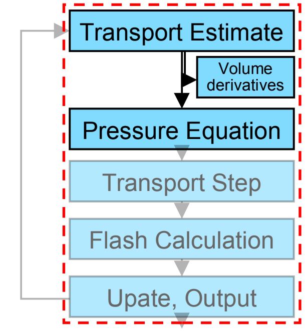
Mass conservation:

- For phases α and components κ , for each component:

$$\sum_\alpha \frac{\partial \phi S_\alpha \varrho_\alpha X_\alpha^\kappa}{\partial t} + \nabla \cdot \left(\sum_\alpha X_\alpha^\kappa \varrho_\alpha \mathbf{v}_\alpha + \mathbf{J}_\alpha^\kappa \right) + \sum_\alpha X_\alpha^\kappa \varrho_\alpha q^\kappa = 0$$

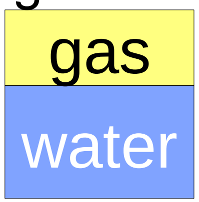
$$\mathbf{v}_\alpha = -\lambda_\alpha \mathbf{K}(\nabla p_\alpha - \varrho_\alpha \mathbf{g})$$

- Solution strategies:
  - Fully implicit
  - Sequential
    - Summation yields one pressure equation.
    - Transport equation
    - Flash calculations



Volume balance:

– Single Phase:



$$c_\alpha \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v}_\alpha = \sum_\kappa \frac{1}{\varrho_\alpha} q^\kappa + \varepsilon,$$

– Multi Phase

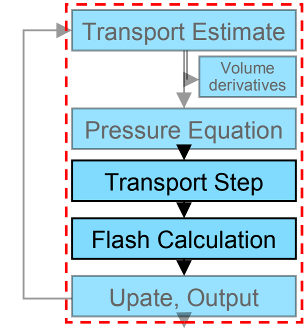
$$c_{total} \frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_{total}}{\partial C^\kappa} \nabla \cdot \left(\sum_\alpha X_\alpha^\kappa \varrho_\alpha \mathbf{v}_\alpha \right) = \sum_\kappa \frac{\partial v_{total}}{\partial C^\kappa} q^\kappa + \varepsilon,$$

Acs et. al (1985)

– If we use non-wetting pressure as primary variable

$$\mathbf{v}_w = -\lambda_w \mathbf{K}(\nabla p_n - \nabla p_c - \varrho_w \mathbf{g}),$$
$$\mathbf{v}_n = -\lambda_n \mathbf{K}(\nabla p_n - \varrho_n \mathbf{g}),$$
$$C^\kappa = \sum_\alpha \varrho_\alpha S_\alpha X_\alpha^\kappa$$

Explicit Step



Transport Equation (explicit):

$$\frac{\partial C^\kappa}{\partial t} = -\nabla \cdot \left(\sum_\alpha X_\alpha^\kappa \varrho_\alpha \mathbf{v}_\alpha \right) + q^\kappa,$$

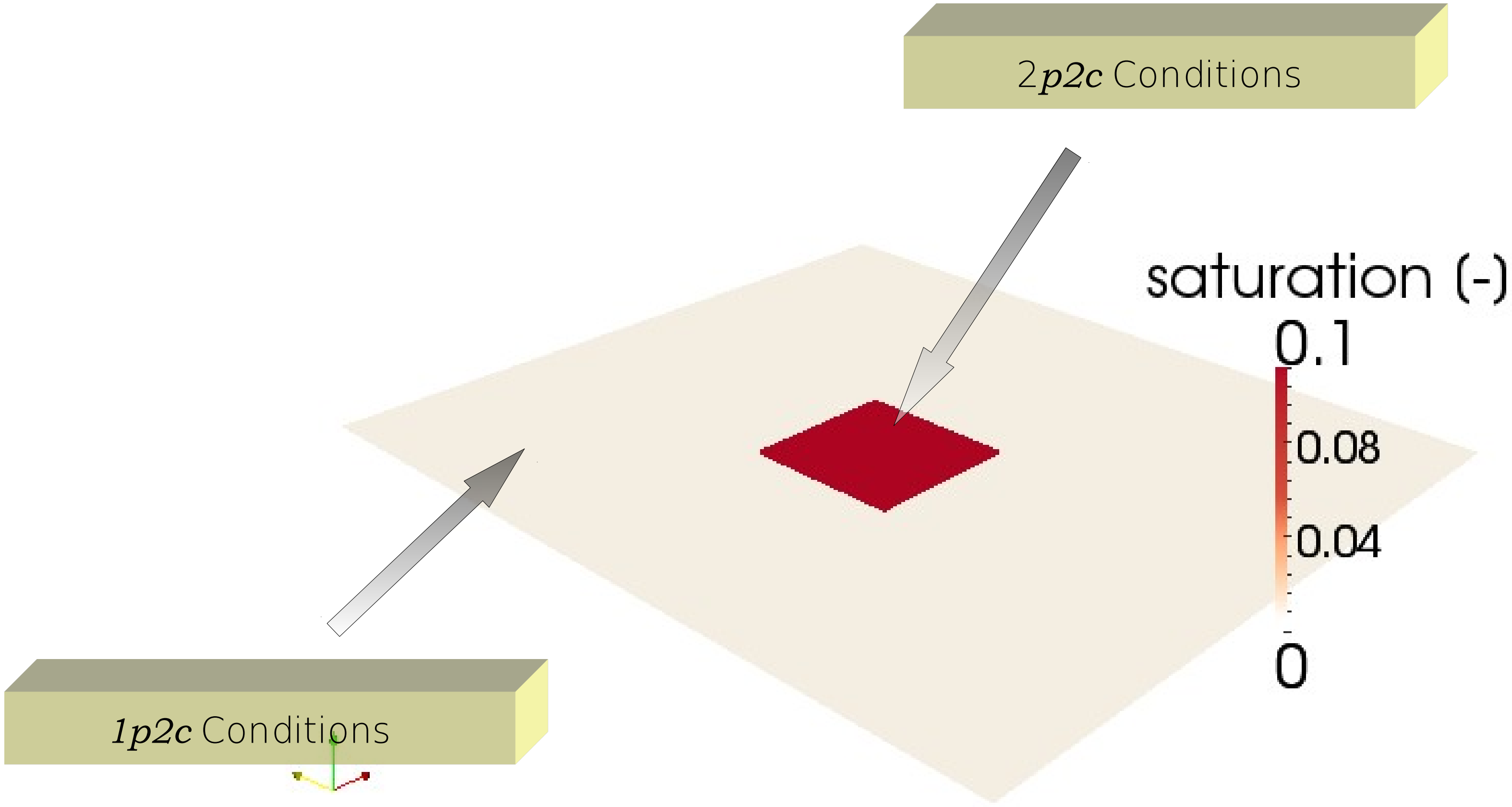
– Determines size of the time step.

Equilibrium (Flash-) Calculation

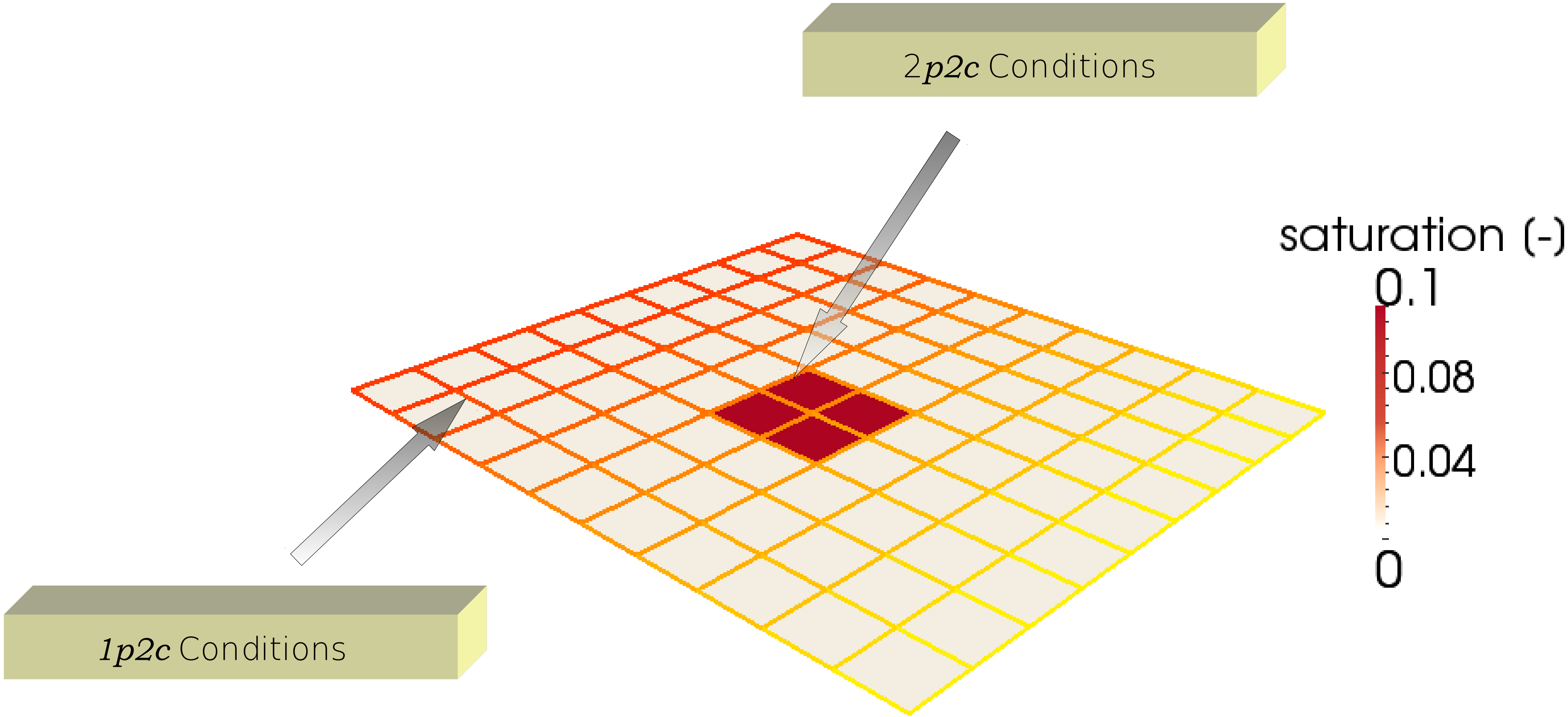

NEVER ADAPT.
Introducing the 2012 Jeep Grand Cherokee.
Jeep

Conditions differ locally.

