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Adaptive methods for multi-phase flow: Grid-adaptivity

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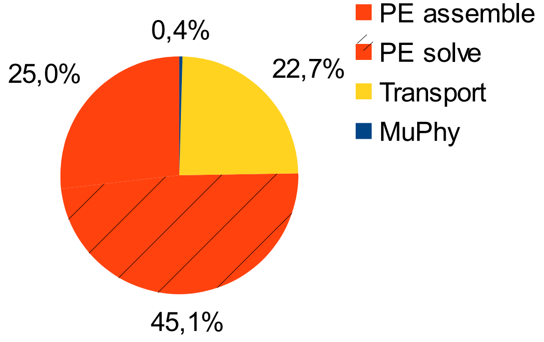
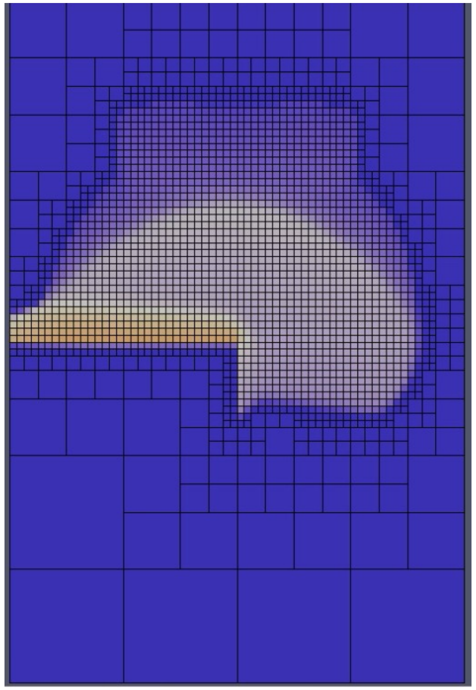
NEVER ADAPT.
Introducing the 2012 Jeep Grand Cherokee.
Jeep

Why adaptive grids?

a) Efficiency reasons:

- Most time spent for the solution of the pressure equation
    - Grid resolution affects assembling and solution time!



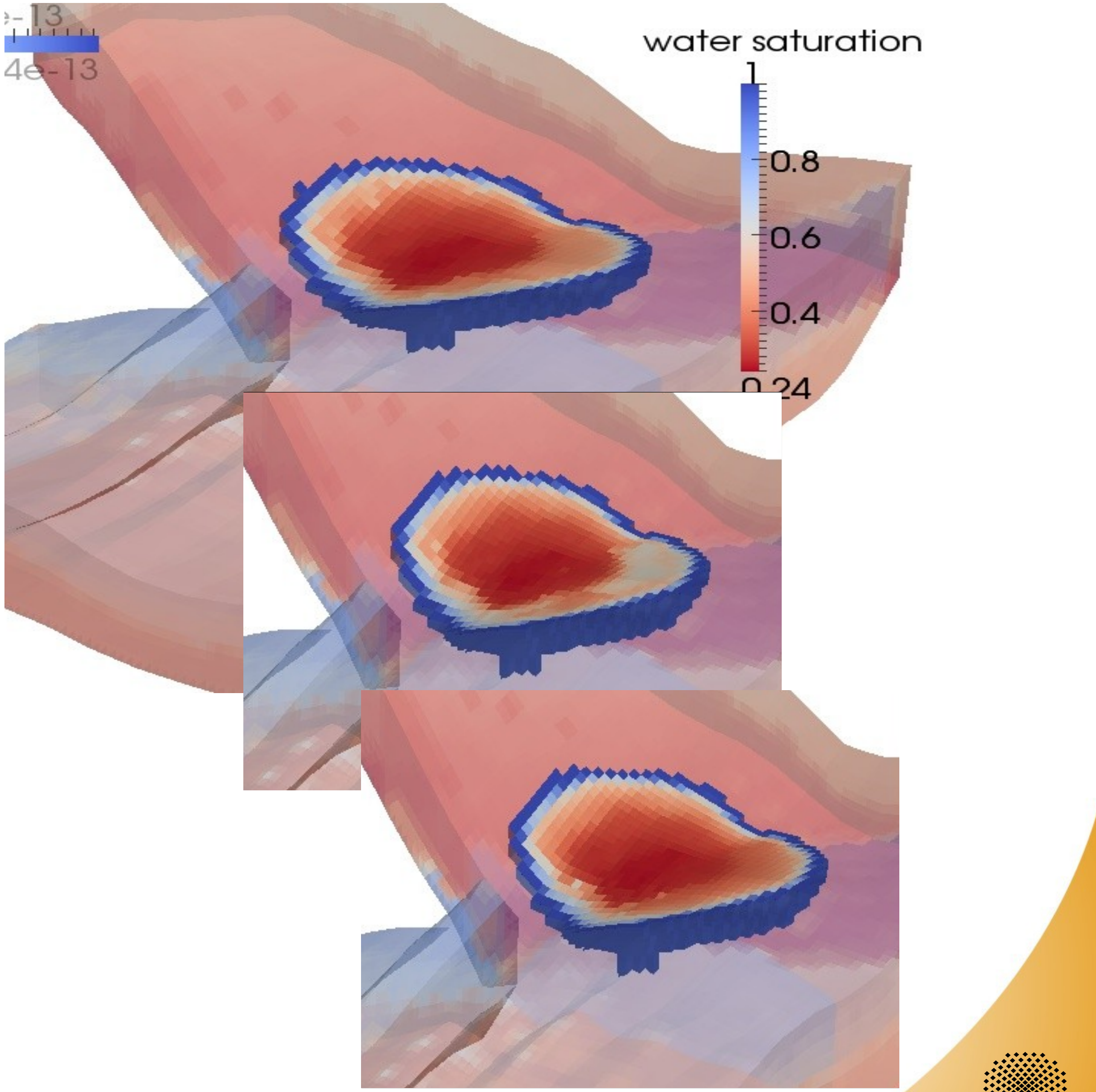


Sinsbeck (2011)

Why adaptive grids?

b) Qualitative reasons:

- Sequential: Solution not converged but approximated: Pressure field should be as good as possible.
- The finer the grid, the less nummerical diffusion.
- Global refinement not always possible.



Numerical Scheme

Mass conservation:

- For phases α and components κ , for each component:

$$\sum_\alpha \frac{\partial \phi S_\alpha \varrho_\alpha X_\alpha^\kappa}{\partial t} + \nabla \cdot \left(\sum_\alpha X_\alpha^\kappa \varrho_\alpha \mathbf{v}_\alpha + \mathbf{J}_\alpha^\kappa \right) + \sum_\alpha X_\alpha^\kappa \varrho_\alpha q^\kappa = 0$$

$$\mathbf{v}_\alpha = -\lambda_\alpha \mathbf{K} (\nabla p_\alpha - \varrho_\alpha \mathbf{g})$$

- Solution strategies:
  - Fully implicit
  - Sequential
    - Summation yields one pressure equation.
    - Transport equation $C^\kappa = \sum_\alpha \varrho_\alpha S_\alpha X_\alpha^\kappa$
    - -Flash calculations

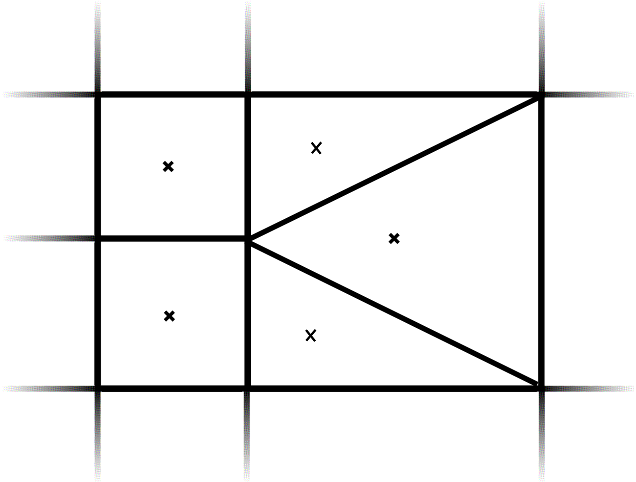
Outline

- Introduction
  - Why using adaptive grids?
- Simulation on adaptive grids
  - How to adapt
    - Representation of fluxes near refined cells
  - Numerical Results
- Outlook

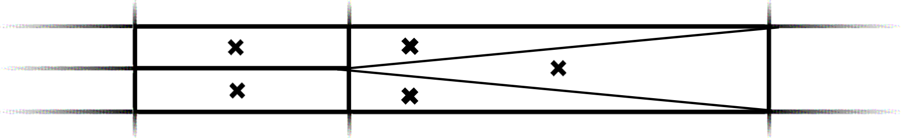
How to refine the grid?