- Here: Remediation scenario

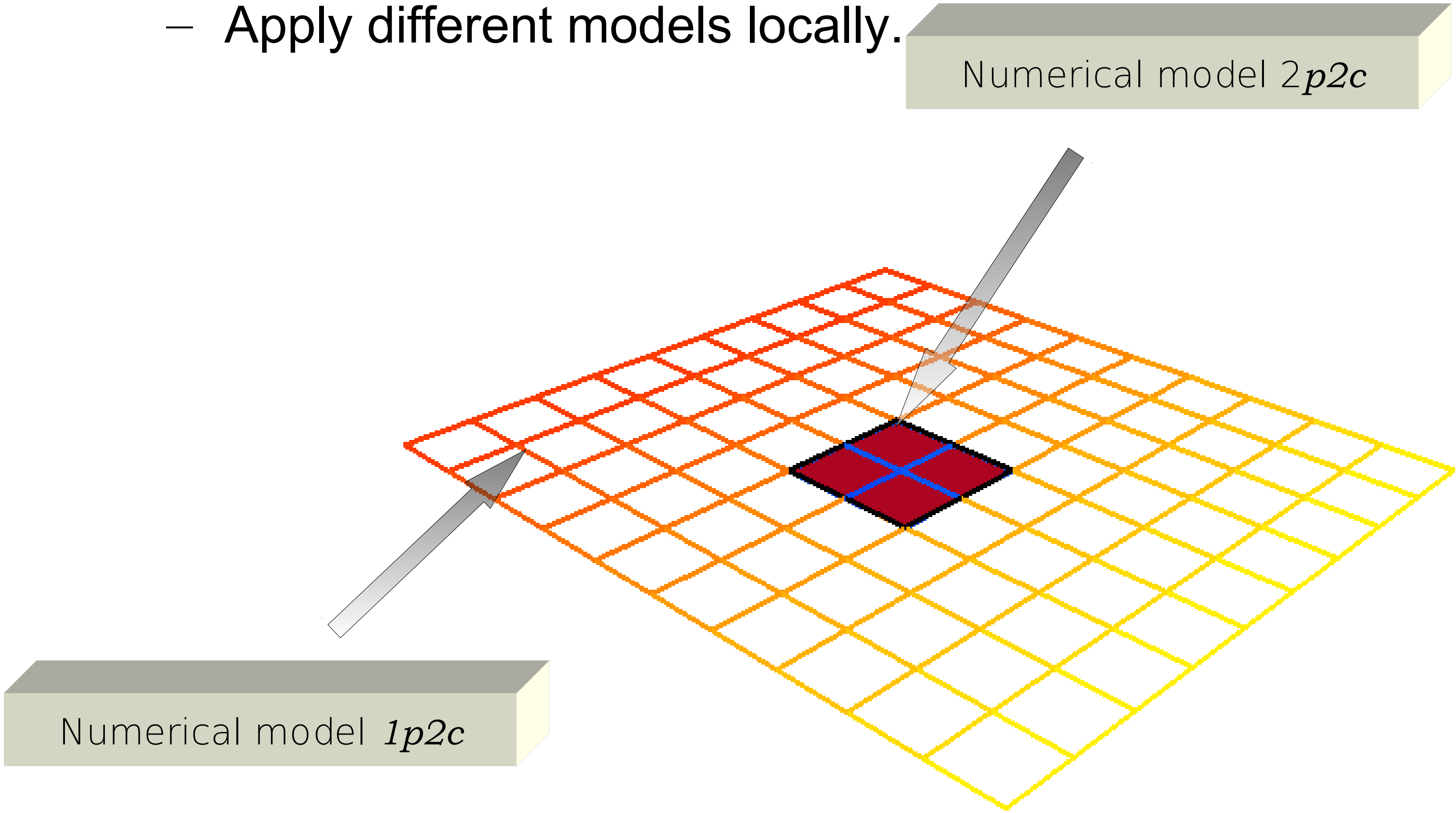


Concept of Multi-Physics



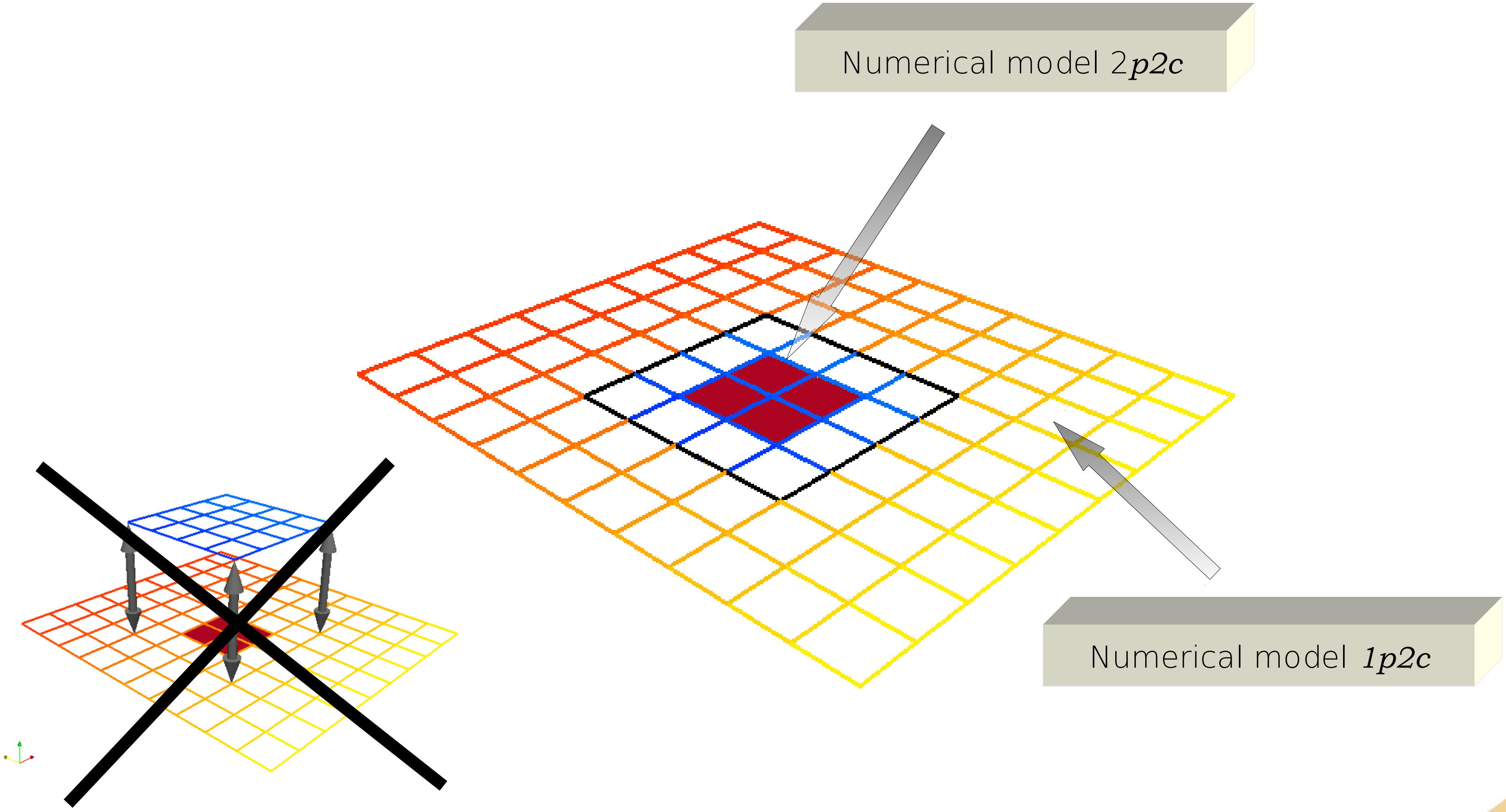
Models differ locally

- Divide into sub-domains.
- Apply different models locally.

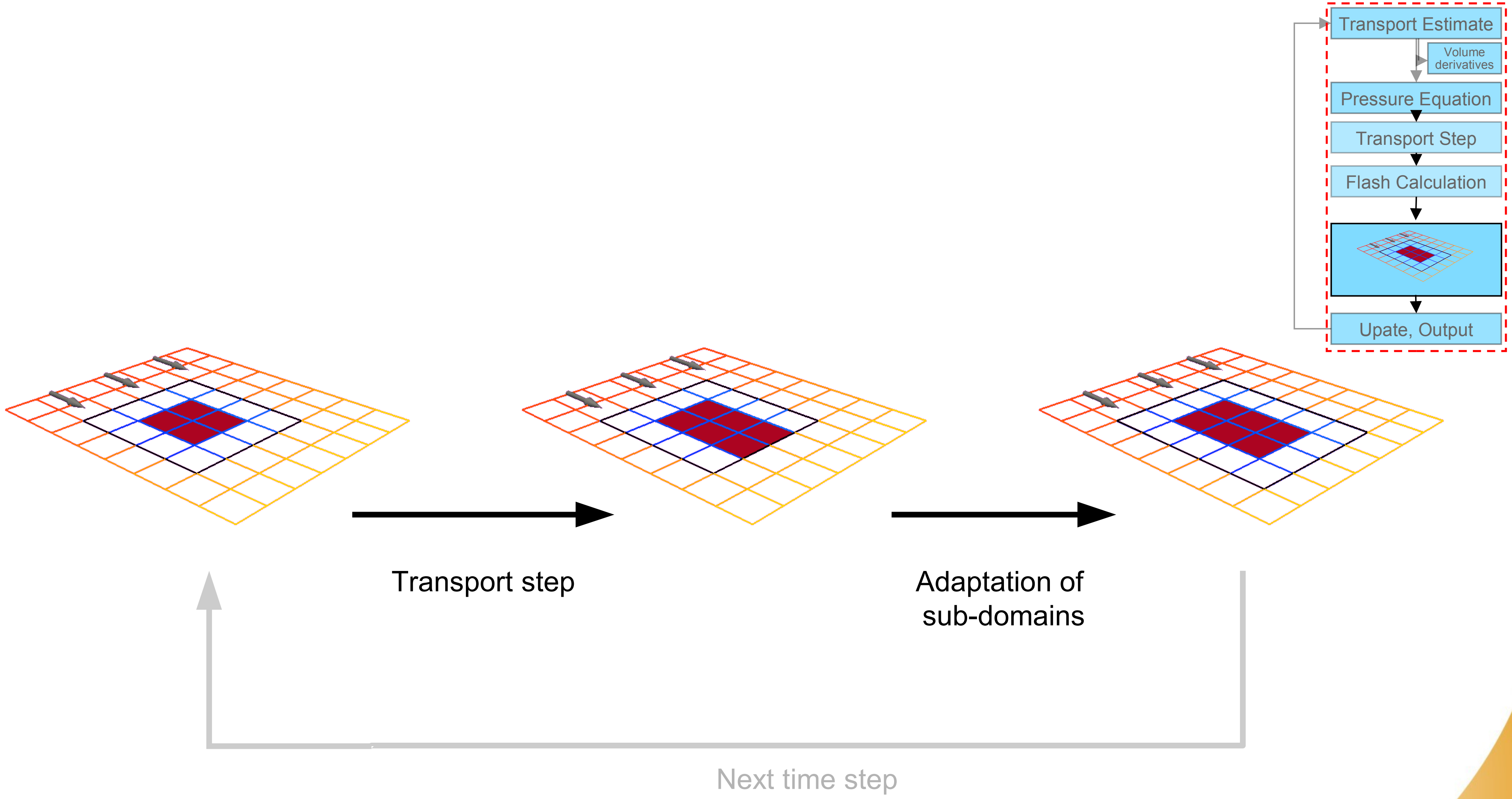


Allocation to Sub-Domains

– Use “safety zone” around complex sub-domains.

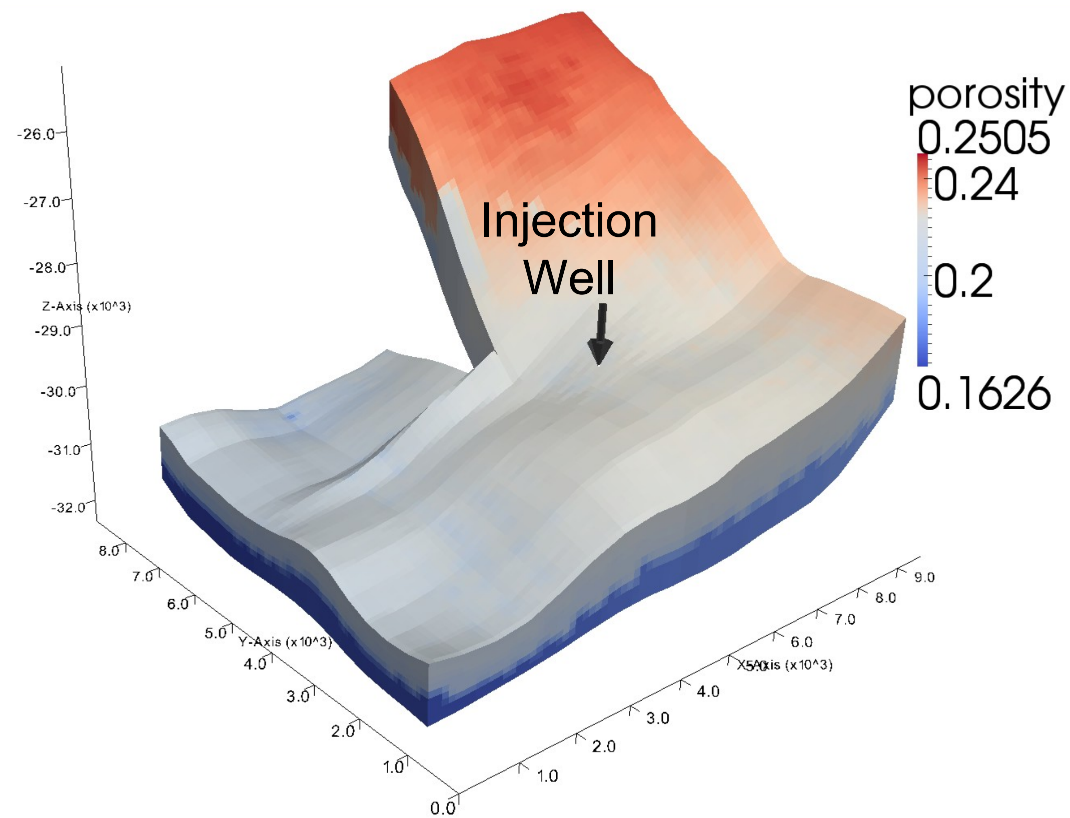


Update of the Sub-Domains

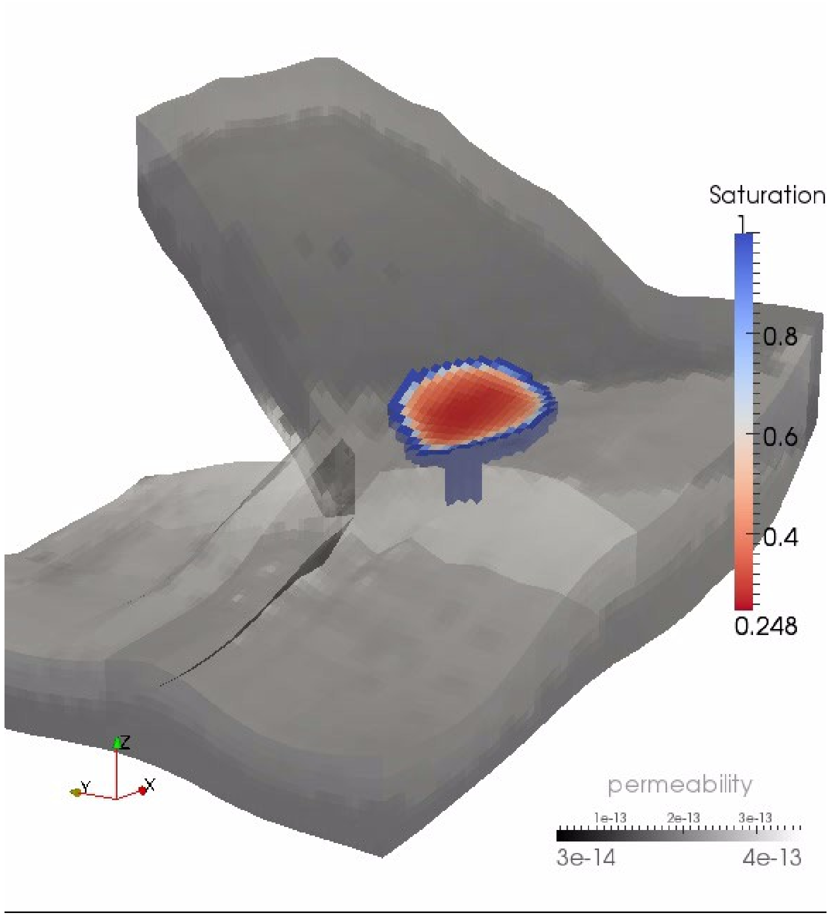


Benchmark: Injection of CO_2: Class et. al (2009)

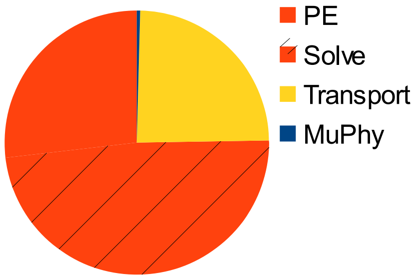
- Injection of CO_2 for 25 years.
- Simulation of 50 years.
- 54756 cells.
- Vertically exaggerated by factor 10.
- Boundary Conditions:
  - Hydrostatic pressure
  - Temperature gradient



Large-scale Benchmark

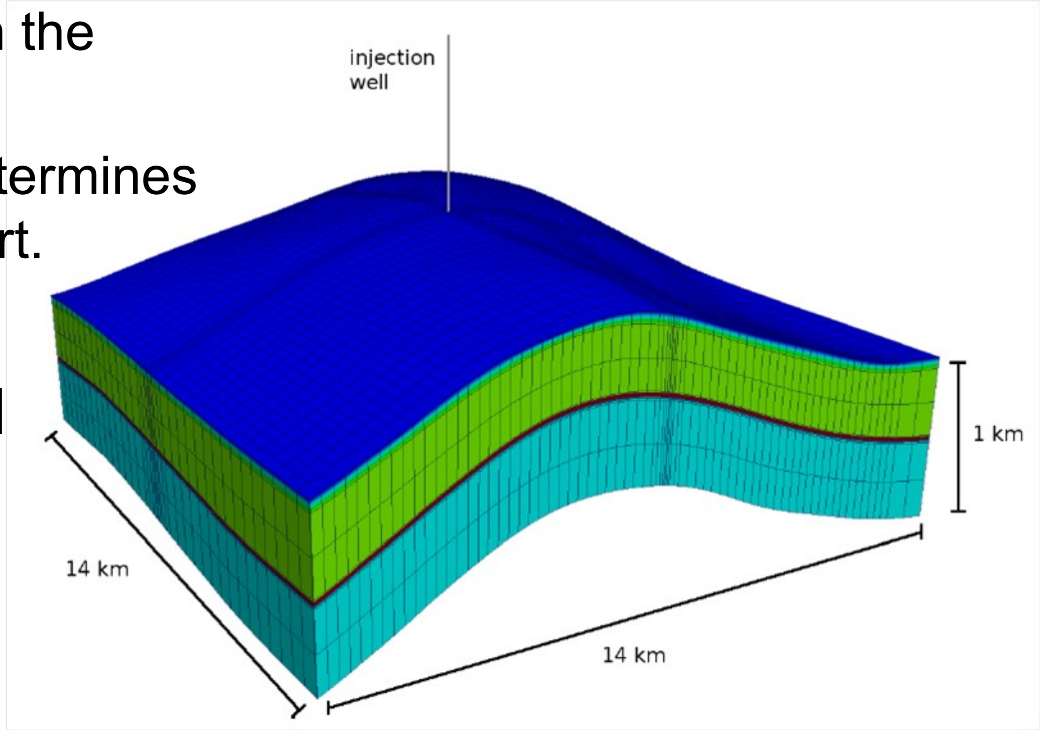


Why adaptive grids?



Sequential IMPEC:

- Most time spent for solution of the pressure equation.
- Most accuracy needed in the pressure field:
  - Pressure field determines explicit transport.