Finite Volume context:

– Refine with closure

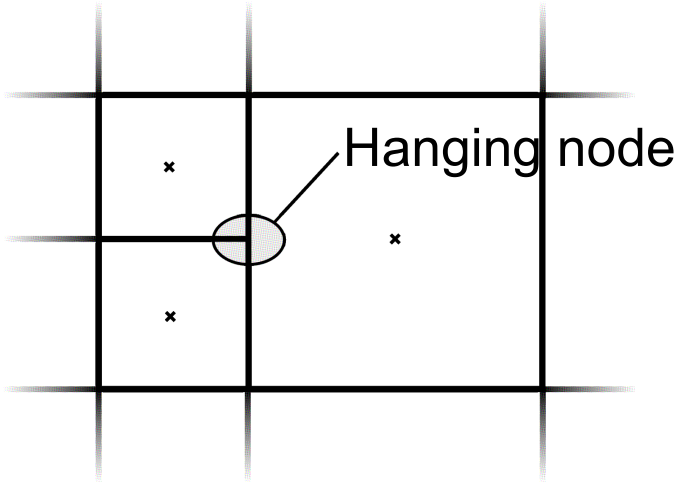


» Problem:



» e.g. Johannsen: Cell 80m x 10m

– Refine with irregular edges



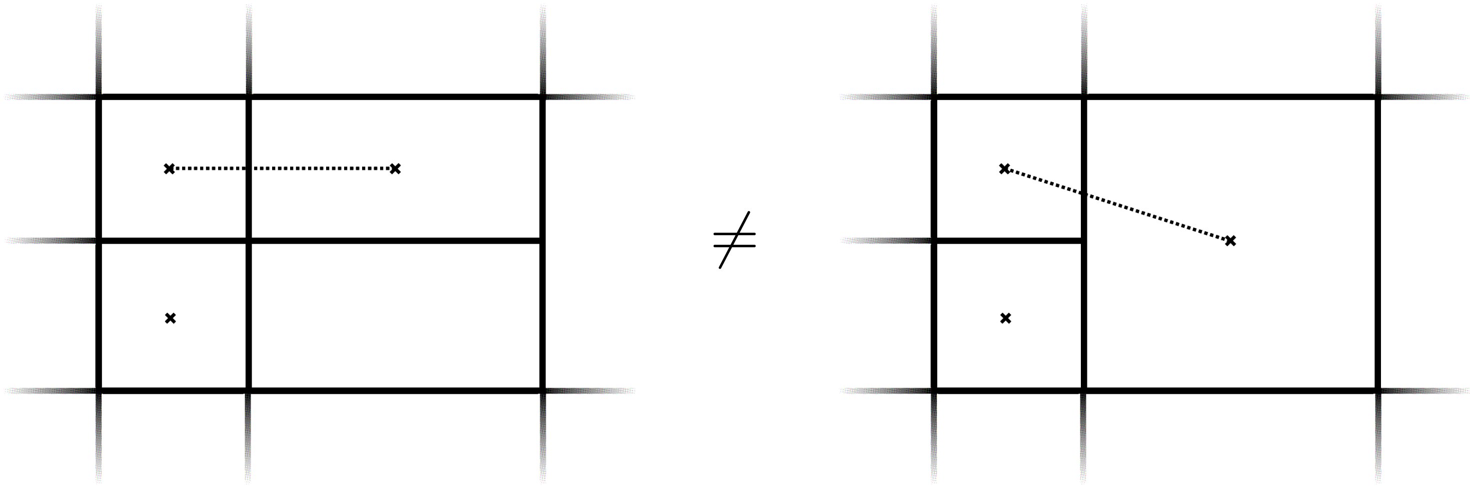
Fluxes: Tpfa

- Standard approach to approximate flux:

  Use a Two-Point flux approximation

$$\nabla \Phi \approx \frac{\Phi_j - \Phi_i}{\Delta x}$$

- Problem:

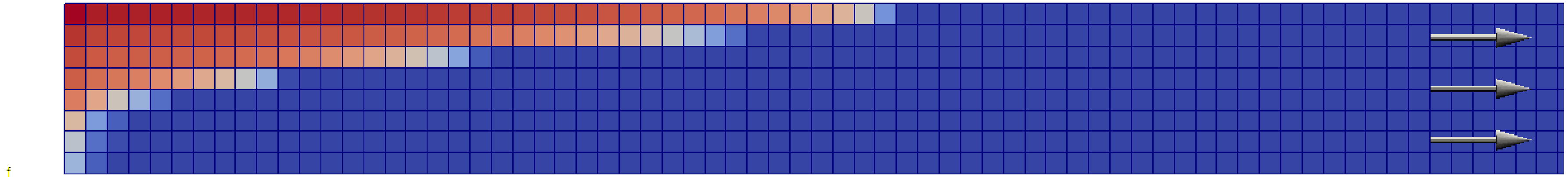


Example Simulation 1

- 2 D, injection of CO_2 into a rectangular domain filled with brine.
- Comparison of fully refined vs. adaptive grids.
- Compositional two-phase system.

Neumann BC:
$q_n = -0.2$ Mt/m yr.
$q_w =$ free flow

Dirichlet BC:
$S_w = 1$
$p_n = 2.5e7\text{Pa} + \varrho g z$



Material data:

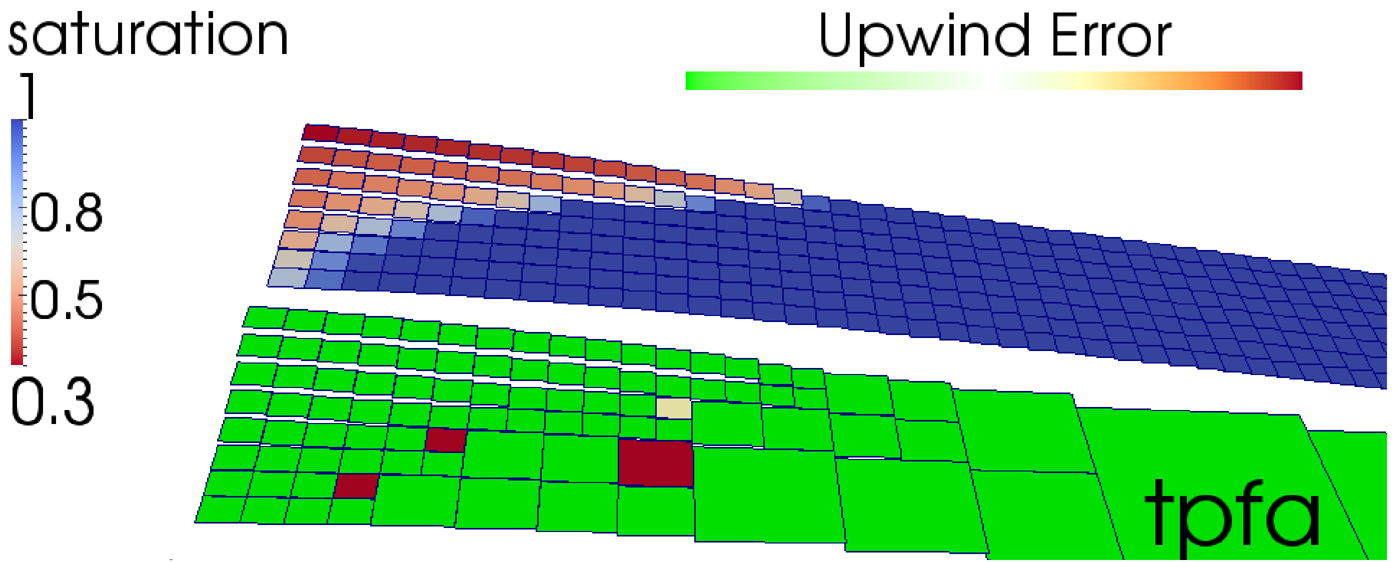
Porosity	Permeability	Entry Pressure	BC-lambda
0.15	10^{-13}m^2	500 Pa	2

Fluxes: Tpfa is inaccurate

– Standard approach to approximate flux:
  Use a Two-Point flux approximation

$$\nabla \Phi \approx \frac{\Phi_j - \Phi_i}{\Delta x}$$

– Problem:
  - Coloured cells expect flux to the left!



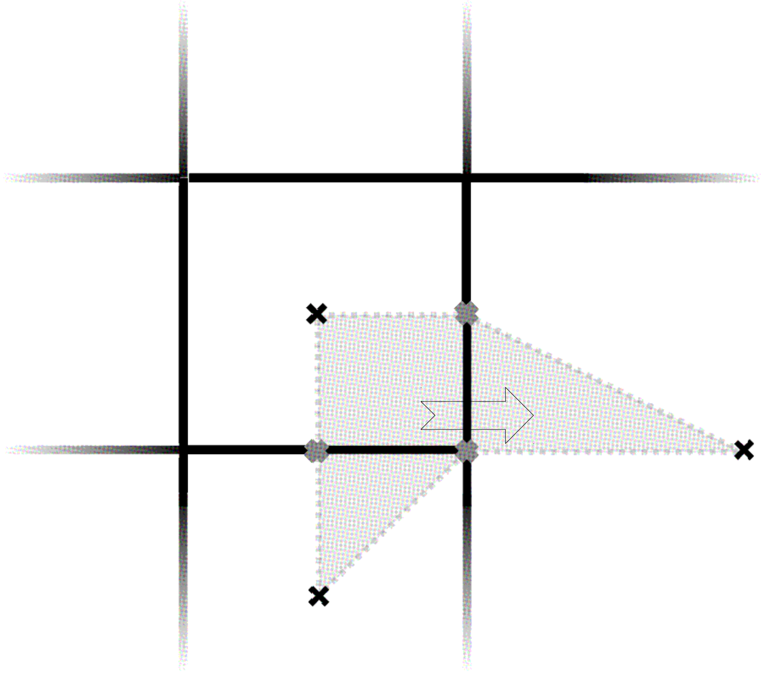
Height: $\Phi_{w,i} = p_{w,i} + \varrho_{w,i} g z_i$

Fluxes: Mpfa

Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.