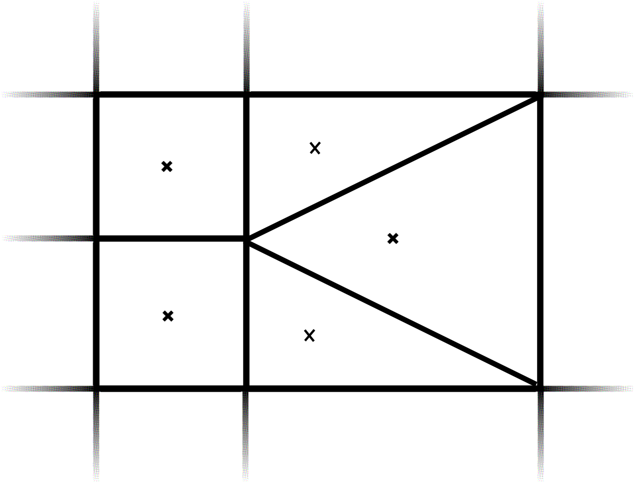


=> local refinement of the grid desired.

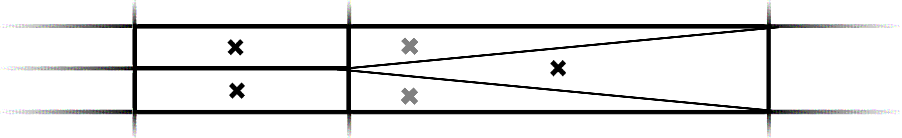
How to refine the grid?

Finite Volume context:

- Refine with closure

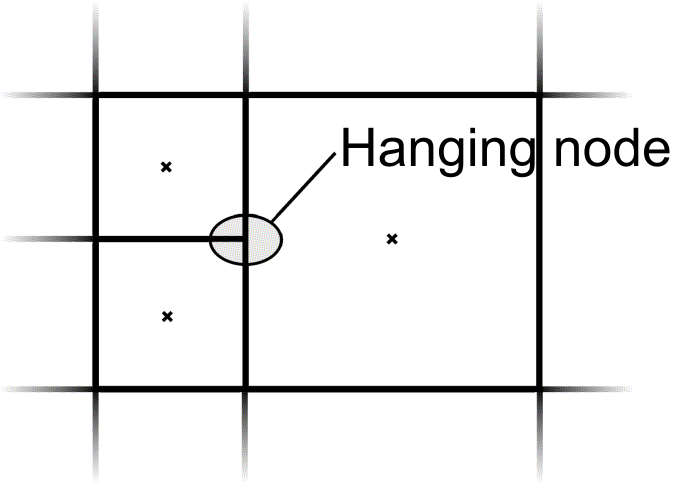


» Problem:



» e.g. Johannsen: Cell 80m x 10m

- Refine with irregular faces

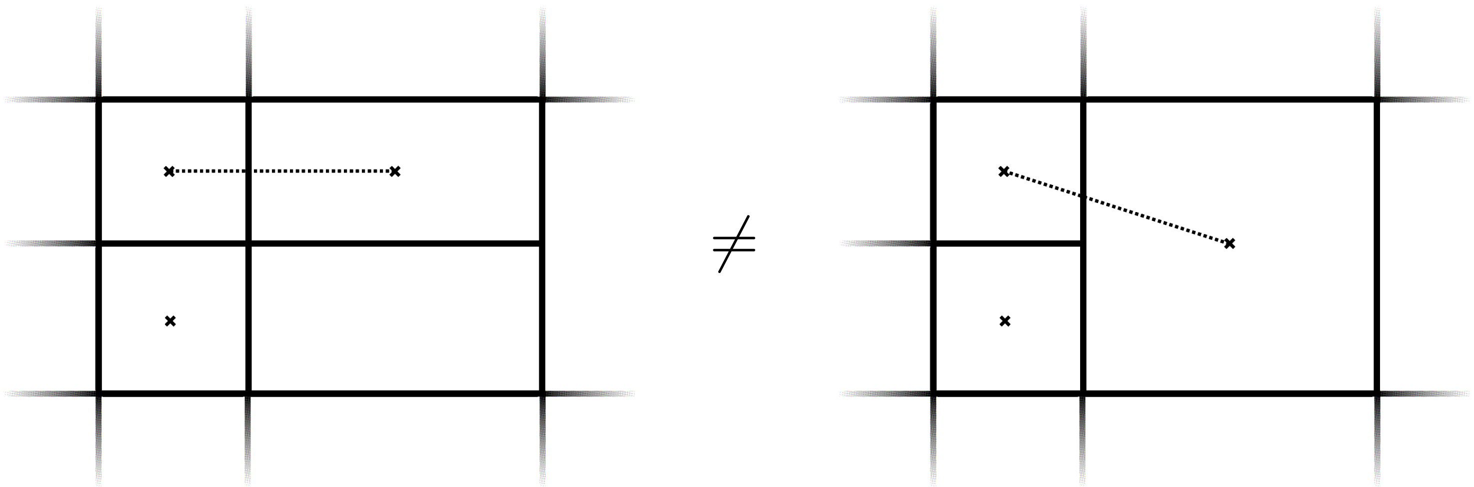


Fluxes: Tpfa

- Standard approach to approximate flux:
  Use a Two-Point flux approximation

$$\nabla\Phi \approx \frac{\Phi_j - \Phi_i}{\Delta x}$$

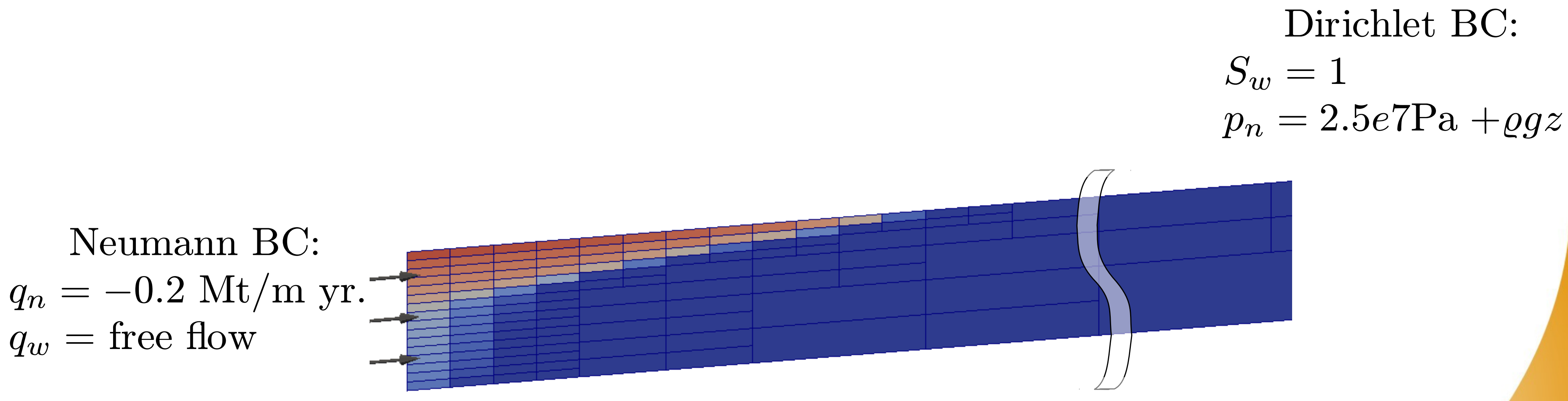
- Problem:



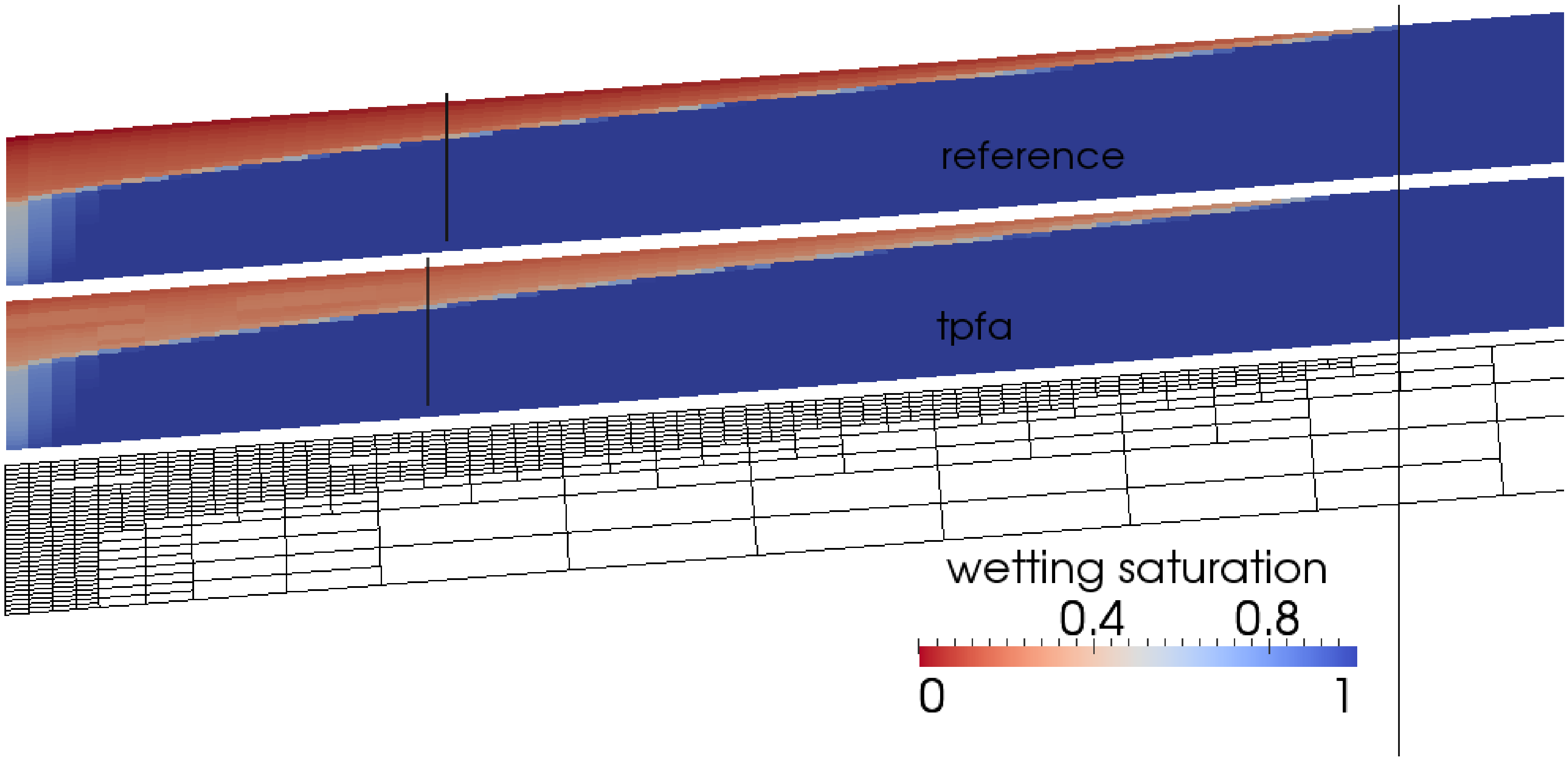
Example Simulation

Injection of CO_2 into Brine:

- Tilted 2D domain, inclined cells .
- Comparison of fully refined reference with adaptively coarsened grid.

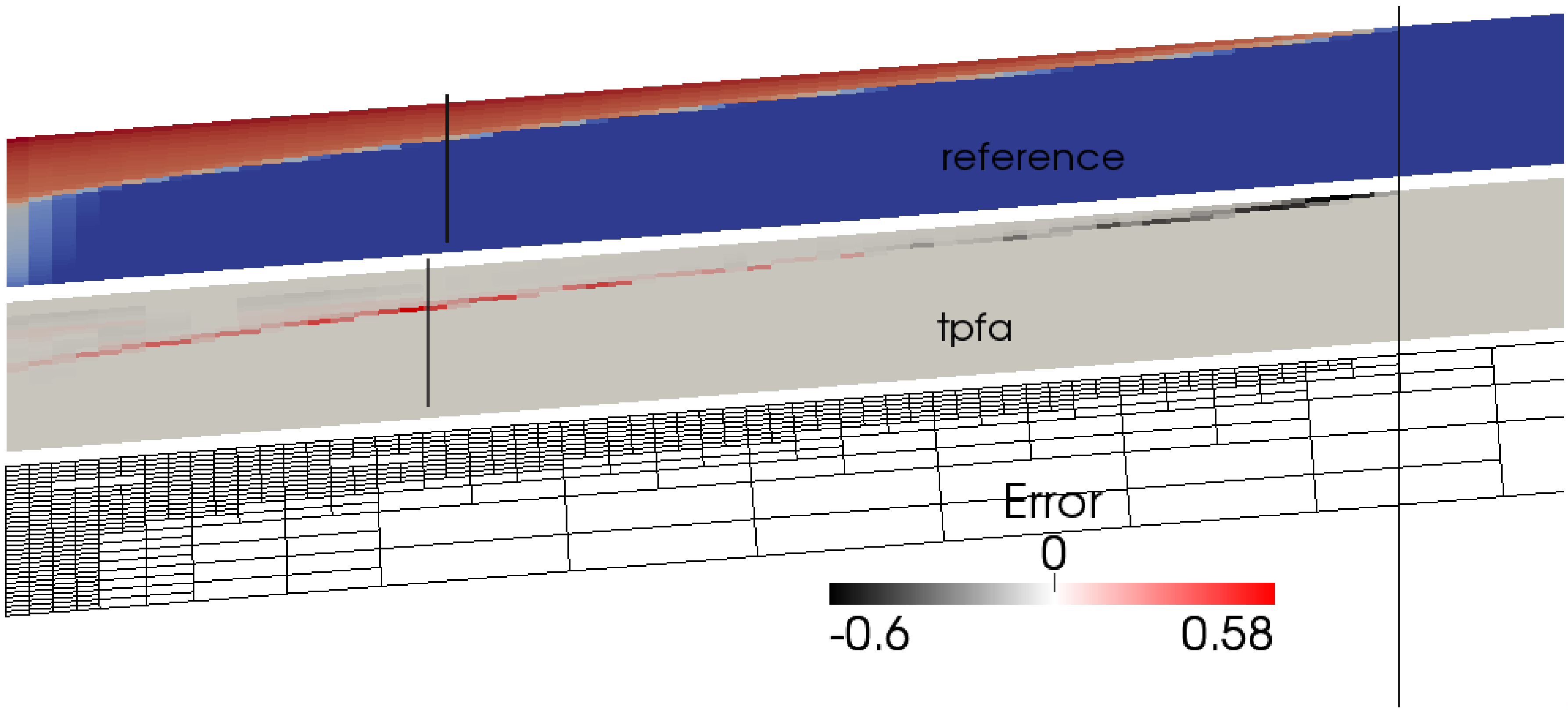


Simulation Results: TPFA



Simulation Results: Error in TPFA

Error in wetting Saturation: $S_{w,ref} - S_w$

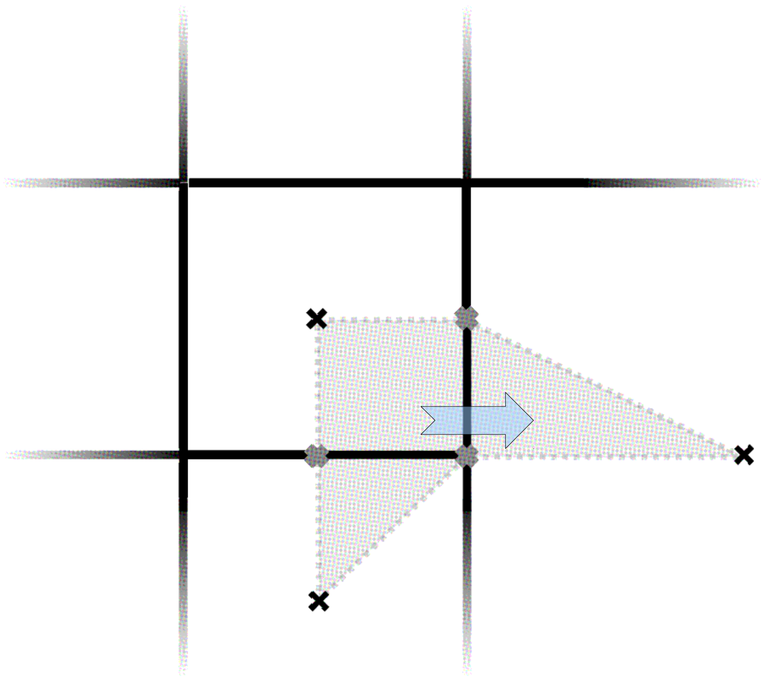


Fluxes: Mpfa

Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.



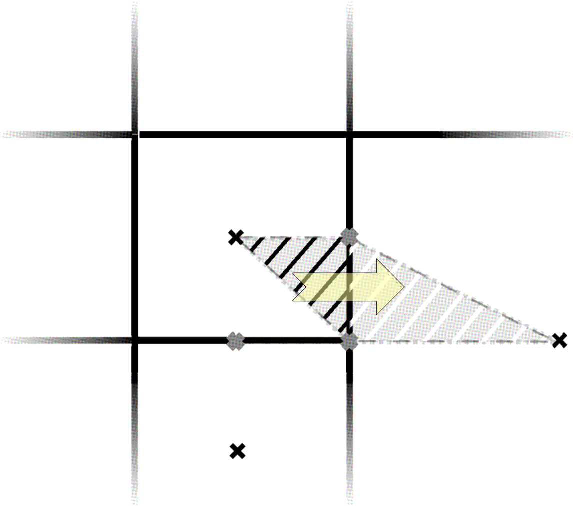
Fluxes: Mpfa

Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.

c) Approximate flux with new points.



$$f_{\gamma;i} = -\mathbf{n}_\gamma^T \mathbf{K}_i \nabla U_i$$

Value on introduced point k

$$\nabla U_i = \frac{1}{T} \sum_{k=1}^{2} \nu_k (u_k^* - u_0)$$

Value at cell center of cell i

Fluxes: Mpfa

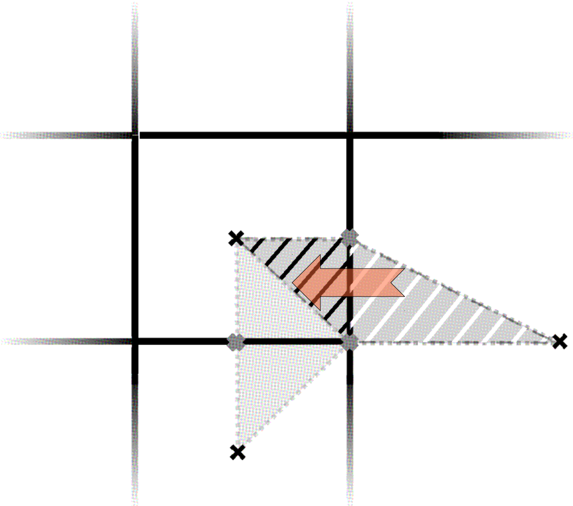
Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.

c) Approximate flux with new points. $f_{\gamma;i} = \ldots$

d) Approximate flux as seen from all cells.



$$
\begin{aligned}
f_{\gamma;j} = & -\mathbf{n_2}^T \mathbf{K}_3 \frac{1}{T_3} \nu_5 (u_2^* - u_3) \\
& -\mathbf{n_2}^T \mathbf{K}_3 \frac{1}{T_3} \nu_6 \left(u_j - u_3 \right. \\
& \qquad + \frac{1}{T_i} \nu_7^T \mathbf{R} \nu_i (u_1^* - u_i) \\
& \qquad \left. + \frac{1}{T_i} \nu_7^T \mathbf{R} \nu_j (u_2^* - u_i) \right)
\end{aligned}
$$

Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.

c) Approximate flux with new points.

d) Approximate flux as seen from all cells.