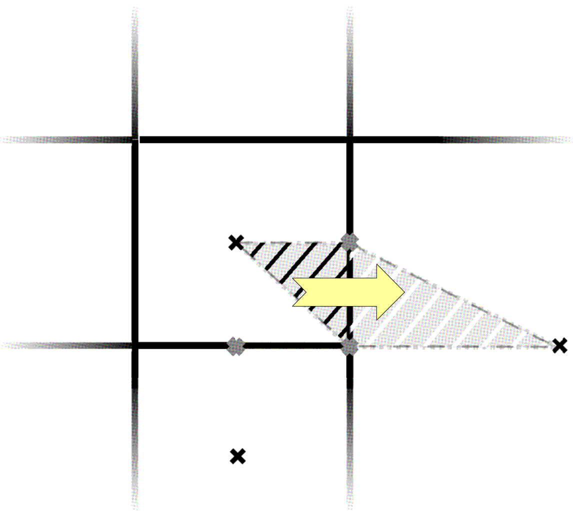
Fluxes: Mpfa

Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.

c) Approximate flux with new points.



$$f_{\gamma;i} = -\mathbf{n}_\gamma^T \mathbf{K}_i \nabla U_i$$

$$\nabla U_i = \frac{1}{T} \sum_{k=1}^{2} \nu_k (u_k^* - u_0)$$

Value on introduced point k

Area of left interaction region

Value at cell center of cell i

Rotated vector connecting the points i,k

Fluxes: Mpfa

Multi-point flux approximation (Mpfa)

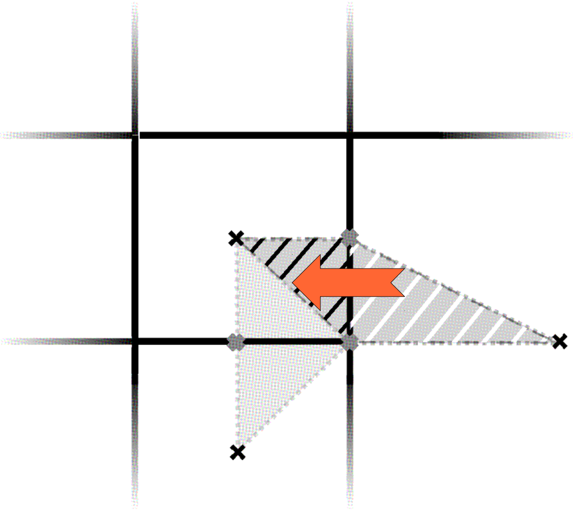
a) Define an “interaction region”.

b) Introduce new points on interface.

c) Approximate flux with new points.

d) Approximate flux as seen from all cells.

$$f_{\gamma;i} = \dots$$



$$\begin{aligned} f_{\gamma;j} = & -\mathbf{n_2}^T \mathbf{K}_3 \frac{1}{T_3} \nu_5 (u_2^* - u_3) \\ & -\mathbf{n_2}^T \mathbf{K}_3 \frac{1}{T_3} \nu_6 \Big(u_j - u_3 \\ & + \frac{1}{T_i} \nu_7^T \mathbf{R} \nu_i (u_1^* - u_i) \\ & + \frac{1}{T_i} \nu_7^T \mathbf{R} \nu_j (u_2^* - u_i) \Big) \end{aligned}$$

Fluxes: Mpfa

Multi-point flux approximation (Mpfa)

a) Define an “interaction region”.

b) Introduce new points on interface.

c) Approximate flux with new points.

d) Approximate flux as seen from all cells.

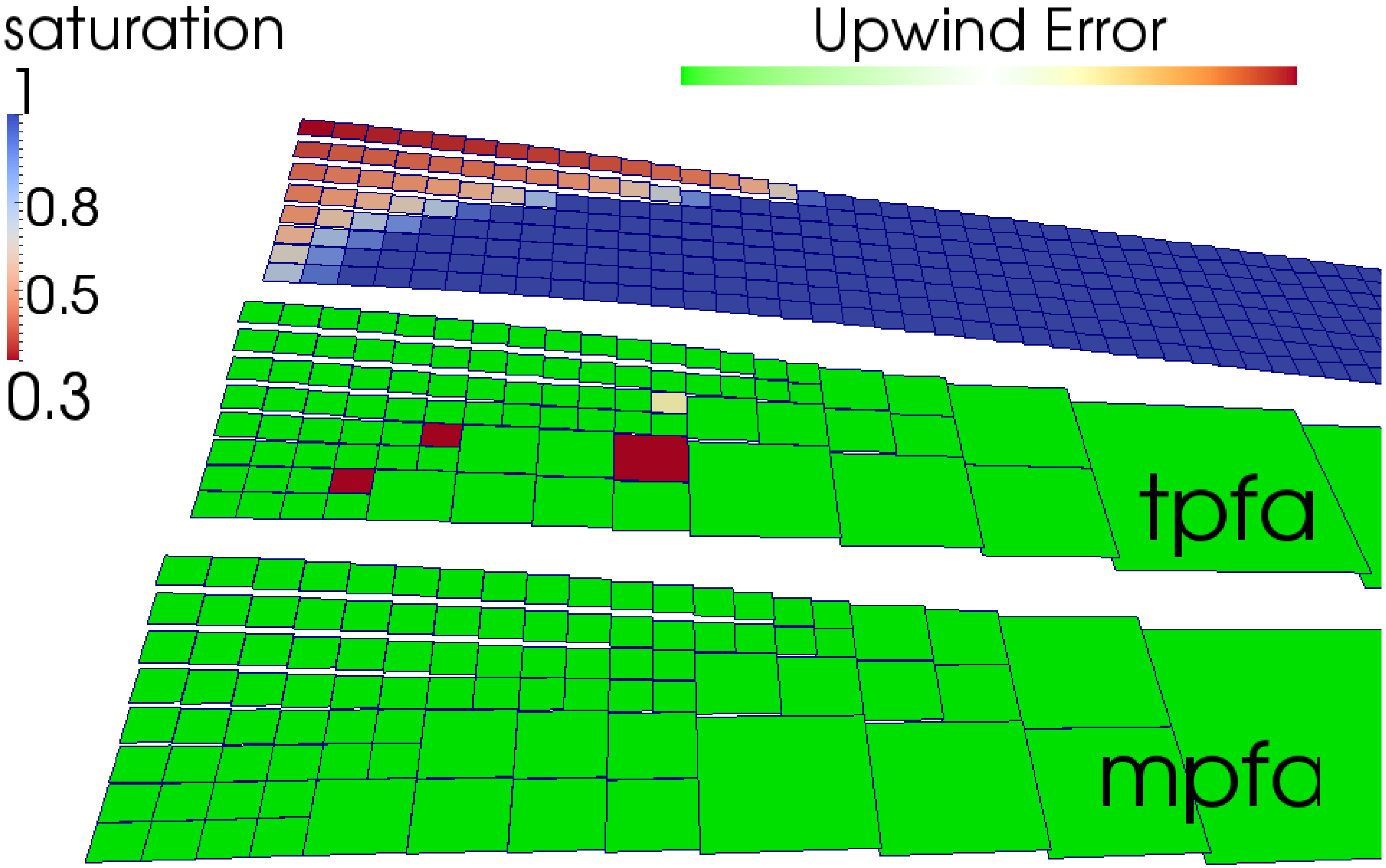
e) Write everything into a large equation system of form

$$\mathbf{f} = \mathbf{T}\mathbf{u}$$

where T contains the „transmissibility coefficients“ (introduced points already eradicated), and u is the vector of unknown cell values

Fluxes with Mpfa

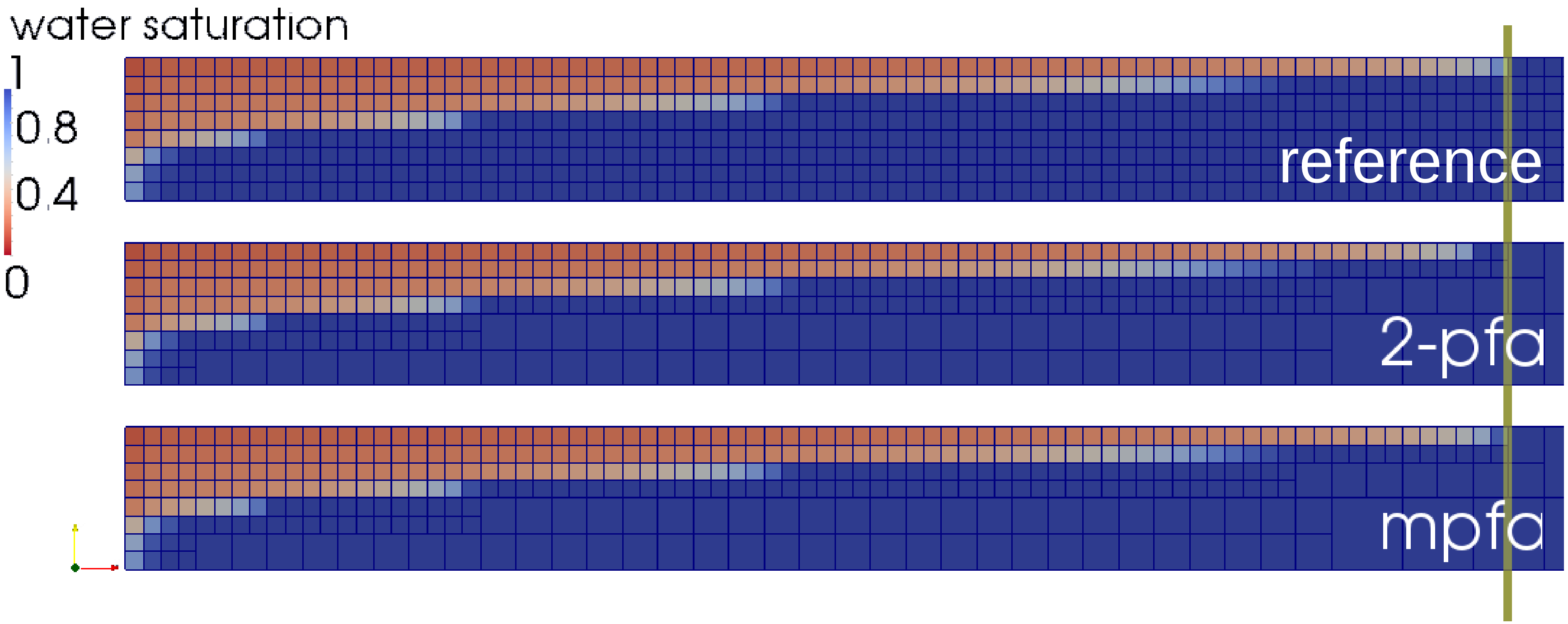
Results: Flux error in horizontal direction



$$z - \text{Coordinate: } \Phi_{w,i} = p_{w,i} + \varrho_{w,i} g \mathbf{x}_i$$

Fluxes with Mpfa

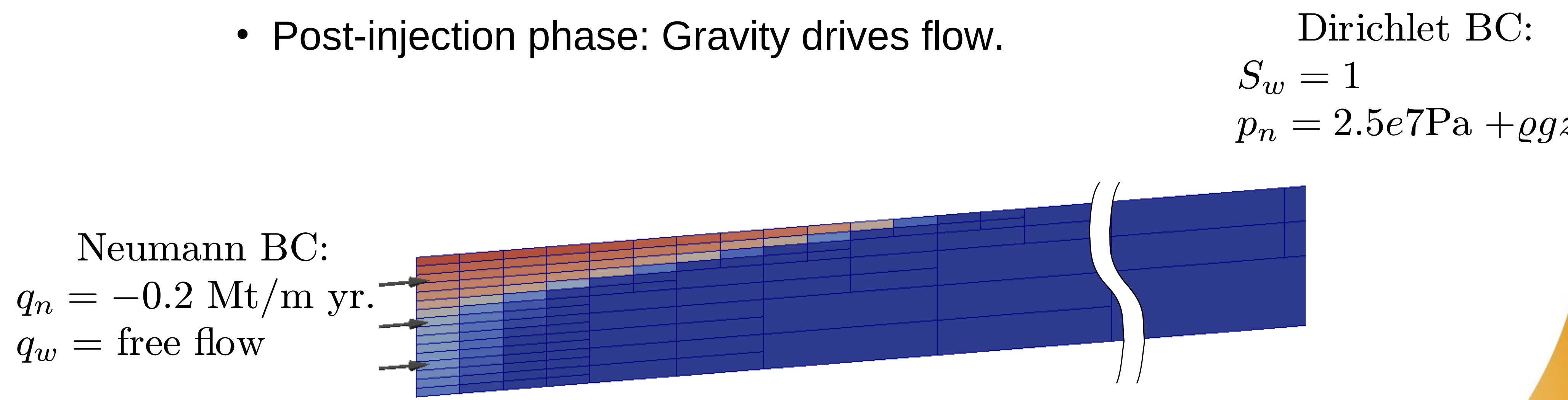
Results after 5 years of injection



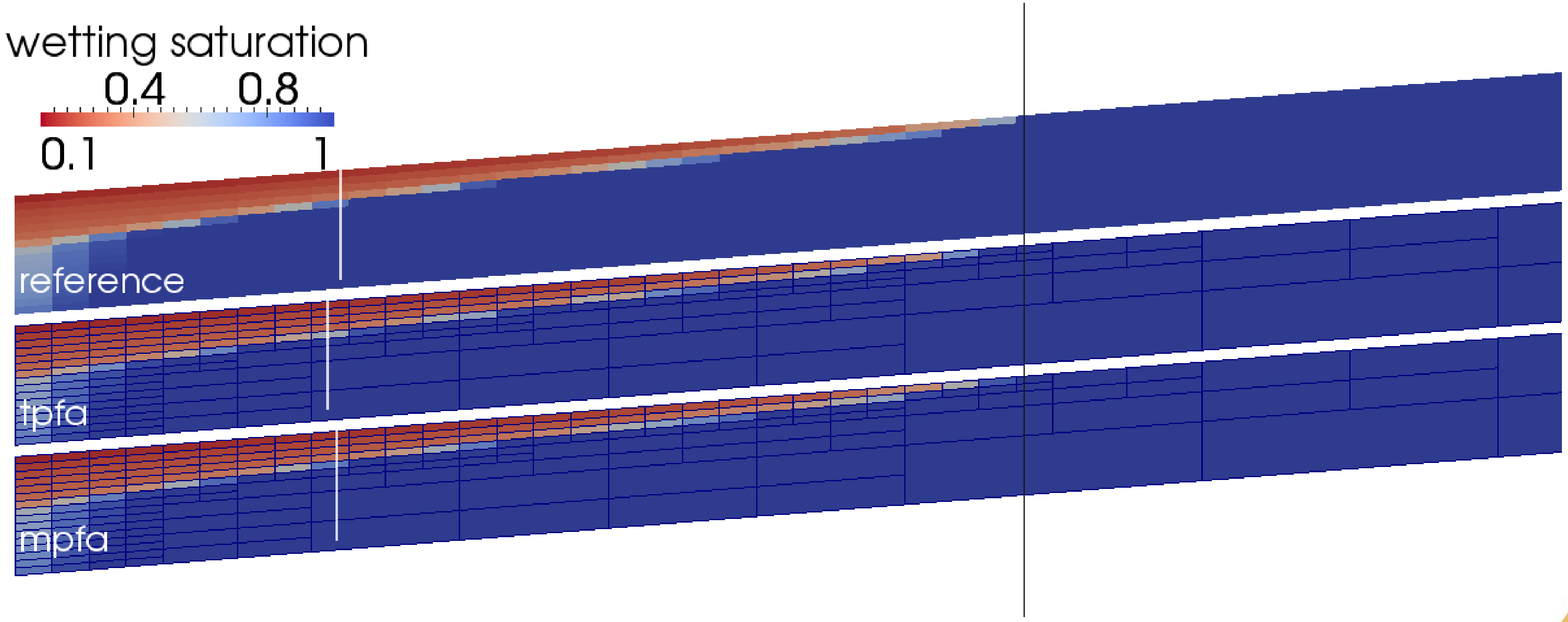
Example Simulation 2

More challenging example:

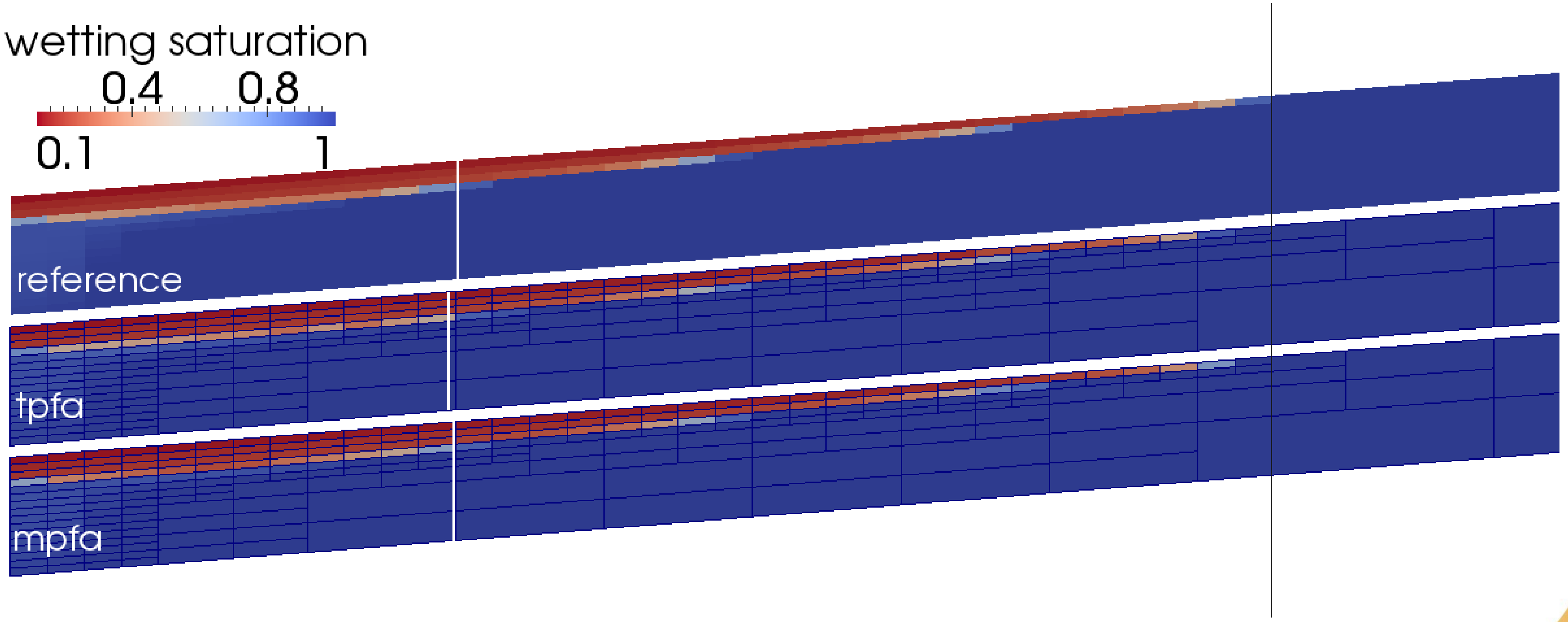
- Tilted domain.
- No quadratic cells.
- Two periods
  - Injection phase: Pressure gradient drives flow.
  - Post-injection phase: Gravity drives flow.