e) Write everything into a large equation system of form

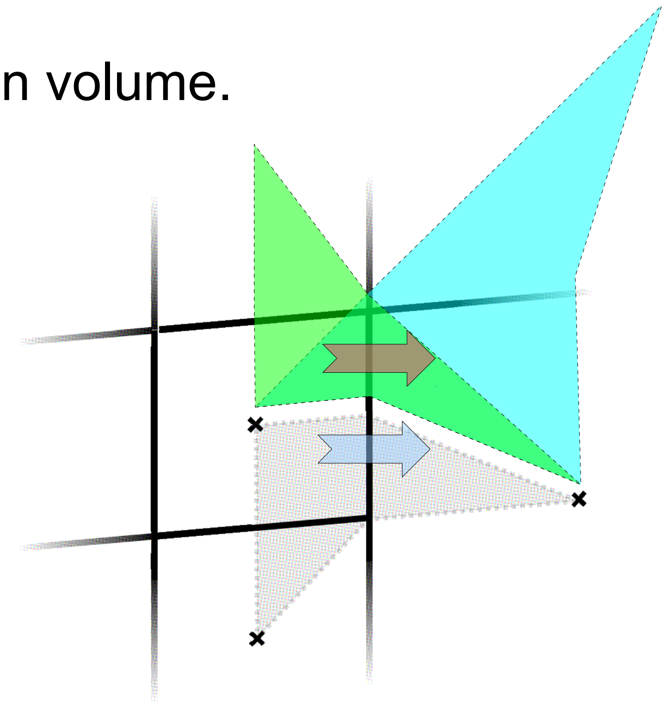
$$\mathbf{f} = \mathbf{T}\mathbf{u}$$

where T contains the „transmissibility coefficients“ (introduced points already eradicated), and u is the vector of unknown cell values

Fluxes: MPFA for Second Half-Edge

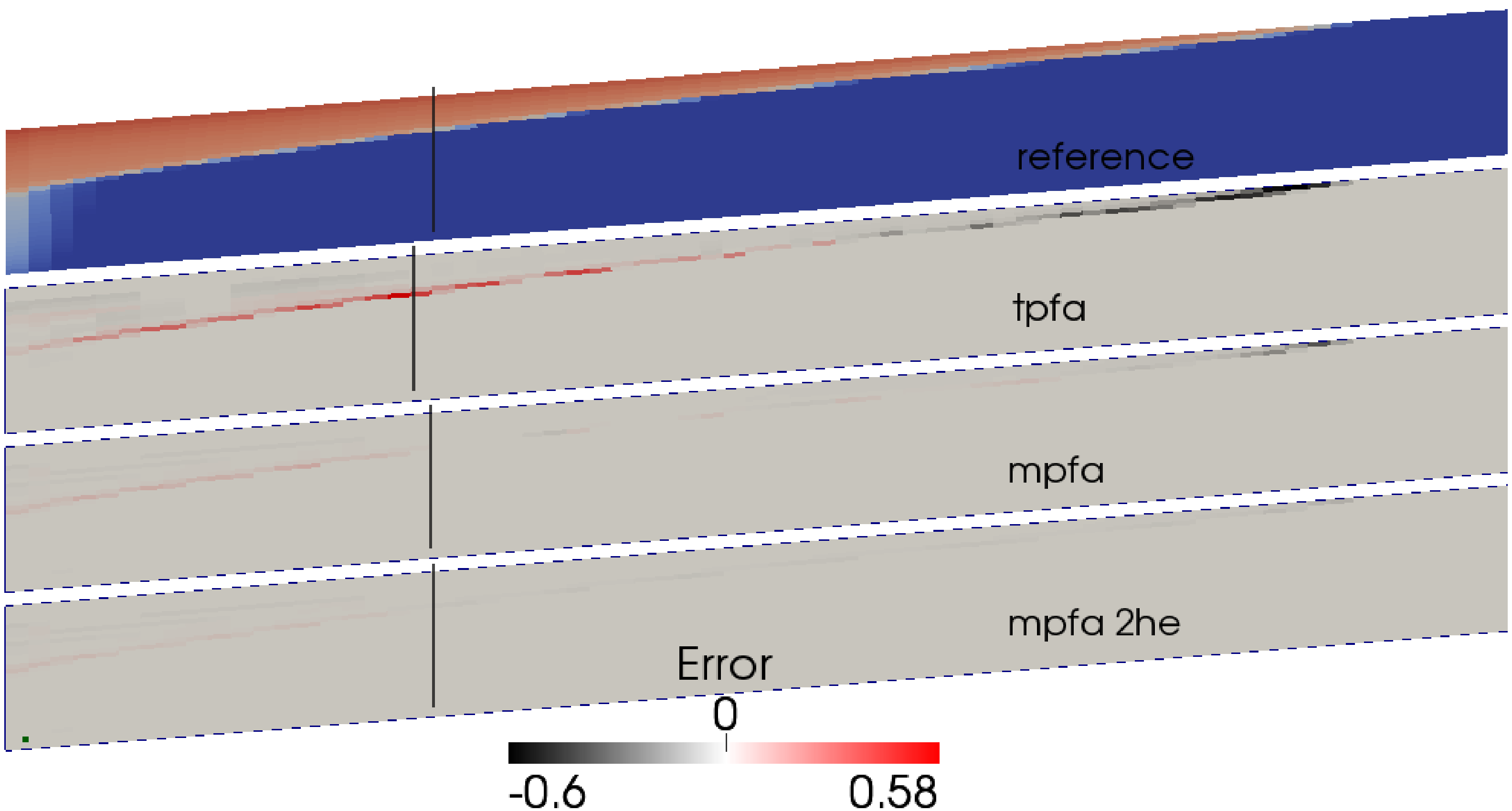
Flux through the edge:

- Twice the flux of first half-edge of the interaction volume.
- Construct second interaction region:
  - Strongly dependent on surrounding cells.
  - Expensive way to "search" the region.
  - Is it "worth the effort"?



MPFA performs much better!