Saturation at the end of the injection period (2 years):



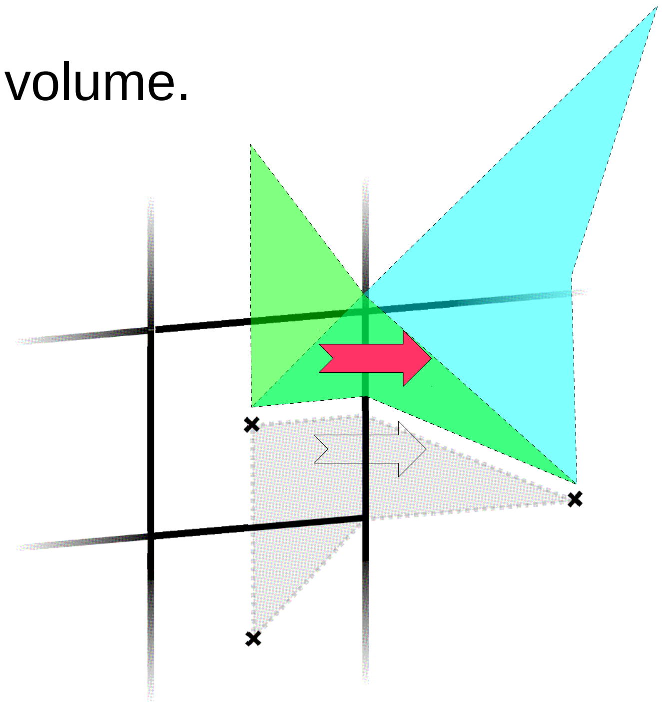
Saturation at the end of the post-injection period (5 years):



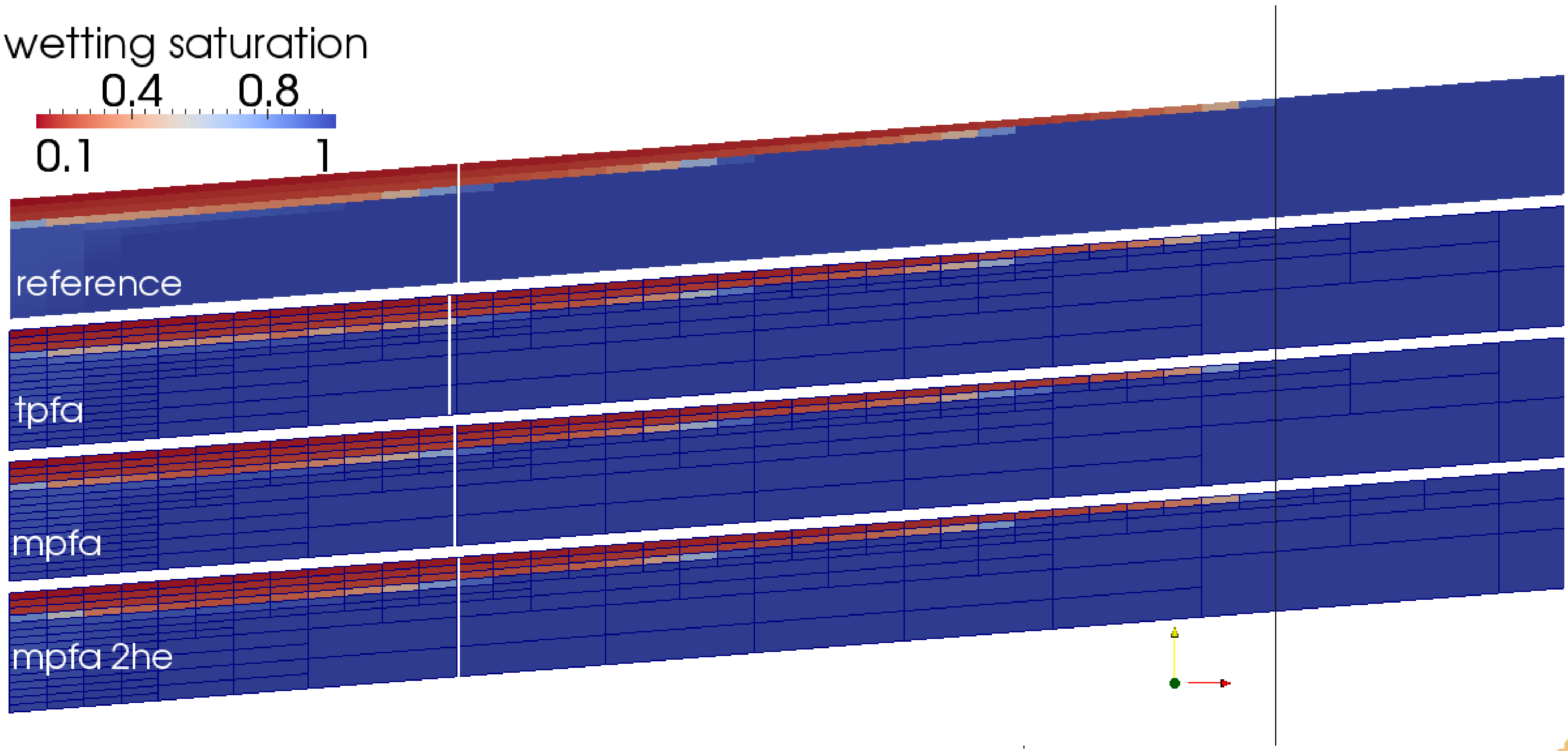
Fluxes with MPFA – Improvements

Flux through the edge:

- Twice the flux of first half-edge of the interaction volume.
- Construct second interaction region:
  - Strongly dependent on surrounding cells.
  - Expensive way to “search” the region.
  - Is it “worth the effort”?



Accurate result with second half edge!



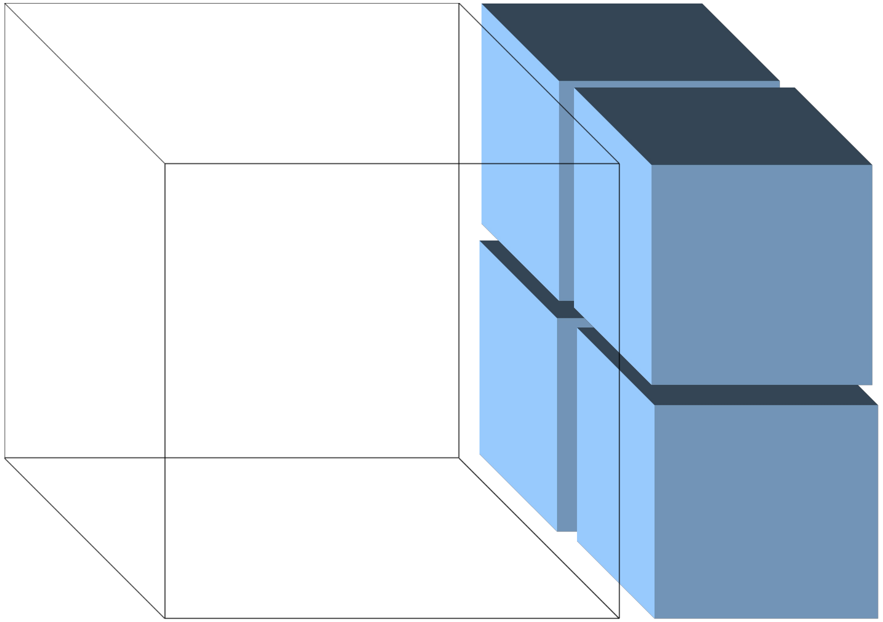
Mpfa – Programming Obstacles

Larger stencil: More inteaction

- Many iterators used to come from intersection to all 3 neighbours and point connecting point.

Per sub-face required:

- 3 neighbour elements
- Point in the middle
- All surrounding Elements??



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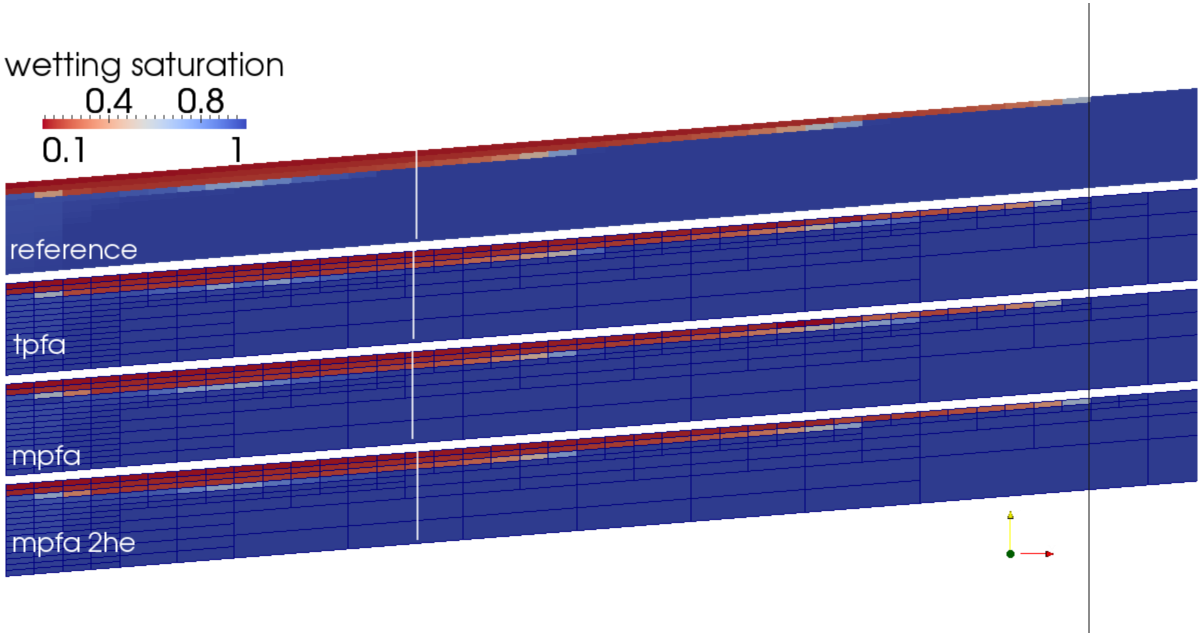
Summary

Mpfa on adaptive grids “does not look so bad”!

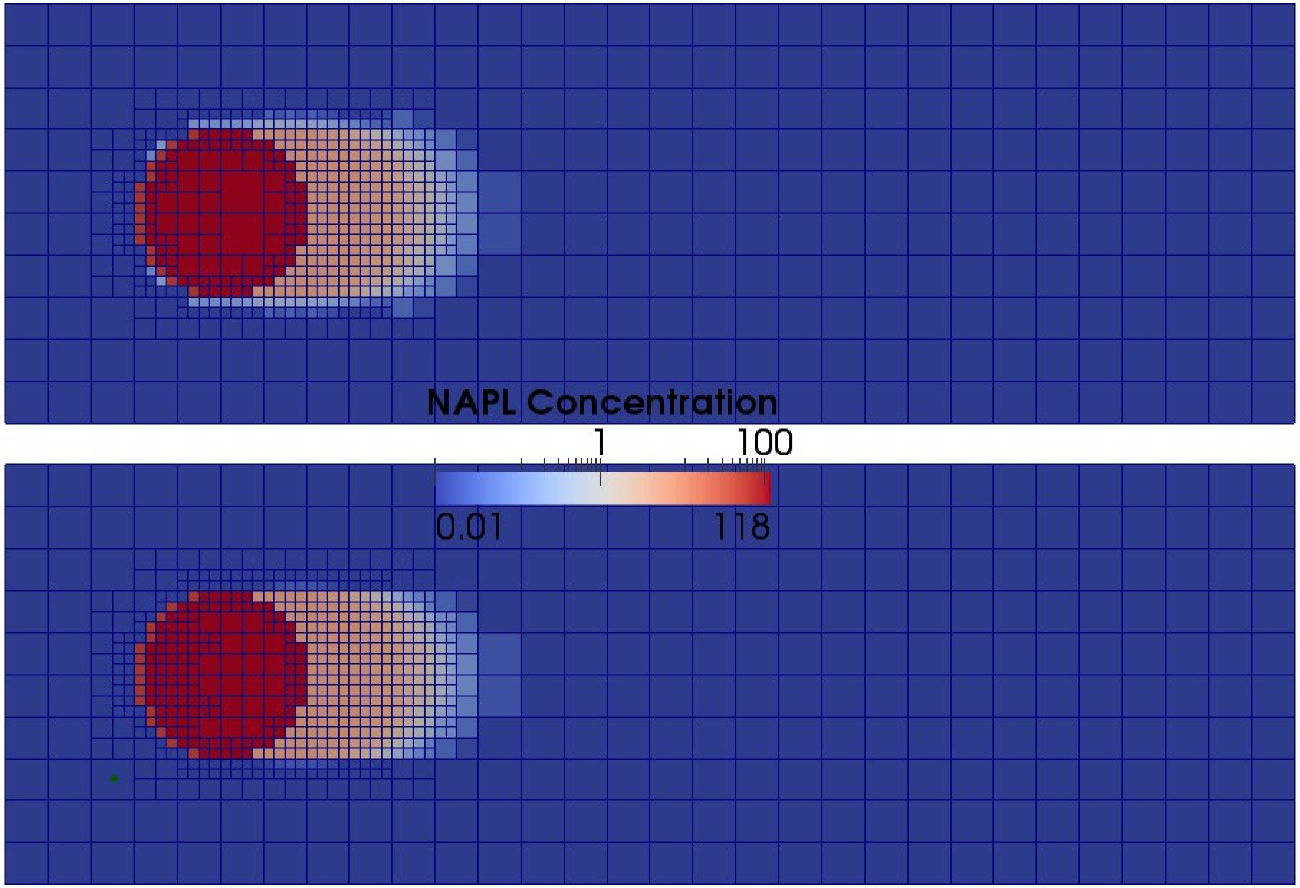
With adaptive grids simulation can be fast and still accurate:

- Tpfa on static, fine grid: 75s
- Tpfa (3 levels): 16s
- Mpfa (3 levels) 16.3s
- Mpfa (", 2 half-edges) 16.9s



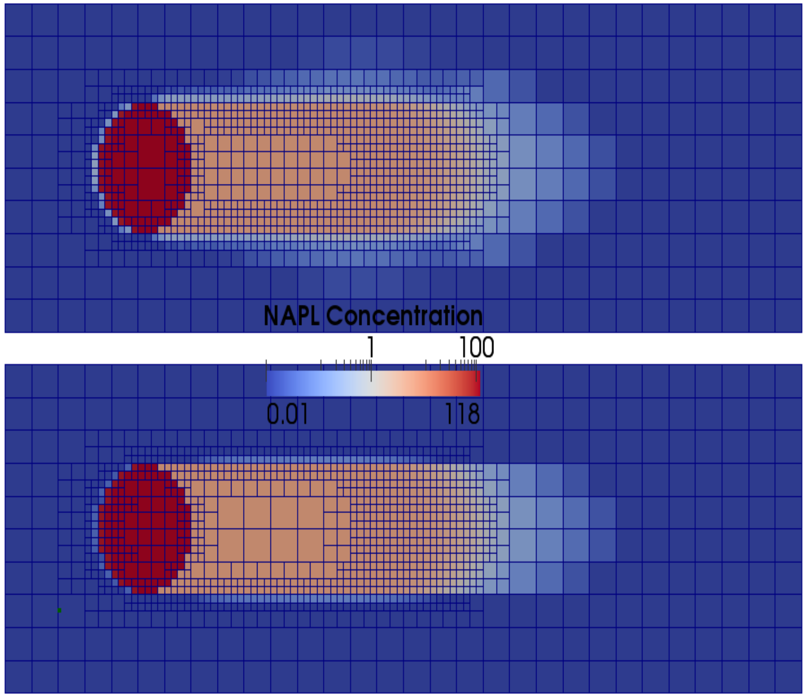
Future work

- Investigate Refinement Indicators
  - Application of several indicators.
  - Dependence of refinement criteria on solution scheme.
  - Influence of Refinement on the solution.



Future work

- Investigate Refinement Indicators
    - Application of several indicators.
    - Dependence of refinement criteria on solution scheme.
    - Influence of Refinement on the solution.
- Extension to three dimensions.
- Applications



Thank you

for your attention!