$$\sum_i (S^i_{w,ref} - S^i_w)^2 / \sum_i (S^i_{w,ref})^2$$

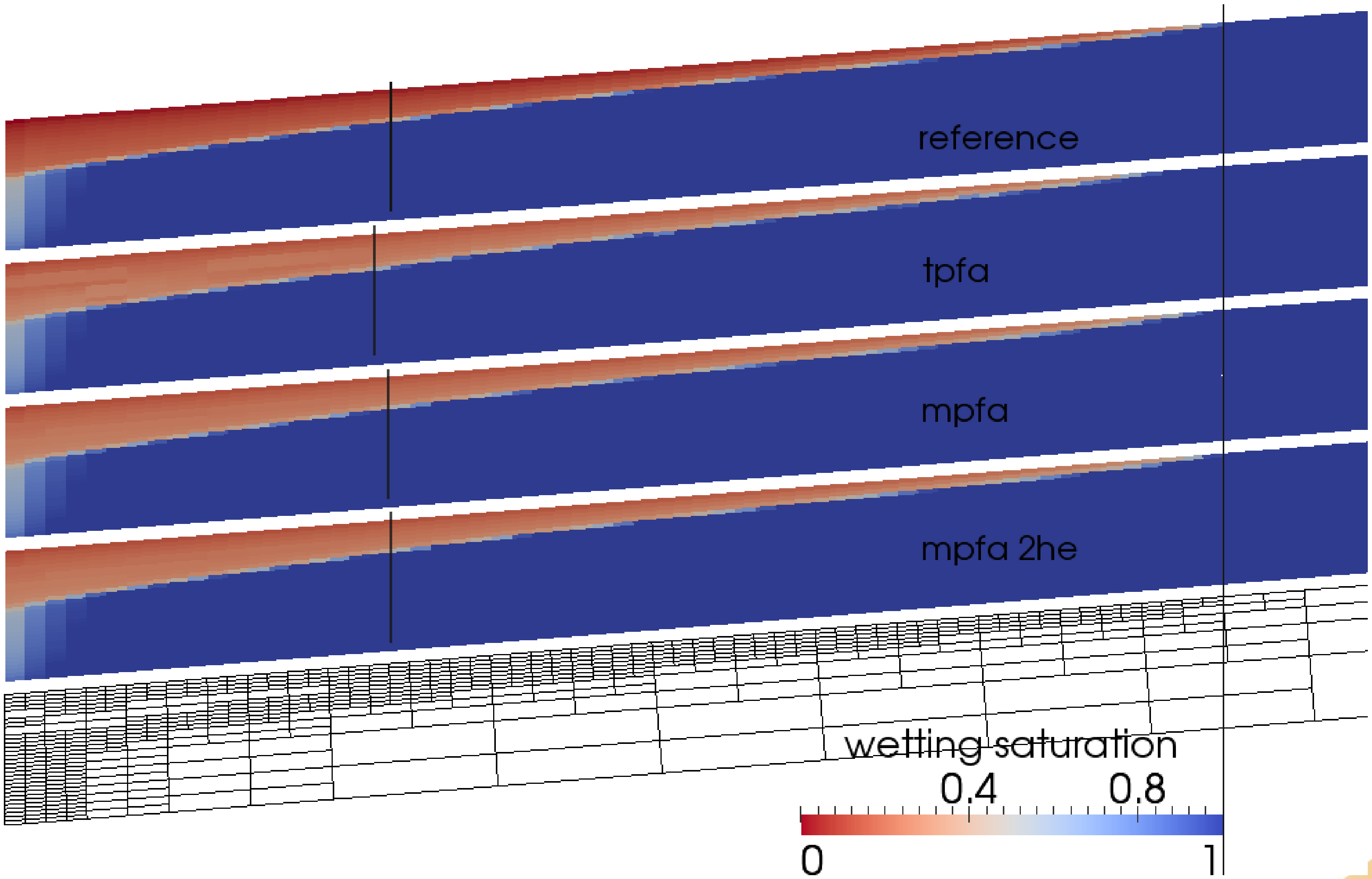


$125.4 \cdot E^{-3}$

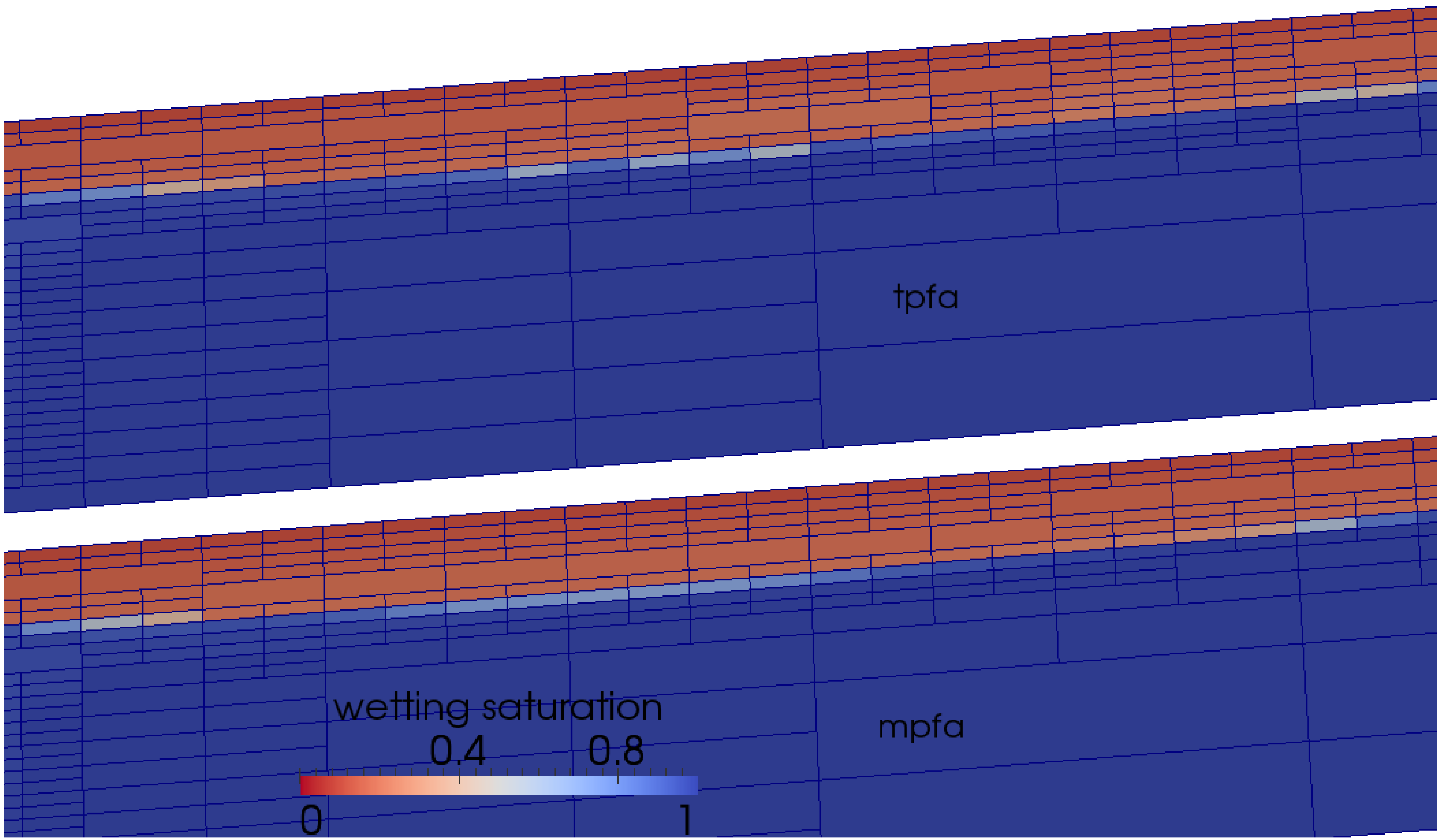
$7.57 \cdot E^{-5}$

$0.82 \cdot E^{-5}$

MPFA performs much better!

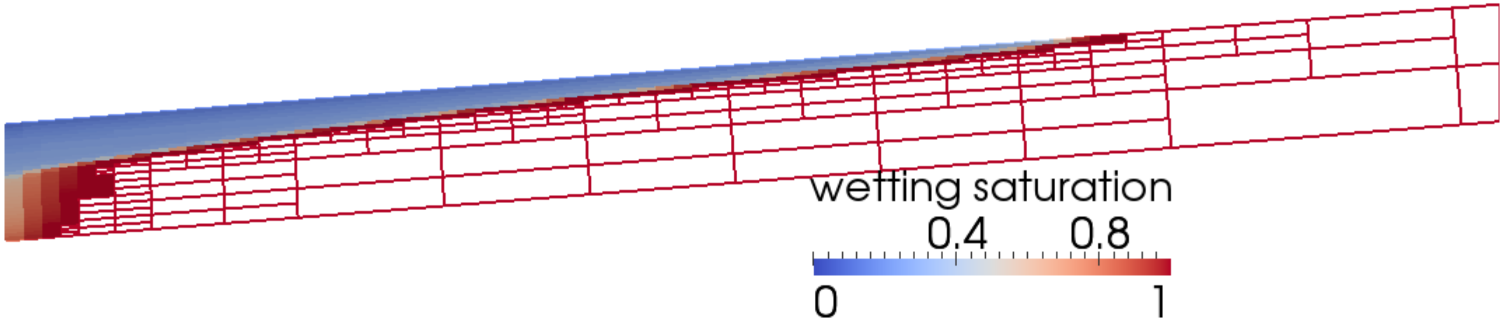


Simulation Results: TPFA



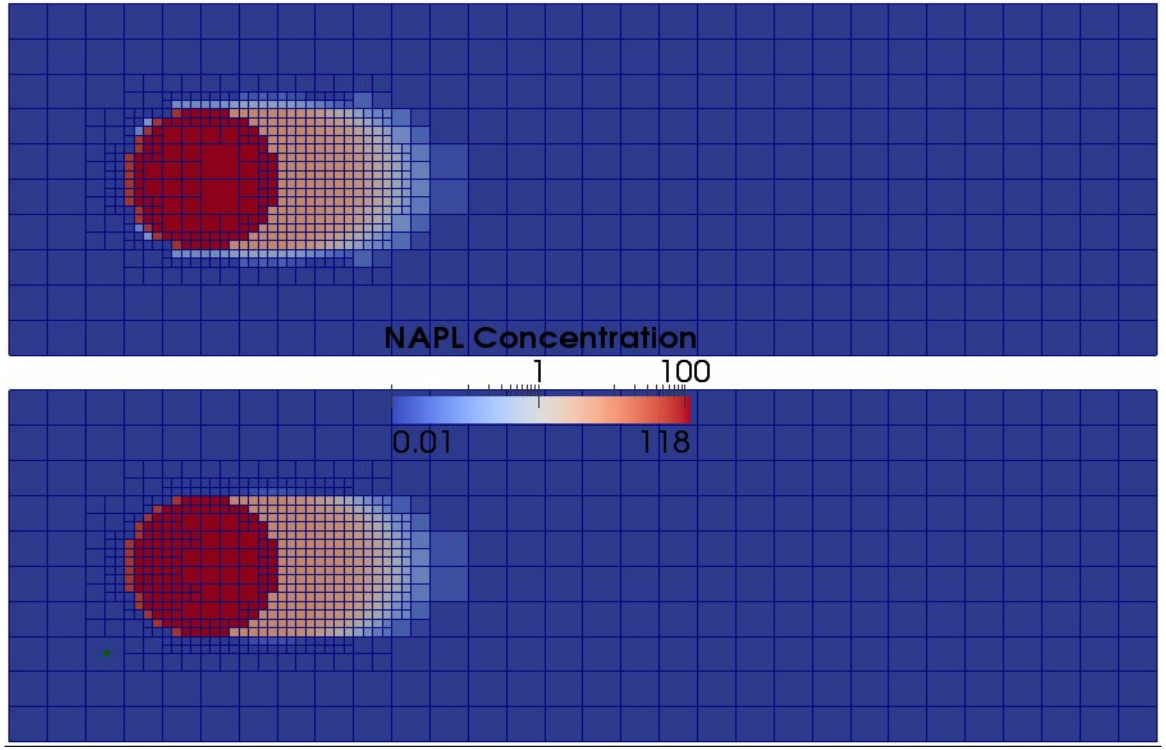
Summary

- An adaptive Multi-physics concept was presented.
- Mpfa can represent fluxes through irregular faces.
- With adaptive grids simulation can be fast and still accurate:
  - Tpfa on static, fine grid: 473s
  - Adaptive Tpfa: 78 s
  - Adaptive Mpfa: 74 s
  - Adaptive Mpfa (both half-edges) 75 s
- Combination of multi-physics framework and adaptive grids is promising and possible.



Future work

- Mpfa in 3 D.
- Investigate Refinement Indicators
  - Application of several indicators.
  - Dependence of refinement criteria on solution scheme.
  - Influence of Refinement on the solution.



Thank you

for your attention!

DuMu^X

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DFG Deutsche Forschungsgemeinschaft

References

I. Aavatsmark et. Al (2008): *A compact multipoint flux approximation method with improved robustness*.

Numerical Methods for Partial Differential Equations, 24:1329-1360, 2008.

G. Acs et. al (1985): *General Purpose Compositional Model*. Society of Petroleum Engineers Journal, 25:543–552.

J. Fritz et. al (2010): *Multiphysics Modeling of Advection - Dominated Two - Phase Compositional Flow in Porous Media*. International Journal of Numerical Analysis & Modeling. (accepted)

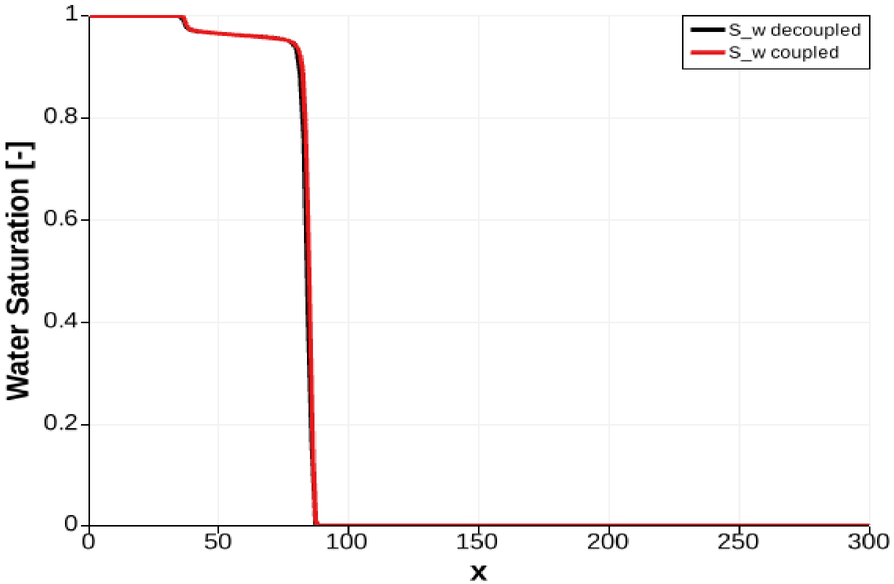
BGR (2010): *Projekt* CO_2 *- Drucksimulation*, final report of Federal Institute of Geosciences and Natural Resources.