DFG Deutsche Forschungsgemeinschaft

References

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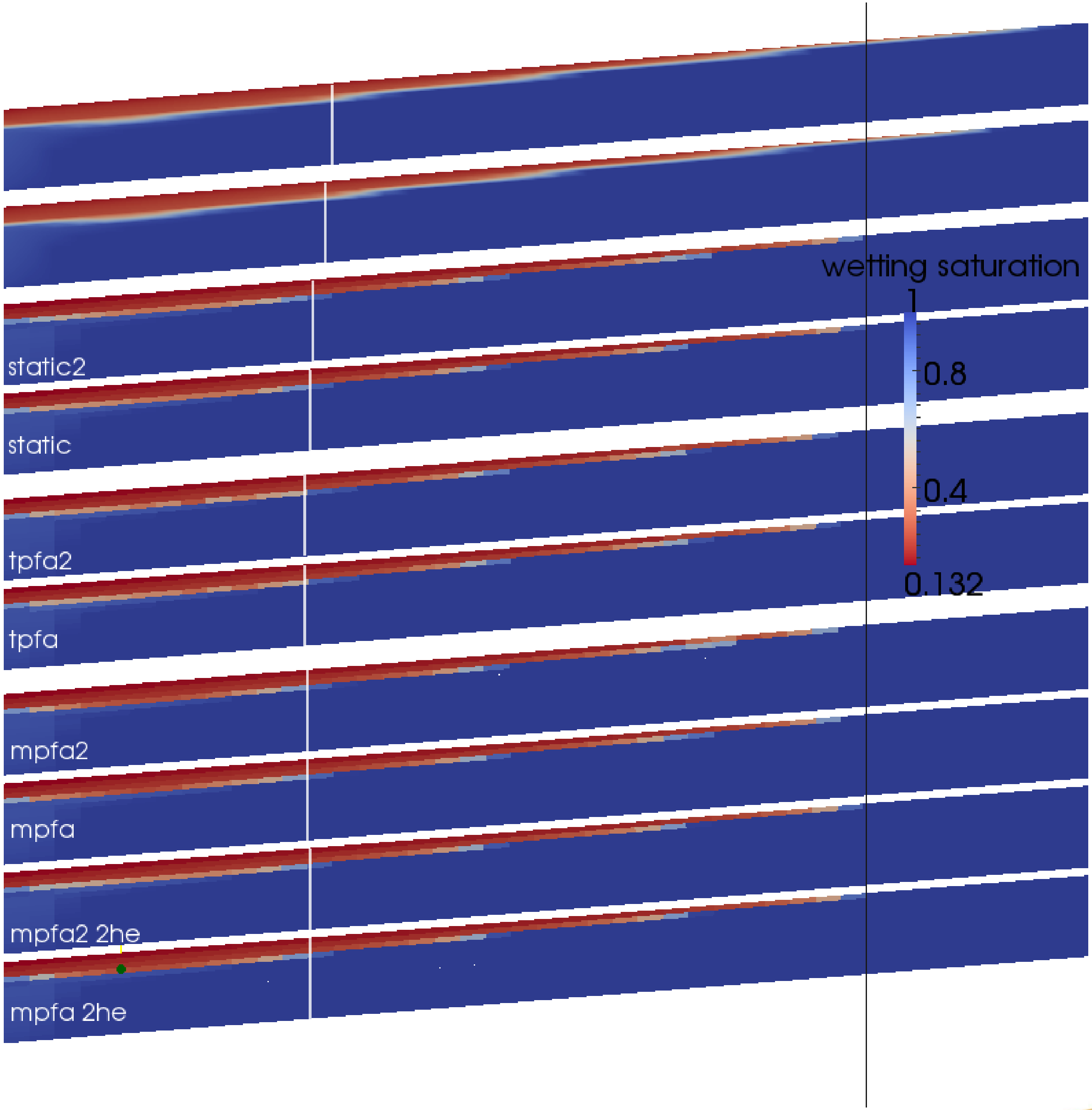
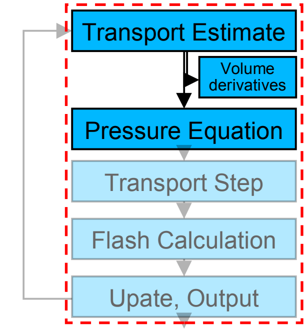
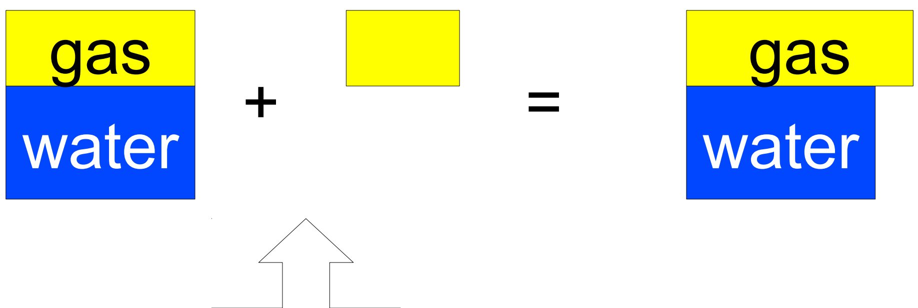

wetting saturation
1
0.8
0.4
0.132
static2
static
tpfa2
tpfa
mpfa2
mpfa
mpfa2 2he
mpfa 2he

Implicit Pressure

Formulation



Volume balance:



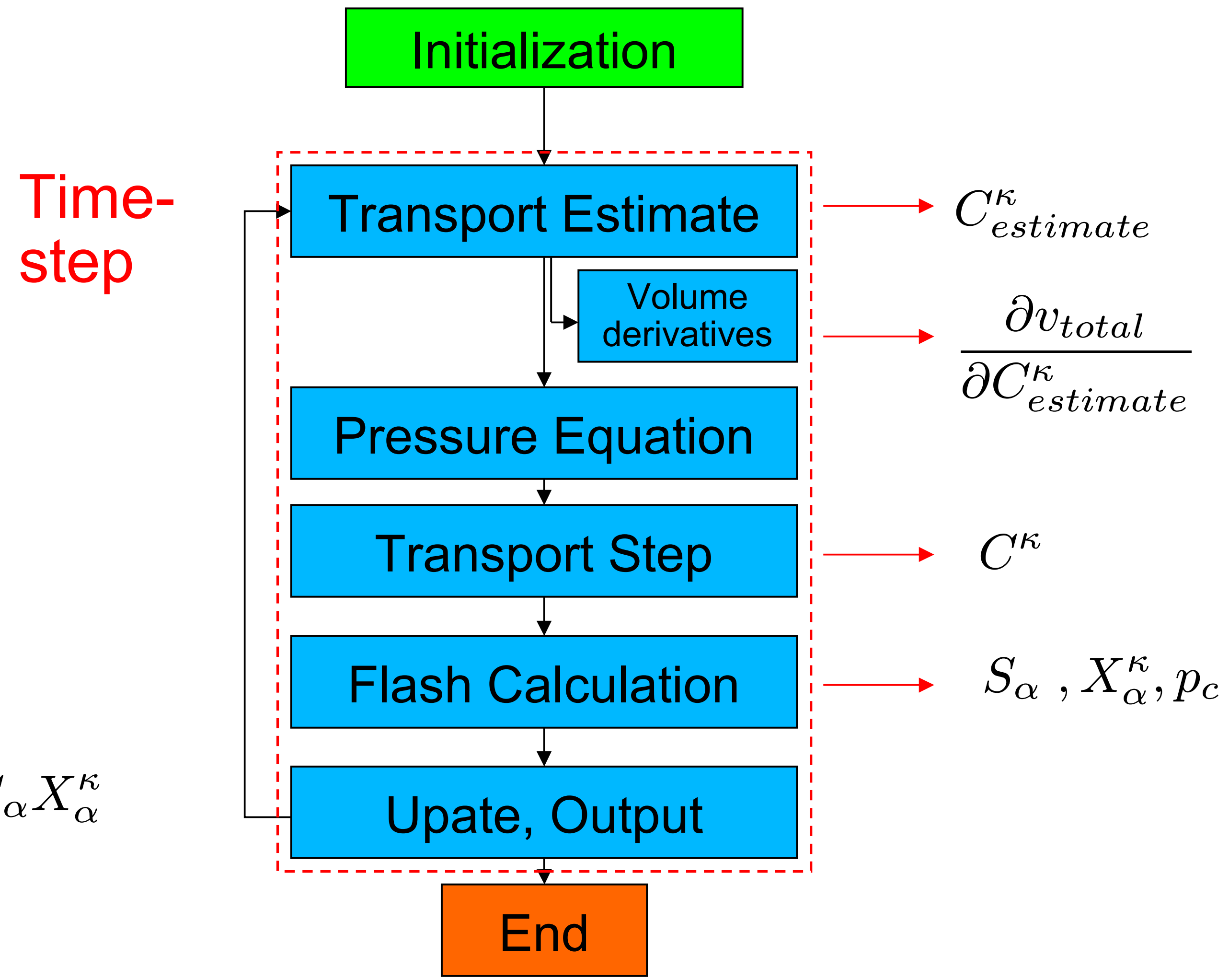
$$c_{total}\frac{\partial p}{\partial t} + \sum_{\kappa} \frac{\partial v_{total}}{\partial C^{\kappa}} \nabla \cdot \left(\sum_{\alpha} X_{\alpha}^{\kappa} \varrho_{\alpha} \mathbf{v}_{\alpha} \right) = \sum_{\kappa} \frac{\partial v_{total}}{\partial C^{\kappa}} q^{\kappa} + \varepsilon,$$

Acs et. al (1985)

- If we use non-wetting pressure as primary variable

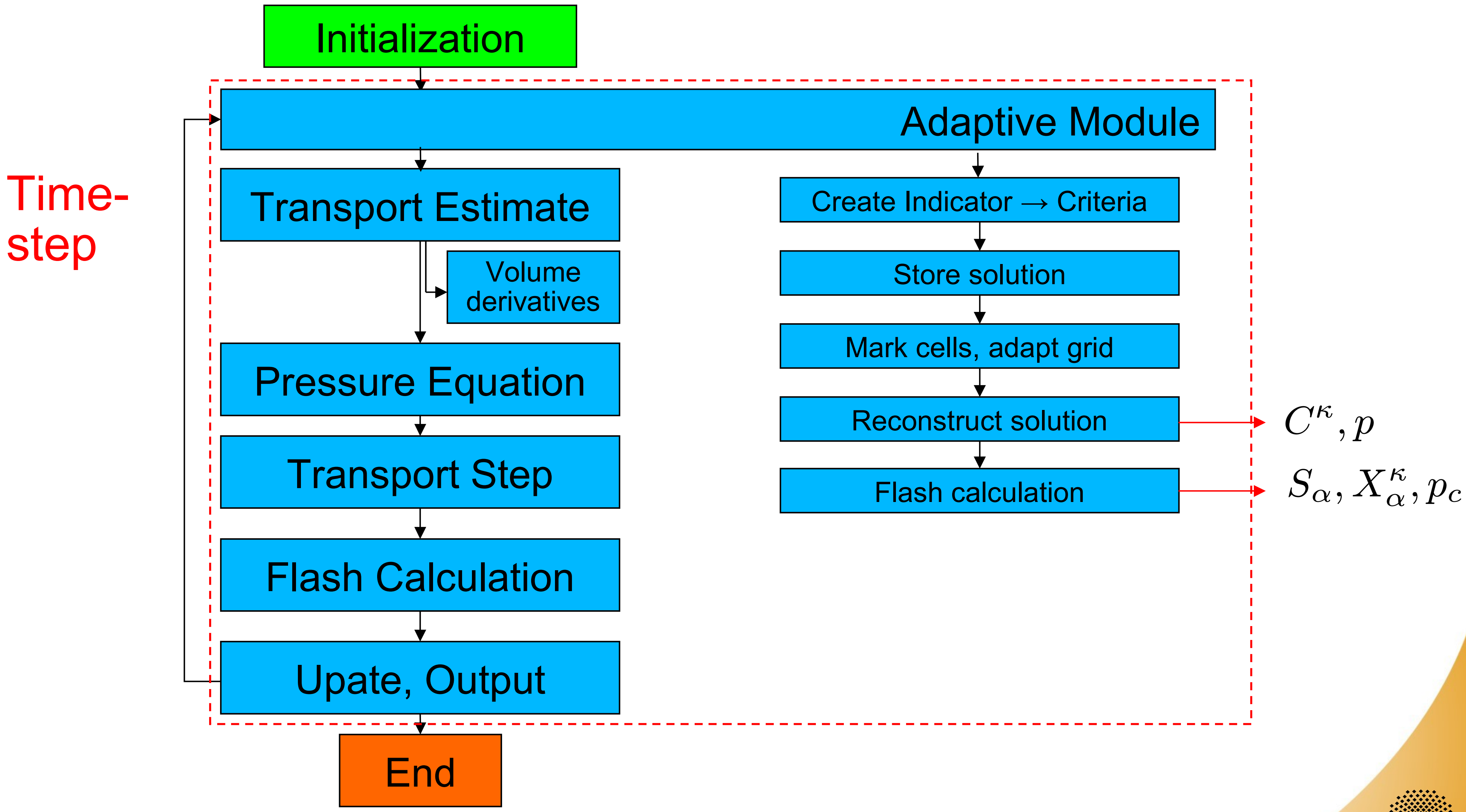
$$\mathbf{v}_w = -\lambda_w \mathbf{K}(\nabla p_n - \nabla p_c - \varrho_w \mathbf{g}),$$
$$\mathbf{v}_n = -\lambda_n \mathbf{K}(\nabla p_n - \varrho_n \mathbf{g}),$$

Solution Scheme: Sequential



$$C^\kappa = \sum_\alpha \varrho_\alpha S_\alpha X^\kappa_\alpha$$

Solution Procedure on Adaptive Grids



Implicit Pressure

Formulation

Discretized (multi-phase):

$$V_i c_{t,i} \frac{p_i^t - p_i^{t-\Delta t}}{\Delta t}$$

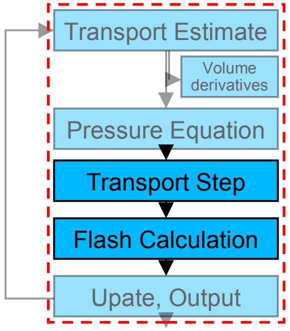
$$- \sum_{\gamma_{ij}} A_{\gamma_{ij}} \mathbf{n}_{\gamma_{ij}} \cdot \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\partial v_t}{\partial C^\kappa} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} (z_j - z_i) \right)$$

$$+ V_i \sum_{\gamma_{ij}} \frac{A_{\gamma_{ij}}}{U_i} \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} (z_j - z_i) \right)$$

$$= V_i \sum_{\kappa} \frac{\partial v_t}{\partial C^\kappa} q_i^\kappa + V_i \alpha_r \frac{v_t - \phi}{\Delta t} \ .$$

Explicit Step

Formulation



Transport Equation (explicit):

$$\frac{\partial C^\kappa}{\partial t} = -\nabla \cdot \left(\sum_\alpha X_\alpha^\kappa \varrho_\alpha \mathbf{v}_\alpha \right) + q^\kappa,$$

– Determines size of the time step.

Equilibrium (Flash-) Calculation

Derivation of the Pressure Equation

- Volume contraint: $v_t = \phi$
- Taylor expansion in time:

$$v_t\left(t\right) + \Delta t \frac{\partial v_t}{\partial t} + \mathcal{O}\left(\Delta t^2\right) = \phi\left(t\right) + \Delta t \frac{\partial \phi}{\partial t} + \mathcal{O}\left(\Delta t^2\right) .$$

$$\frac{\partial v_t}{\partial t} = \frac{\partial v_t}{\partial p}\frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_t}{\partial C^\kappa}\frac{\partial C^\kappa}{\partial t} \qquad \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p}\frac{\partial p}{\partial t}$$

- Reordering:

$$\left(\frac{\partial v_t}{\partial p} - \frac{\partial \phi}{\partial p}\right)\frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_t}{\partial C^\kappa}\frac{\partial C^\kappa}{\partial t} = \frac{\phi - v_t}{\Delta t}$$

$$c_t \frac{\partial p}{\partial t} + \sum_\kappa \frac{\partial v_t}{\partial C^\kappa} \sum_\alpha \nabla \cdot \left(\mathbf{v}_\alpha \varrho_\alpha X_\alpha^\kappa\right) = \sum_\kappa \frac{\partial v_t}{\partial C^\kappa} q^\kappa + \varepsilon$$

Pressure Equation with Mpfa

Adaptive Grid

$$
\begin{aligned}
& V_i c_{total} \frac{p_i^t - p_i^{t-\Delta t}}{\Delta t} \\
& - \sum_{\gamma_{ij,irregular}} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\partial v_{total}}{\partial C^\kappa} X_\alpha^\kappa \left(\left(t_{2i} p_{\alpha,i}^t + \sum_j t_{2j} p_{\alpha,j}^t \right) + \varrho_\alpha \mathbf{g} \left(t_{2i} z_i + \sum_j t_{2j} z_j \right) \right) \\
& - \sum_{\gamma_{ij,regular}} A_{\gamma_{ij}} \mathbf{n}_{\gamma_{ij}} \cdot \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\partial v_{total}}{\partial C^\kappa} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right) \\
& + V_i \sum_{\gamma_{ij,irregular}} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} X_\alpha^\kappa \left(\left(t_{2i} p_{\alpha,i}^t + \sum_j t_{2j} p_{\alpha,j}^t \right) + \varrho_\alpha \mathbf{g} \left(t_{2i} z_i + \sum_j t_{2j} z_j \right) \right) \\
& + V_i \sum_{\gamma_{ij,regular}} \frac{A_{\gamma_{ij}}}{U_i} \mathbf{K} \sum_{\alpha} \varrho_\alpha \lambda_\alpha \sum_{\kappa} \frac{\frac{\partial v_{t,j}}{\partial C_j^\kappa} - \frac{\partial v_{t,i}}{\partial C_i^\kappa}}{\Delta x} X_\alpha^\kappa \left(\frac{p_{\alpha,j}^t - p_{\alpha,i}^t}{\Delta x} + \varrho_\alpha \mathbf{g} \frac{z_j - z_i}{\Delta x} \right) \\
& = V_i \sum_{\kappa} \frac{\partial v_t}{\partial C^\kappa} q_i^\kappa + V_i \alpha_r \frac{v_t - \phi}{\Delta t} \ . \qquad (1)
\end{aligned}
$$