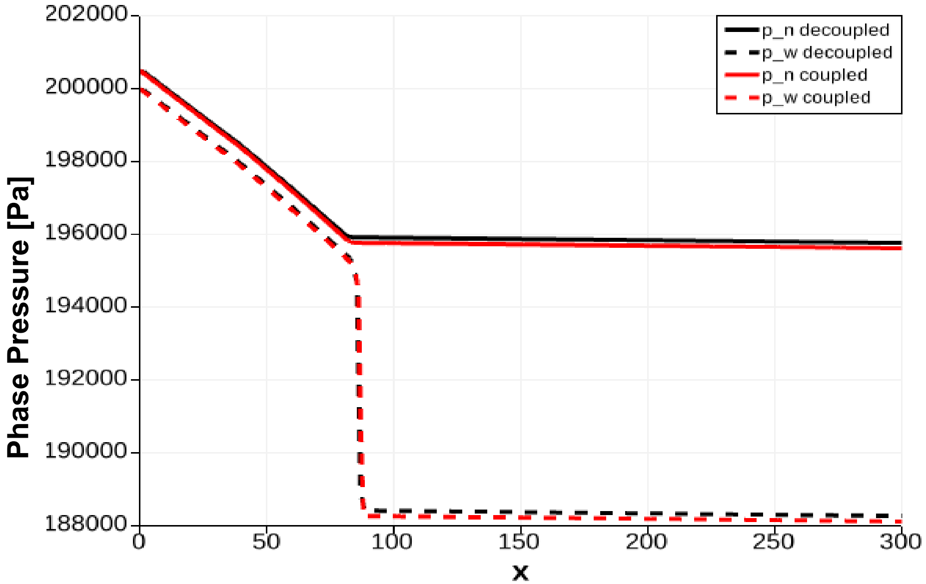
Testcase: Water Injection

Injection of gas into gas column: Fully Implicit vs Sequential.

Dirichlet BC:
$S_w = 1$
$p_w = 2e5$ Pa

Neumann BC:
$q_n = 0.01 \text{ kg/m}^2\text{s}$
$q_w =$ free outflow





Derivation of the Pressure Equation

- Volume contraint: $$v_t = \phi$$
- Taylor expansion in time:

$$v_t\left(t\right) + \Delta t \frac{\partial v_t}{\partial t} + \mathcal{O}\left(\Delta t^2\right) = \phi\left(t\right) + \Delta t \frac{\partial \phi}{\partial t} + \mathcal{O}\left(\Delta t^2\right) .$$

$$\frac{\partial v_t}{\partial t} = \frac{\partial v_t}{\partial p}\frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_t}{\partial C^\kappa}\frac{\partial C^\kappa}{\partial t} \qquad \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p}\frac{\partial p}{\partial t}$$

- Reordering:

$$\left(\frac{\partial v_t}{\partial p} - \frac{\partial \phi}{\partial p}\right)\frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_t}{\partial C^\kappa}\frac{\partial C^\kappa}{\partial t} = \frac{\phi - v_t}{\Delta t}$$

$$c_t \frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_t}{\partial C^\kappa} \sum_\alpha \nabla \cdot \left(\mathbf{v}_\alpha \varrho_\alpha X_\alpha^\kappa\right) = \sum_\kappa \frac{\partial v_t}{\partial C^\kappa} q^\kappa + \varepsilon$$

$$
\begin{aligned}
& V_i c_{total} \frac{p_i^t - p_i^{t-\Delta t}}{\Delta t} \\
& - \sum_{\gamma_{ij,irregular}} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\partial v_{total}}{\partial C^\kappa} X_\alpha^\kappa \left(\left(t_{2i} p_{\alpha,i}^t + \sum_j t_{2j} p_{\alpha,j}^t \right) + \varrho_\alpha \mathbf{g} \left(t_{2i} z_i + \sum_j t_{2j} z_j \right) \right) \\
& - \sum_{\gamma_{ij,regular}} A_{\gamma_{ij}} \mathbf{n}_{\gamma_{ij}} \cdot \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\partial v_{total}}{\partial C^\kappa} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right) \\
& + V_i \sum_{\gamma_{ij,irregular}} \frac{1}{U_i} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} X_\alpha^\kappa \left(\left(t_{2i} p_{\alpha,i}^t + \sum_j t_{2j} p_{\alpha,j}^t \right) + \varrho_\alpha \mathbf{g} \left(t_{2i} z_i + \sum_j t_{2j} z_j \right) \right) \\
& + V_i \sum_{\gamma_{ij,regular}} \frac{A_{\gamma_{ij}}}{U_i} \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right) \\
& = V_i \sum_{\kappa} \frac{\partial v_t}{\partial C^\kappa} q_i^\kappa + V_i \alpha_r \frac{v_t - \phi}{\Delta t} \,. \qquad (1)
\end{aligned}
$$

Implicit Pressure

Discretized (multi-phase):

$$V_i c_{t,i} \frac{p_i^t - p_i^{t-\Delta t}}{\Delta t}$$

$$- \sum_{\gamma_{ij}} A_{\gamma_{ij}} \mathbf{n}_{\gamma_{ij}} \cdot \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\partial v_t}{\partial C^\kappa} \qquad X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right)$$

$$+ V_i \sum_{\gamma_{ij}} \frac{A_{\gamma_{ij}}}{U_i} \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} \qquad X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right)$$

$$= V_i \sum_{\kappa} \frac{\partial v_t}{\partial C^\kappa} q_i^\kappa + V_i \alpha_r \frac{v_t - \phi}{\Delta t} \; .$$

Discretized (single phase):

$$V_i c_{t,i} \frac{p_i^t - p_i^{t-\Delta t}}{\Delta t}$$

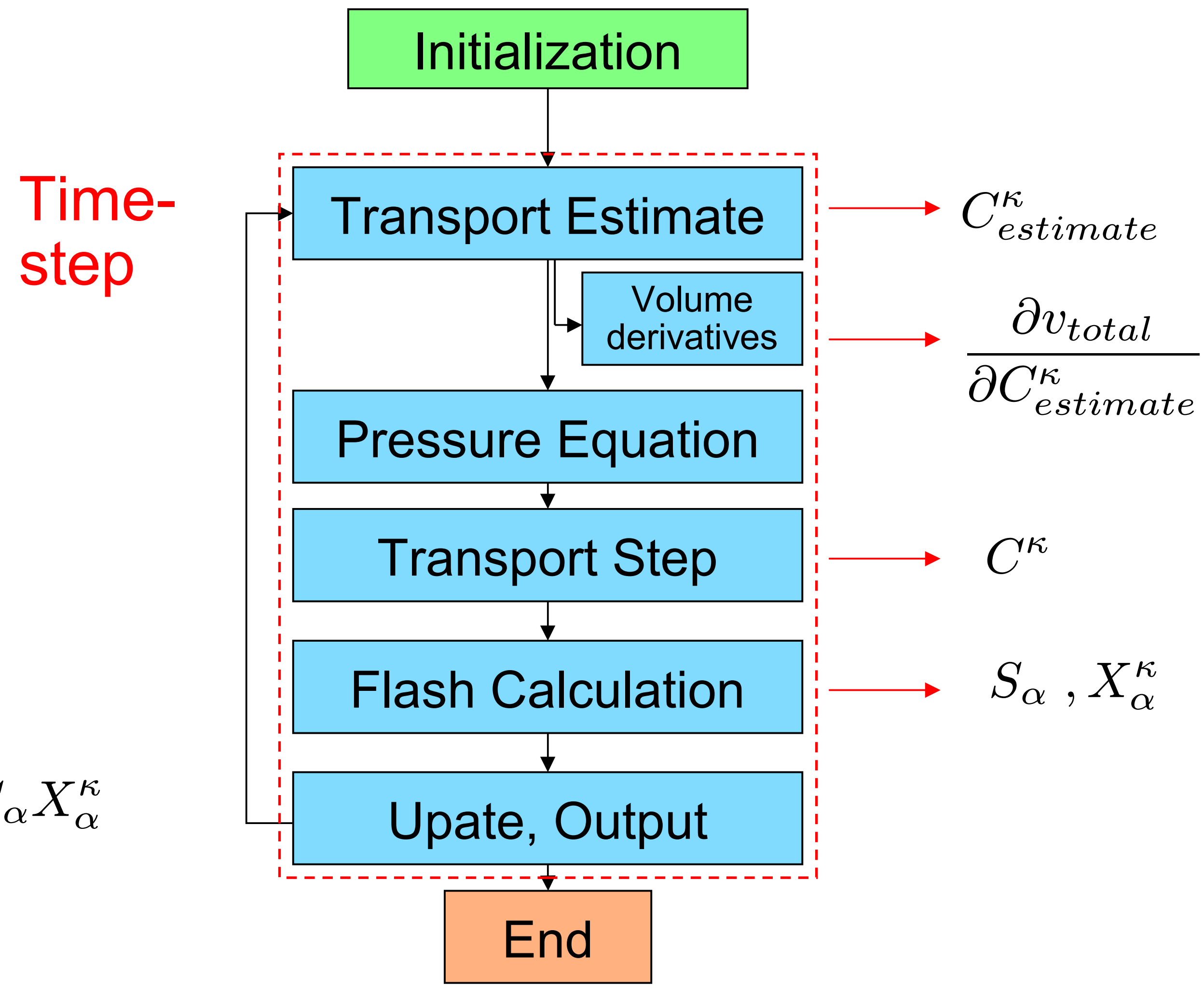
$$- \sum_{\gamma_{ij}} A_{\gamma_{ij}} \mathbf{n}_{\gamma_{ij}} \cdot \mathbf{K} \sum_\alpha \varrho_\alpha \lambda_\alpha \sum_\kappa \frac{\partial v_t}{\partial C^\kappa} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right)$$

$$+ V_i \sum_{\gamma_{ij}} \frac{A_{\gamma_{ij}}}{U_i} \mathbf{K} \sum_\alpha \varrho_\alpha \lambda_\alpha \sum_\kappa \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right)$$

$$= V_i \sum_\kappa \frac{\partial v_t}{\partial C^\kappa} q_i^\kappa + V_i \alpha_r \frac{v_t - \phi}{\Delta t} \, .$$

$$\frac{1}{\varrho_\alpha}$$

Sequential solution scheme:



$$C^\kappa = \sum_\alpha \varrho_\alpha S_\alpha X^\kappa_\alpha$$

Solution Procedure on Adaptive Grids